

# Music Similarity: Improvements of Edit-Based Algorithms by Considering Music Theory

Matthias Robine, Pierre Hanna and Pascal Ferraro  
LaBRI - University of Bordeaux 1  
351 cours de la Libération  
33405 Talence cedex, France  
firstname.name@labri.fr

## ABSTRACT

Estimating the symbolic music similarity is one of the major open problems in the music information retrieval research domain. Existing systems consider sequences of notes characterized by pitches and durations. Similarity estimation is mainly based on variations of pitches and durations and does not consider any other musical elements. However, musical elements such as tonality or rhythm are particularly important in the perception of music. In this paper we propose to investigate some algorithmic improvements that allow edit-based systems to take into account important musical elements: tonality, passing notes, strong and weak beats. These elements are illustrated with a few monophonic musical examples which lead to important errors in usual systems. First experiments with these examples show that the improvements induced are significant. Furthermore, experimental results obtained with the MIREX 2005 database are very good. All the results are thus very promising since they confirm that considering musical information improves the accuracy of music retrieval systems.

## Categories and Subject Descriptors

H.3.3 [Information storage and retrieval]: Information Search and Retrieval

## General Terms

Algorithms

## Keywords

Music similarity, Edit-based systems, Information retrieval, Music theory

## 1. INTRODUCTION

Music information retrieval research area involves new methods for classification, indexation and retrieval of musical audio signals. One of the main open problems of this

area is the estimation of music similarity between symbolic musical pieces. Measuring similarity between sequences is a well-known problem in computer science which has applications in many fields [10, 6]. However, musical sequences are characterized by specific properties. That is the reason why developing efficient and accurate algorithms implies to take into account sound perception and music analysis. In this paper, we propose an investigation of algorithmic improvements based on musical analysis. In section 1, we introduce edit-based systems on which the studies described are based. In section 2, we present and describe the musical properties we propose to take into account. We finally show in section 3 the results of experiments performed on both examples and collections.

### 1.1 Edit-based Systems

Several techniques for evaluating symbolic music similarity have been introduced during the last few years. Geometric algorithms consider geometric representations of melodies and compute the distance between objects. Some systems [16] are closely linked to the well-known piano-roll representation. Other geometric systems represent notes by weighted points [14].

Another algorithm adapted from string matching domain has been proposed in [2]. N-grams techniques involve counting the distinct terms that the query and a potential answer have in common. It is important to note that this algorithm applied for the computation of the similarity measure (counting the matching subsequences) does not take into account the musical properties of the music.

In the early seventies, [8] and then [17] proposed algorithms which compute a similarity measure between two strings of symbols as the minimum score sequence of elementary operations needed to transform one of the strings into the other. Given two strings of symbols  $S_1$  and  $S_2$  of respective lengths  $|S_1|$  and  $|S_2|$  (for example the number of notes in melodies), a set of elementary operators on strings, called edit operations, and a score associated to each edit operation, a score between these two strings is defined as the score of the sequence of edit operations that transforms  $S_1$  into  $S_2$  with minimum score. This similarity measure makes use of the dynamic programming principle to achieve an algorithm with quadratic complexity, *i.e.* in  $O(|S_1| \times |S_2|)$ .

This approach has been applied in the monophonic musical context by Mongeau and Sankoff [7] and a few improvements have been recently proposed [4]. Edit-based systems are very flexible: they permit to take into account musical

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

MIR'07, September 28–29, 2007, Augsburg, Bavaria, Germany.  
Copyright 2007 ACM 978-1-59593-778-0/07/0009 ...\$5.00.

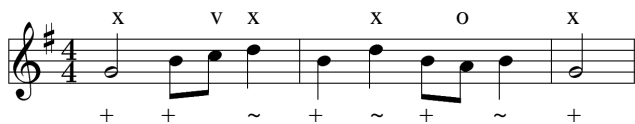


Figure 1: Analysis of a musical piece allows to identify the different functions of the notes and their placement inside the bar. Above the notes, “x” tags the importance of the note regarding the tonality limited to the tonic and the dominant tones (respectively G and D for a G Major tonality here). “v” is used to identify the passing note and “o” for a note on the weak part of the beat (which is not a passing note). Under the staff, “+” stands for the strong beats and “~” for the weak ones.

properties by adapting the computation of the scores of edit operations. That is why all the studies presented here are based on these edit-based systems.

## 1.2 Musical Information

Even if edit-based systems are very precise in the monophonic context, several other improvements have to be investigated. One of the major proposals is to take into account musical information to adapt these algorithms to music context for being more efficient in this case. It is also the aim of [5] for example which proposes to use cognitive structures based on the Implication-Realization model of Eugene Narmour. We prefer here to use musical analysis to propose simple rules that can improve edit-based systems with musical theory elements, by considering the tonality or the harmonic function of the notes in a musical piece for example. These notions are linked to the traditional western music and focus our studies and results on the popular and traditional western music.

Thus, we think that a preliminary music analysis may highlight the properties that help listeners to perceptually discriminate two musical patterns, because it allows to identify the important notes in the melody regarding the tonality for example. This analysis may therefore lead to the modification of edit operations specific to music. The notes located on the stronger beats in a bar could therefore be considered as more important than the other ones and can be weighted more than the notes placed on the weak beats in an edit-based algorithm

## 2. MUSICAL ELEMENTS

We expose in this section some elements of music theory. These notions explain why some notes can be considered as more important than others in a musical piece. It allows us to propose some rules for melodic similarity which are intuitive to humans since they are sensible to western music codes. Indeed, we consider that few persons are familiar with harmonic rules of western music but notions like tonality or pulse are perceptively pertinent for the most of them, with the permanent influence of pop music, ring tones or music in video games.

As we want to improve an existent edit-based algorithm, our method consists in implementing first some simple musical notions with an evaluation of the results. A more com-

plex and entire approach of the tonality will then be study later and can be motivated by the works exposed in [12] or [1]. Figure 1 is a musical piece useful to highlight the different elements of music theory we introduce in the following.

### 2.1 Tonality

One of the most important characteristics of the traditional western music is certainly the tonality. Several algorithms have already been proposed to estimate the tonality of a musical piece, for example [9, 1]. It is a system of music in which certain hierarchical pitch relationships are based on a note called tonic. The tonic is the pitch upon which all the other pitches of a piece are hierarchically centered. The scale associated to a tonality begins by the tonic. Each note of the scale is denoted by a degree and the the fifth degree is called dominant. The triad formed on the tonic note, the tonic chord, is the most important chord for the tonality, but only the tonic and the fifth does not change with the mode. In western tonal music, these two degrees are the most important. They are often used and their succession composes for example the perfect cadence that commonly ends a musical piece. In the G major or in the G minor one, tonic is the note G and dominant is the note D, like in the example of the Figure 1.

We propose some music examples to demonstrate the interest of this notion for similarity systems based on *Day Tripper* from *The Beatles* (Figure 2). Even if we have removed and changed a lot of notes in the piece 2(b), we have preserved the tonic and dominant notes in the variation. The resulting sound is thus close to the original one. The reader may want to hear this sound and the followings on <http://dept-info.labri.fr/~hanna/MIR07/>.

To implement this tonality rule, a difference in semi-tones (modulo 12) of each note with the tonic is computed in our system. A note is assumed to be important and is therefore marked when the difference equals 0 (it is a tonic note) or 7 (dominant), Computation of edit scores takes into account these marked notes by increasing the score when two marked notes match: a fixed value  $s_{key}$  is added. Thus, the musical sequence alignment encourages matches between these marked notes.

### 2.2 Passing Notes

The second rule we propose concerns the passing notes in a musical piece. A passing note is generally defined in a polyphonic context as a non-harmonic tone, a note which is not a part of the chord that is formed by the other notes sounding at the time. It is a note of a part starting at one chord tone and moved up or down by diatonic or chromatic intervals through one or more non-chord tones until resolving to another chord tone. This definition can be used with monophonic music if the tonality is known, where the passing note is not part of the chord of this tonality. There is one occurrence of a passing note in Figure 1. The musical example from *Day Tripper* presented by Figure 2(c) is perceptually close to the original piece even if it differs because of the insertion of passing notes.

The algorithm we propose detects the passing notes of musical pieces. As explained before, a passing note is a note between two others in a constant movement (ascending or descending) which is diatonic or chromatic. We decided



**Figure 2:** (a) Piece of *Day Tripper* (The Beatles) (b) Variation with conservation of the tonic and the dominant of the E Major tonality, respectively E and B (c) Variation with insertion of passing notes between the notes of the melody (d) Variation with conservation of the notes placed on the beats.

therefore to mark all the notes whose pitch interval with its neighbours is less or equals to two semi-tones, and whose the pitch variations with these neighbours are opposed, like with a positive variation with the precedent note and negative one with the following note. The edit scores are computed according to the information about the passing notes by changing the insertion/deletion score associated to marked notes. This edit score is less important than the usual substitution/insertion score. Thus, the insertion or the deletion of notes is less penalized by the similarity system.

### 2.3 Strong and Weak Beats

The bar is a segment of time in a musical piece defined as a given number of beats of a given duration. In function of their position in the bar, the beats can be strong or weak with parts that are also strong or weak. These differences are indicated in western music notation by the time signature. It means that the notes placed on strong beats or on the strong part of a beat are more important than the others. Most popular music is in 4/4 time, where the strong beats are the first and the third in the bar. In triple meter music there is only one strong beat, the first. The readers could refer to [11] for other types of time signature. Even they are placed on weak beats in the bar, we can distinguish that the notes are on the strong *part* of the beat, as opposite to the weak part. Two quavers beginning on a beat are therefore not perceptually equal and the first one is more important. An example of the different strengths is illustrated by Figure 1. As done for the other rules, we have built a musical piece perceptually close to the *Day Tripper* song by conserving almost all the notes that are on the beats (Figure 2(d)).

Our implementation assumes the knowledge of the time

signature of the compared musical pieces. This time signature is often indicated in MIDI files. We also assume the knowledge of the duration of a beat (also indicated in MIDI files). The algorithm proposes to mark the notes placed on the beats. A weight is associated to each of these notes, depending of the strength of the beat. As described just above, in 4/4 time, the strong beats are the first (a weight 4 is given), and the third (weight 2) of the bars. Other beats are weighted with 1, and the other notes, which are not on the beats, are not weighted. The algorithm takes into account these weighted notes by favouring matches between notes on strong beats, and by not penalizing insertion or deletion of notes on the weak part of the beat.

## 3. EXPERIMENTS

In this section, we propose some experiments in order to evaluate the improvements induced by the elements of music theory explained in the previous section. The edit-based system studied is the one presented in [4], and is very similar to the Mongeau and Sankoff algorithm [7].

### 3.1 MIREX 2005 Collection

First experiments concern the evaluation of the improved system. One of the main problem in the music information retrieval domain is the problem of the evaluation of the system proposed. The first Music Information Retrieval Evaluation eXchange (MIREX) [3] is a contest whose goal is to compare state-of-the-art algorithms and systems relevant for Music Information Retrieval. During this first contest, an evaluation topic about symbolic melodic similarity has been performed. Participants have discussed the process of

evaluation and proposed an evaluation procedure. The experiments presented in this section are based on these procedures.

The RISM A/II (International inventory of musical sources) collection is composed of one half-million notated real world compositions. The incipits are symbolically encoded music. They are monophonic and contain between 10 and 40 notes. During MIREX 2005, 11 incipits have been randomly chosen from the RISM collection. A ground truth has been established [13] by combining ranked lists that were created by 35 music experts. A tested system returns a ranked list of incipits estimated melodically similar to the query proposed. A few measures are then used to compute a score according to the corresponding ground truth. A specific measure has been proposed: the Average Dynamic Recall (ADR) [15]. It takes into account the ranked groups of the ground truth by indicating how many of the documents that should have appeared before or at a given position in the result list actually have appeared. The higher the ADR measure is, the more accurate the tested system is.

The improved algorithm has been tested with the MIREX 2005 database according to the ADR measure. These data are composed of 11 queries and 580 incipits in the database collection. Table 1 shows the results obtained.

Algorithm	Author	Average ADR
<b>Improved edit-based system</b>		<b>77</b>
Edit distance I/R	Grachten	66
N-grams	Orio	65
Simple N-grams	Uitdenbogerd	64
Geometric	Typke	57
Geometric	Lemström	56
Edit distance	Lemström	54
Hybrid	Frieler	52

**Table 1: Results of the evaluation of some retrieval systems during MIREX 2005 compared to the results obtained by the improved retrieval system.**

The improved edit-based system obtains very impressive results compared to the ones obtained by similar systems during MIREX 2005. For example, the average ADR is 77 whereas it is only 54 for similar system, and 66 for the system that takes into account cognitive structures. Moreover, the algorithm proposed is significantly the most precise. In the following, we investigate some musical examples to illustrate and explain this global increase of precision.

### 3.2 Musical Examples

In this section, the experiments proposed are based on the musical examples presented in the previous section. We also consider the RISM A/II collection, but the number of incipits constituting the collection is about 17000. All the musical examples described in this paper have also been symbolically encoded as MIDI files.

Similarity between musical pieces is estimated as a similarity score. It is important to note that these scores are not distances, and thus are not normalized. The higher the score is, the more similar are the musical pieces compared. The maximum score is the score obtained when comparing

a musical piece with itself. In the following, tables show the different results of the similarity estimation system with and without the algorithmic improvements by considering music theory elements. The similarity scores are presented and can be compared to the maximum score (denoted *max score*). Variations of *Day Tripper* melody (Figure 2) are considered as the query pieces. They are compared to each piece of the RISM collection and to the original melody. The rank of the corresponding piece, *i.e.* the original Beatles melody, is shown (denoted *RISM rank*, ideally 1). The maximum score obtained with a piece from the RISM collection is also indicated (denoted *RISM max*).

#### 3.2.1 Tonality

The first experiments estimate the improvements due to the tonality information. As previously explained in section 2, the musical piece 2(b) (Figure 2) sounds very similar to the original piece of The Beatles. However, since the sequence of notes is strongly different, the similarity estimated by existing edit-based systems is low. Table 2 shows the different results of the similarity estimation system with and without the algorithmic improvement based on tonality information. The piece 2(b) is considered as the query and table 2 shows the similarity score obtained when comparing this query to the original piece of The Beatles.

Algorithm	Original system	Improved system
Score	19.8	<b>21.1</b>
Max score	29.1	30.4
RISM max	21.2	21.2
RISM rank	3	<b>3</b>

**Table 2: Improvements of the similarity edit-based system when considering tonality information.**

Results show that the similarity score between the piece 2(b) and the original Beatles melody increases when the similarity system consider tonality properties. As expected, the modification of the score values for matching operation leads to the increase of score (21.1 instead of 19.8) if the number of important tonal notes (tonic or dominant) is important in both the query and the piece tested. The amount of this increase highly depends on the value of the coefficient  $s_{key}$  (see section 2). However, the results with the RISM collection is a bit disappointing. The Beatles original piece reaches only the third rank and the score 21.1 obtained is far from the maximum score 30.4 whereas the two pieces sound very similar. One possibility would be to increase the value of the coefficient  $s_{key}$  but experiments show that it degrades the general accuracy of the similarity system. Other musical information like harmony for example could be useful.

#### 3.2.2 Passing Notes

Other experiments have been performed in order to evaluate the improvement of the system when considering passing notes. As previously explained in section 2, the musical piece 2(c) (Figure 2) sounds very similar to the original piece of the Beatles. The piece 2(c) is composed of several passing notes which are not as perceptually important as the other notes. Table 3 shows the different results of the similarity estima-

tion system with and without the algorithmic improvement based on passing notes. The piece 2(c) is considered as the query.

Algorithm	Original system	Improved system
Score	18.1	<b>22.3</b>
Max score	29.1	29.1
RISM max	18.6	22.2
RISM rank	2	<b>1</b>

**Table 3: Improvements of the similarity edit-based system when considering passing notes.**

Here again, the results show that the score increases (22.3 instead of 18.1) if the insertion and the deletion of passing notes are less penalized. In the example of the piece 2(c), the score becomes higher than the score 22.2 obtained by the most similar piece of the RISM collection, which does not sound similar to the Beatles piece. The algorithmic improvement proposed, based on passing notes, thus permits to correct this potential error.

### 3.2.3 Strong and Weak Beats

The experiments presented in this section evaluate the improvement based on the analysis of the strong and weak beats. As previously explained in section 2, the musical piece 2(d) (Figure 2) sounds very similar to the original piece of The Beatles. The piece 2(d) is composed of the notes corresponding to the strong beats. These notes are perceptually important. Table 4 shows the different results of the similarity estimation system with and without the algorithmic improvement based on strong and weak beats. The piece 2(d) is considered as the query.

Algorithm	Original system	Improved system
Score	15.6	<b>20.7</b>
Max score	29.1	30.4
RISM max	15.8	18.1
RISM rank	2	<b>1</b>

**Table 4: Improvements of the similarity edit-based system when considering strong and weak beats.**

The results show that considering strong and weak beats permits to increase the similarity score obtained for the comparison of the piece 2(d) tested and the original Beatles melody (20.7 instead of 15.6). Furthermore, by considering strong and weak beats, the similarity scores obtained with the pieces of the RISM database (maximum 18.4) are lower than the score obtained with the Beatles melody (20.7). In the case of the piece 2(d), the algorithmic improvement results in correcting a potential error.

Concerning the strong and weak beats, we propose another musical example. One of the main applications of music similarity systems is the query-by-humming/singing systems. If users would have to sing or whistle a melody with fast notes, several notes would be missing. The example we choose to illustrate this limitation is the *Toccata and*

*Fugue in D minor* by J.S. Bach. Figure 3 shows a melody extracted from this musical piece and two pieces that are assumed to be queries. The first query 3(b) is very similar to what a lot of people would probably sing: several notes of the original piece are not present. The second query 3(c) has been created by deleting the same number of notes, but not the same. Although the number of missed notes is important, the first query 3(b) sounds very similar to the original piece 3(a), but the second one 3(c) sounds very different. Nevertheless, existing systems may consider the second one as more similar to the original piece.

Piece (b)		
Algorithm	Original system	Improved system
Score	13.3	<b>27.7</b>
Max score	30.9	34.3
RISM max	21.7	23.4
RISM rank	> 20	<b>1</b>

Piece (c)		
Algorithm	Original system	Improved system
Score	<b>19.2</b>	23.4
Max score	30.9	34.3
RISM max	21.8	26.1
RISM rank	14	5

**Table 5: Improvements of the similarity edit-based system when considering strong and weak beats.**

Table 5 shows the different results of the similarity estimation system with and without the algorithmic improvement based on strong and weak beats. The pieces 3(b) and 3(c) of the figure 3 are considered as the queries. Without improvement, the similarity system estimates that the query 3(c) is more similar than the query 3(b) to the original Bach melody (19.2 instead of 13.3), contrary to the listening tests. But the results clearly show that the improvement proposed permits to estimate the piece 3(b) as more similar to the original Bach piece. The score obtained is 27.7 instead of 23.4 for the piece 3(c) which sounds less similar. Furthermore, the improvement allows the original Bach piece to reach the first rank in the RISM collection (the most similar piece of the RISM collection is only 23.4).

### 3.2.4 General Improvement

In the previous experiments, algorithmic improvements have been successively tested. In this section, we propose to consider the similarity system with all the improvements previously described: tonality, passing notes, strong and weak beats. The computation of the scores associated to edit operations takes into account all the information described in section 2. For now the combination of these modifications for the calculation of scores is rather naive. The bonus scores for important notes are added (strong beats, tonal notes) whereas the lack of penalization is considered in the case of passing notes or in the case of notes on the weak part of beats. In the future, this combination scheme will certainly have to be investigated in order to be improved.

Table 6 presents different results obtained by the similar-



Figure 3: (a) Piece of the *Toccata and Fugue in D minor* (Bach) (b) Here is a variation where a lot of notes have been removed, except the notes which are on the beats. (c) On the opposite, this variation is built by removing several notes that were on the beats. The number of the notes is nearly the same in (b) and (c) but the results are perceptually very different: only the variation (b) sounds similar to the original version.

Without improvement			
Query	Score	Rank correct piece	RISM top rank
2(b)	19.8	3	21.2
2(c)	18.1	2	18.6
2(d)	15.6	2	15.8
3(b)	13.3	> 20	21.7
3(c)	<b>19.2</b>	14	21.8

With improvement			
Query	Score	Rank correct piece	RISM top rank
2(b)	<b>22.3</b>	1	21.7
2(c)	<b>25.1</b>	1	22.7
2(d)	<b>18.1</b>	1	22.0
3(b)	<b>28.3</b>	1	23.5
3(c)	24.5	3	26.1

Table 6: Evaluation of the improvements of the similarity edit-based system.

ity estimation system with and without all the algorithmic improvements. Results clearly shows that the improvements increase the accuracy of the similarity system in the case of the musical examples presented in this paper. For each query, the algorithm allows now to retrieve the correct corresponding musical piece (Beatles or Bach piece) from the RISM collection whereas it is not always the case without the improvements. For example, without improvement, the most similar variation 3(b) of Bach melody obtains a lower score than the other variation 3(c) (13.3 instead of 19.2). But with improvements, the score associated to the variation 3(b) reaches the highest value (28.3).

## 4. CONCLUSION

In this article we indicate how musical information can improve musical similarity edit-based systems. As a first approach, we have introduced new rules which consider basic

music theory elements to match in a better way two musical pieces which are perceptually close but possibly very different regarding their number of notes or their rhythm. We have implemented three new rules about the tonality, the passing notes and the position of the notes in the bars of a musical piece. If the results of the tonality rule is not so impressive for the moment, other rules improve clearly the existing systems and allow us to better match similar musical pieces. After these first good results, we plan now to evaluate the new rules on a great number of music documents.

In the future, we aim at studying if the analysis of the harmony of a musical piece – especially the notes composing the chords for each bar of the piece – could be more efficient than using the tonality of the whole piece. The idea is to highlight the root note and the fifth of these chords instead of the tonic and the dominant of the tonality. One other perspective is to propose some new rules. A note surrounded by a big interval ascending before and a big one descending after, like note C in the sequence G C G is generally an important note, and it is not a non-chord tone. Figure 4 illustrates this notion.

Furthermore, we think that some new experiments about the rules proposed are necessary and require a new monophonic database with various pieces of traditional or popular western music. We are convinced that our new approach of music similarity will help us to deal successfully with polyphonic music. Finally we imagine also to match polyphonic music documents at least on three different levels : one for comparing the melodic sequences, one for the chord sequences and the third for the key sequences which would take modulations into account. The chord or key sequences could be obtain with the *Spiral Array* model for tonality [1] for example. Let us imagine what could be the main interest of using this hierarchy of levels for detecting similarities: only the pieces of a database that would be similar on higher level would be compared at the melodic level. It would give also a fair way to deal easily with polyphonic sounds reduced to a monophonic sequence of chords.

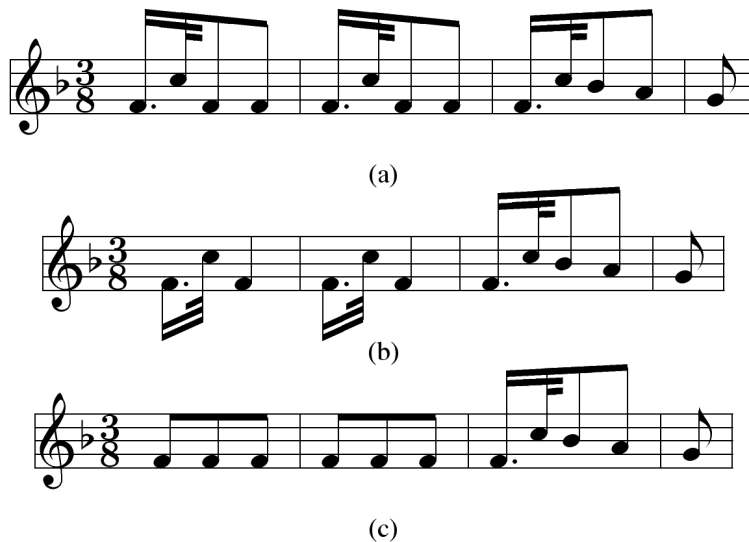


Figure 4: (a) Piece of *L'automne* from *The Four Seasons* (Vivaldi). Variation with conservation of the note C rather than the last note F in the bar in (b) and the inverse in (c). We removed the same number of notes in the variations (b) and (c) but (b) is much more similar to the original piece (a). We project to add a rule for this case: a note surrounded by a big ascending interval before and a big descending one after (or the inverse) is musically and perceptually more important than others.

## 5. REFERENCES

- [1] E. Chew. *Towards a Mathematical Model of Tonality*. PhD thesis, MIT Cambridge, MA, 2000.
- [2] S. Doraisamy and S. Ruger. Robust Polyphonic Music Retrieval with N-grams. *Journal of Intelligent Information Systems*, 21(1):53–70, 2003.
- [3] J. S. Downie, K. West, A. F. Ehmann, and E. Vincent. The 2005 Music Information retrieval Evaluation Exchange (MIREX 2005): Preliminary Overview. In *ISMIR*, pages 320–323, 2005.
- [4] P. Ferraro and P. Hanna. Optimizations of Local Edition for Evaluating Similarity Between Monophonic Musical Sequences. In *Proceedings of the International Conference on Large-Scale Semantic Access to Content (RIA0)*, Pittsburgh, PA, USA, 2007.
- [5] M. Grachten, J. L. Arcos, and R. L. de Mantaras. Melodic Similarity: Looking for a Good Abstraction Level. In *Proceedings of the International Conference on Music Information Retrieval (ISMIR)*, 2004.
- [6] D. Gusfield. *Algorithms on Strings, Trees and Sequences - Computer Science and Computational Biology*. Cambridge University Press, 1997.
- [7] M. Mongeau and D. Sankoff. Comparison of Musical Sequences. *Computers and the Humanities*, 24(3):161–175, 1990.
- [8] S. Needleman and C. Wunsch. A General Method Applicable to the Search for Similarities in the Amino Acid Sequences of Two Proteins. *Journal of Molecular Biology*, 48:443–453, 1970.
- [9] D. Rizo, J. Iesta, P. Ponce de Leon, and A. Pertusa. Tree Model of Symbolic Music for Tonality Guessing. In *Proceedings of the IASTED International Conference on Artificial Intelligence and Applications*, pages 299–304, 2006.
- [10] D. Sankoff and J. B. Kruskal, editors. *Time Wraps, Strings Edits, and Macromolecules: the Theory and Practice of Sequence Comparison*. Addison-Wesley Publishing Company Inc, University of Montreal, Quebec, Canada, 1983.
- [11] M. Simon. *Cours complet theorique et pratique des principes de la musique*. ed. Jacquot, 1947. in french.
- [12] D. Temperley. A bayesian approach to key-finding. In *Proceedings of the International Conference on Music and Artificial Intelligence (ICMAI)*, pages 195–206, London, UK, 2002. Springer-Verlag.
- [13] R. Typke, M. den Hoed, J. de Nooijer, F. Wiering, and R. C. Veltkamp. A Ground Truth For Half A Million Musical Incipits. *Journal of Digital Information Management*, 3(1):34–39, 2005.
- [14] R. Typke, R. C. Veltkamp, and F. Wiering. Searching Notated Polyphonic Music Using Transportation Distances. In *Proceedings of the ACM Multimedia Conference*, pages 128–135, New-York, USA, 2004.
- [15] R. Typke, R. C. Veltkamp, and F. Wiering. A Measure for Evaluating Retrieval Techniques Based on Partially Ordered Ground Truth Lists. In *Proceedings of the International Conference on Multimedia and Expo (ICME)*, pages 128–135, Toronto, Canada, 2006.
- [16] E. Ukkonen, K. Lemstrom, and V. Makinen. Geometric Algorithms for Transposition Invariant Content-Based Music Retrieval. In *Proceedings of the International Conference on Music Information Retrieval (ISMIR)*, pages 193–199, Baltimore, USA, 2003.
- [17] R. A. Wagner and M. J. Fisher. The String-to-String Correction Problem. *Journal of the association for computing machinery*, 21:168–173, 1974.