ON THE FINITE SUBGROUPS OF A CONTEXT-FREE GROUP

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Abstract. Let G be a group, X_G a finite set of generators of G and Γ the Cayley-graph of G on X_G. Let us suppose that Γ is κ-triangulable for some integer $\kappa \geq 0$. Using geometric considerations, we compute an upper bound on the cardinality of every finite subgroup of G. As a corollary we obtain an upper bound on the accessibility-length of G. Both bounds are exponential functions of κ.

1. Introduction

1.1. General presentation. The notion of *pushdown-automaton* is a classical notion from automata theory which extends the notion of *finite automaton*.The class of formal languages recognized by such automata is known as the class of *context-free languages* and coincides with the class of languages generated by *context-free* grammars ([Har78, HU79]).

A.V. Anisimov defined in [Ani71] the notion of *context-free group*:
a group G, given by some presentation $< X \cup \bar{X}; R >$ (where $X \cup \bar{X}$ is a finite paired alphabet and R is some set of relations over the free monoid $(X \cup \bar{X})^*$) is called *context-free* iff the word-problem for $< X \cup \bar{X}; R >$ can be solved by some p.d.a. $\mathcal{M}$, in the following sense:

$$\{w \in (X \cup \bar{X})^* \mid w \xleftrightarrow[R]{*} \epsilon\} = \mathrm{L}(\mathcal{M}).$$

D. Muller and P. Schupp showed [MS83] that the class of context-free groups coincides with the class of *virtually free* groups (see also [LS76, Smi76]). Their proof involves geometric considerations on the Cayley graph Γ of G over the set of generators X and proves the following equivalence.

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Theorem 1.1. The following properties are equivalent

1. G is a context-free group
2. there exists some integer κ such that Γ is κ-triangulable
3. G is the fundamental group of some finite graph of groups $\mathcal{G}$ with only finite vertex groups
4. G is virtually free.

The proof of (2) $\Longrightarrow$ (3) involves

- on one hand Stallings' decomposition theorem ([Sta68],[Coh72],[DD90]) which implies that, if G is c.f. and infinite, then it can be decomposed either as a non-trivial amalgamated product (with finite amalgamated subgroups) or as an HNN-extension (with finite associated subgroups)
- on the other hand Dunwoody's theorem ([Dun85]) which implies that, if G is c.f., then it can be decomposed only a *finite* number of times.

We have proved ([Sén93]) that each decomposition can be effectively performed in elementary time (namely five-fold exponential time).

1.2. New results. We prove here that the accessibility length of G (i.e. the maximum number of successive decompositions of G or, equivalently, the maximum number of edges of $\mathcal{G}$) is also elementary. In order to obtain such an upper bound on the accessibility-length, we first establish the two following upper bounds on finite subgroups of G.

Proposition 1.2. Let Γ be the Cayley graph on X of a group G and let us suppose that Γ is κ-triangulable. Then every finite subgroup F of G fulfills

$$\text{Card }(F) \leq (2\cdot \mid X \mid)^{12\cdot\kappa+10}.$$

Proposition 1.3. Let Γ be the Cayley graph on X of a group G and let us suppose that Γ is κ-triangulable. Then every finite subgroup F of G has a conjugate inside the ball $\mathrm{B}(1,\, 252\cdot\kappa+168)$.

By means of a theorem of Linnel ([Lin83]) we deduce from proposition 1.2 the following

Theorem 1.4. Let Γ be the Cayley graph on X of a group G and let us suppose that Γ is κ-triangulable. Then every minimal graph of groups $\mathcal{G}$ admitting G as fundamental group has a number of edges N_e such that

$$N_e \leq (2\cdot \mid X \mid)^{12\cdot\kappa+11}.$$

Let us recall that κ is at most exponential as a function of the size of a pushdown-automaton recognizing the word-problem of G on X.

Combining these results with the theorem of S.Krstic ([Krs89]) showing decidability of the isomorphism-problem for virtually free groups and the previous result of the author ([Sén93]) mentioned in section 1.1 we get the following theorem.

Theorem 1.5. The isomorphism-problem for virtually free groups G_1, G_2 defined by pushdown-automata $\mathcal{M}_1, \mathcal{M}_2$ is primitive recursive.

2. Preliminaries

We make precise here our notation and terminology. We also recall a lemma of [MS83] concerning κ-triangulations and cocycles.

2.1. Cayley graphs. Given a finitely generated group G and a finite set X_G of generators of G, the Cayley-graph of G over X_G is the following oriented, labeled graph $\Gamma = (V_\Gamma, E_\Gamma)$:
let X be an alphabet in one-to-one correspondence with X_G, $\overline{X}$ a disjoint alphabet in one-to-one correspondence with X_G^{-1}, such that for every $x_G \in X_G$, if $x \in X$ denotes x_G, $\overline{x} \in \overline{X}$ denotes x_G^{-1} ;

- V_Γ, the set of vertices, is equal to G
- $E_\Gamma = \{(g, x, g') \mid g, g' \in G, x \in X, g.x_G = g'\} \cup \{(g, \overline{x}, g') \mid g, g' \in G, x \in X, g.x_G^{-1} = g'\}$.

Given a subset $E \subseteq V_\Gamma$, we define the *frontier* ΔE of E by :

$$\Delta E = \{v \in E \mid \exists v' \in V_\Gamma - E, \exists x \in X \cup \overline{X}, (v', x, v) \in E_\Gamma\}.$$

The *cofrontier* $\vec{\partial} E$ of E is defined by :

$$\vec{\partial} E = \{(v', x, v) \in E_\Gamma \mid v' \in V_\Gamma - E, x \in X \cup \overline{X}, v \in E\}.$$

A *cocycle* $\vec{\partial}$ of G is any subset $\vec{\partial}$ of E_Γ such that there exists some subset E of V_Γ fulfilling $\vec{\partial} = \vec{\partial} E$. For every $\vec{\partial} \subseteq E_\Gamma$, by $\vec{\partial}^{-1}$ we denote the set

$$\vec{\partial}^{-1} = \{(v, \overline{x}, v') \mid (v', x, v) \in \vec{\partial}\}$$

(here we extend the notation $\overline{x}$ to every $x \in X \cup \overline{X}$ by setting $\overline{\overline{x}} = x$).

By $d(v, v')$ we denote the *distance* between two vertices v, v' in G : it is the minimal length of a path from v to v' in G.

We call *diameter of a cocycle* $\vec{\partial}$ the number

$$\begin{aligned} \operatorname{diam}(\vec{\partial}) \quad = \max \quad & \{d(v_1, v_2) \mid \exists x_1, x_2 \in X \cup \overline{X}, \exists v_1', v_2' \in V_\Gamma, (v_1, x_1, v_1') \in \vec{\partial} \cup \vec{\partial}^{-1} \\ & \text{and } (v_2, x_2, v_2') \in \vec{\partial} \cup \vec{\partial}^{-1}\} \end{aligned}$$

By $B(v, R)$ we denote the ball $\{v' \in V_\Gamma \mid d(v, v') < R\}$. Here we call an *end* of Γ any subset $E \subseteq V_\Gamma$ which is a connected component of $\Gamma - B(1, R)$ for some radius R (see footnote [1]).

We call *spherical* cocycle the cofrontier of any end in the above sense.

Γ is said κ*-triangulable* iff, (roughly speaking) every planar diagram D representing some closed path in Γ, admits a triangulation such that:

- the vertices of every triangle belong to D
- the sides of every triangle represent a path in Γ whose length is $\leq \kappa$ (a completely precise definition can be found in [MS85, p.302]).

[1] This definition corresponds essentially to the notion of "end" used in [MS83] exept that we allow here *finite* connected components; the notion of end defined in [Coh72] is another (related) notion which is not used explicitly here.

A key argument of [MS85] is the following

Lemma 2.1 ([MS85]). If $\vec{\partial}$ is a spherical cocycle, then $\mathrm{diam}(\vec{\partial}) \leq 3 \cdot \kappa + 2$.

Proof. Let E be an end of Γ. It is proved in [MS85, proof of theorem 2.9 p. 64] that

$$\mathrm{diam}(\Delta E) \leq 3 \cdot \kappa.$$

As every vertex of an edge belonging to $\vec{\partial} E$ is at distance ≤ 1 of ΔE, we obtain

$$\mathrm{diam}(\vec{\partial} E) \leq 3 \cdot \kappa + 2.$$

□

In the following we define the constant κ' by

$$\kappa' = \max \{\mathrm{diam}(\vec{\partial}) \mid \vec{\partial} \text{ is a spherical cocycle of } \Gamma\}$$

Lemma 2.1 can then be rephrased as:

$$\kappa' \leq 3.\kappa + 2. \tag{1}$$

2.2. Presentations. Given a finitely generated group G, a finite set X_G of generators of G, and an associated paired alphabet $X \cup \overline{X}$ as defined in section 2.1, we get an associated *presentation* of G over the alphabet $X \cup \overline{X}$:
this is just the surjective (monoid-) homomorphism φ: $(X \cup \overline{X})^* \longrightarrow G$ defined by

$$\forall x \in X, \varphi(x) = x_G$$

$$\forall x \in X, \varphi(\overline{x}) = x_G^{-1}.$$

From now on we assume some fixed presentation is given.

2.2.1. *Length.*

Given any word $w \in (X \cup \overline{X})^*$, we denote its length by $\mid w \mid$. Given any $g \in G$, by $\ell(g)$ we denote

$$\ell(g) = \min \{\mid w \mid \mid \varphi(w) = g\},$$

we call $\ell(g)$ the *length* of g. A word $w \in (X \cup \overline{X})^*$ is said *minimal* iff $\mid w \mid = \ell(\varphi(w))$ or, equivalently, iff

$$\mid w \mid = \min \{\mid w' \mid \mid \varphi(w') = \varphi(w)\}.$$

By $\mathrm{Min}((X \cup \bar{X})^*)$ we denote the set of all minimal words of $(X \cup \bar{X})^*$.
Given some subset $S \subseteq G$, the *norm* of S, denoted by $\|S\|$ is defined by

$$\|S\| = \sum_{g \in S} \ell(g).$$

For every $x, g \in G$ by x^g we denote the effect of the right action of g on x by conjugation:

$$x^g = g^{-1} x g.$$

This notation is extended to subsets of G in the obvious way:

$$S^g = \{s^g \mid s \in S\}.$$

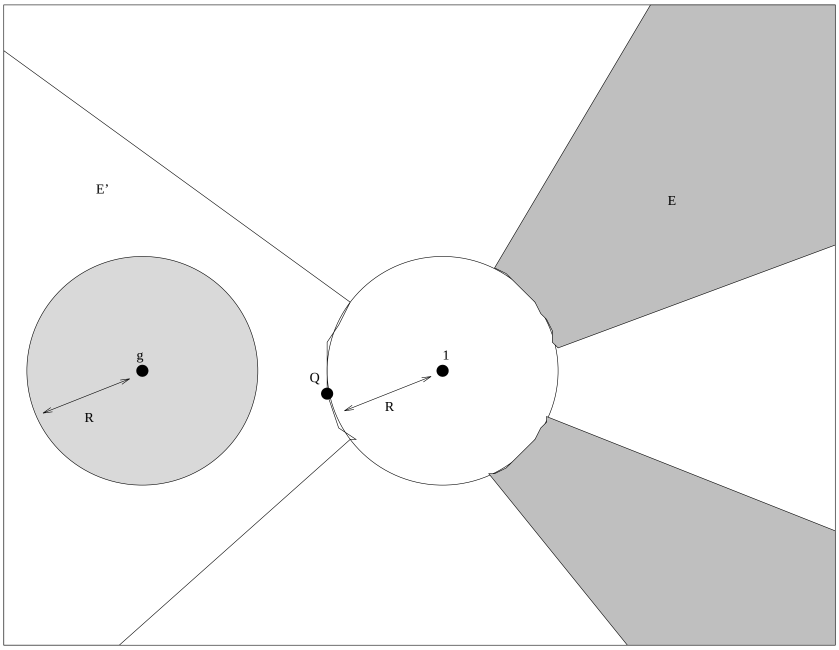

FIGURE 1

The subset S is said *cyclically reduced* iff

$$\forall g \in G, \| S \| \leq \|S^g\|.$$

2.2.2. *Weight.*

Let us fix now some finite subgroup F of G. The *weight* $W(S)$ of a subset S of G is defined by

$$W(S) = \sharp(S \cap F)/\sharp(F)$$

(where $\sharp(T)$ denotes the cardinality of T, for any subset T of G).

3. Cardinality of finite subgroups

We establish here proposition 1.2.
Let G be a group with set of generators X_G. Γ is the Cayley graph of G over X_G. We suppose that Γ is κ-triangulable for some $\kappa > 0$. We then consider some finite subgroup F of G (which is fixed throughout the section). Without loss of generality we can assume that F is cyclically reduced.

Let us consider the ball $B(1, R)$ where $R = n \cdot \kappa'$, for some integer $n \geq 2$.

Let E be some spherical end associated to $B(1, R)$ which has maximal weight (among all ends associated to the same ball).

Lemma 3.1. Let E' be an end associated to $B(1, R), E' \neq E$.
Then $E' \cap F \subseteq B(1, 2R)$.

Proof. Let us suppose there exists $g \in F \cap E'$ such that $d(g,1) \geq 2R$ (see figure 1). Then, one end E'' associated with $B(g,R)$, contains $B(1,R) \cup E$. As $B(1,R)$, $B(g,R)$ are isomorphic under left-multiplication by an element of F, $E'' = gE_1$ where E_1 is some end associated with $B(1,R)$. As

$$W(E_1) = W(E'') > W(E) \text{ while } E \text{ has maximal weight,}$$

this is impossible. □

Hence

$$F \subseteq B(1,2R) \cup E. \tag{2}$$

Let $\frac{1}{q} = W(B(1,2R))$. Let us show now that $q \leq \frac{2n-1}{n-1}$.

Let $h \in \Delta E$ and let u be a minimal word representing h. Let $F' = F \cap E - B(1,2R)$. By inclusion 2

$$F = F' \cup (B(1,2R) \cap F) \text{ (where the union is disjoint).} \tag{3}$$

Lemma 3.2. Let w be a minimal word representing an element of F' . There exist $u_1, v, u_1' \in Min((X \cup X^{-1})^*)$ such that

$$w = u_1 v u_1', \mid u_1 \mid = \mid u_1' \mid = R, \mid u^{-1} u_1 \mid \leq \kappa' \text{ and } \mid u_1' u \mid \leq \kappa'.$$

Proof. As w is minimal, so is w^{-1}. Let u_1 (resp. u_1') be the prefix (resp. suffix) of w of length R. As $\mid w \mid \geq 2R, w = u_1 v u_1'$, for some minimal word $v \in (X \cup \overline{X})^*$.

As $w \in E, u_1 \in \Delta E$. But $\mathrm{diam}(\Delta E) \leq \kappa'$. Hence $d(u, u_1) \leq \kappa'$, or, equivalently

$$\mid u^{-1} u_1 \mid \leq \kappa'.$$

As well, $w \in E^{-1} \Rightarrow \mid u^{-1} {u'_1}^{-1} \mid \leq \kappa'$, hence

$$\mid u_1' u \mid \leq \kappa'.$$

□

Lemma 3.3. For every integer $n \geq 2$, $\sharp(F) \leq \frac{2n-1}{n-1} \sharp(B(1, 2n\kappa') \cap F)$.

Proof. Let $R = n \cdot \kappa'$. By lemma 3.2

$$\| {F'}^u \| \leq \| F' \| - 2\sharp(F')(R - \kappa'). \tag{4}$$

It is also clear that, as $\mid u \mid \leq R$

$$\| (B(1,2R) \cap F)^u \| \leq \| B(1,2R) \cap F \| + 2\sharp(B(1,2R) \cap F) \cdot R. \tag{5}$$

Hence, by adding (4) and (5)

$$\frac{\| F^u \| - \| F \|}{\sharp(F)} \leq 2[(-1+\frac{1}{q})(R-\kappa')+\frac{1}{q}R] \tag{6}$$

$$= 2[(-1+\frac{1}{q})(n-1)\kappa' + \frac{n}{q}\kappa'] \tag{7}$$

$$= 2\kappa'[-(n-1)+\frac{2n-1}{q}]. \tag{8}$$

As F is cyclically reduced, the right-hand-side of (8) is non-negative hence :

$$q \leq \frac{2n-1}{n-1}. \tag{9}$$

□

Let us prove now proposition 1.2:
Let us choose $n=2$ in lemma 3.3. We obtain

$$\sharp(F) \leq 3 \cdot \sharp(B(1, 4\cdot\kappa') \cap F)$$

hence

$$\sharp(F) \leq 3 \cdot (2 \cdot \mid X \mid)^{4\kappa'}$$

and by lemma 2.1,

$$\sharp(F) \leq 3 \cdot (2 \cdot \mid X \mid)^{12\kappa+8} \leq (2 \cdot \mid X \mid)^{12\kappa+10}$$

which proves proposition 1.2.

4. Conjugacy classes of finite subgroups

We establish here proposition 1.3

Lemma 4.1. Let K be some subgroup of G such that $K = < S >$ where S is a subset of G fulfilling:

(1) $1 \in S, S = S^{-1}$
(2) $\sharp(K)/\sharp(S) \leq 3$.

Then $K = S^7$.

Proof. If $K = S^2$ then $K = S^7$. If $K \neq S^2$, then $S^3 \neq S^2$. Let $t \in S^3 - S^2$: $tS \cap S = \emptyset$. Hence $\sharp(K)/\sharp(S \cup tS) \leq \frac{3}{2}$.
If $K \neq S^5$ then $S^6 \neq S^5$. Let $u \in S^6 - S^5$: $uS \cap S = \emptyset$ and $uS \cap tS = \emptyset$.
Hence $\sharp(K)/\sharp(S \cup tS \cup uS) \leq 1$
$\Rightarrow K = S \cup tS \cup uS \Rightarrow K = S^7$. □

Lemma 4.2. Let F be a finite cyclically reduced subgroup of G. Then F has a subgroup K such that $K \subseteq B(1, 42\kappa')$ and $[F : K] \leq 2$.

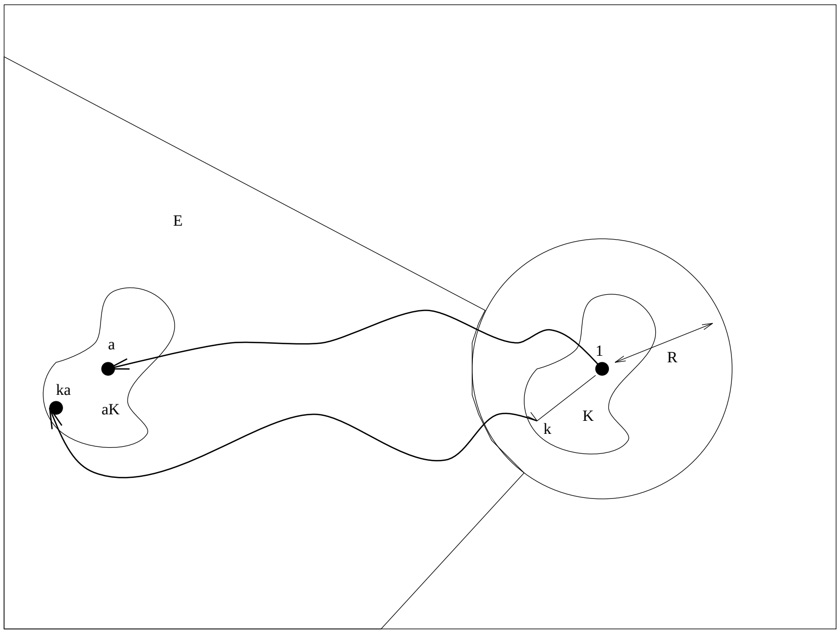

Figure 2

Proof. Let us choose $n = 3$ in lemma 3.3:

$$\sharp(F) \leq \frac{5}{2}\sharp(B(1, 6\kappa') \cap F). \tag{10}$$

Let us choose $K =< S >$ where $S = B(1, 6\kappa') \cap F$. By inequality (10) and lemma 4.1

$$K \subseteq S^7 \subseteq B(1, 42\kappa').$$

Moreover

$$[F : K] \leq \sharp(F)/\sharp(S) \leq \frac{5}{2} \text{ (by inequality (10))}$$

hence

$$[F : K] \leq 2.$$

□

Lemma 4.3. Let F be a finite subgroup of G and K a subgroup of F such that

(1) $K \subseteq B(1, R)$ (where $R > 2\kappa'$)
(2) $[F : K] = 2$.

Then F has a conjugate inside $B(1, 2R)$.

Proof. Let a be an element of minimal length in $F - K$ (we shall not distinguish between $a \in G$ and a fixed minimal word $a \in (X \cup \bar{X})^*$ representing a).

We show that if $| a | \geq 2R$, then there exists some $g \in G$ with $K^g \subseteq B(1, R)$ and $d(1, (F - K)^g) < d(1, F - K)$(see figure 2). The result then follows by an induction on $d(1, F - K)$.

Let us suppose $| a | \geq 2R$.

Hence aK is inside some end E associated with B$(1, R)$ (see figure 2).

Let us denote by a_i the prefix of a of length i.

Let $k \in K$, let i be such that $\ell(ka_i) = R$ and for $i' > i, \ell(ka_{i'}) > R$.

Then

$$ka_i, a_R \in \Delta E, \text{ hence } d(ka_i, a_R) \leq \kappa'. \tag{11}$$

Let $e \in G$ such that $ka_i e = a_R$.

By (11)

$$\ell(e) \leq \kappa' \text{ and } K(a_i e) = K(a_R). \tag{12}$$

Let s_i, s_R be the words such that $a = a_i s_i = a_R s_R$. By (12) $(a_i e)s_R$ and $(a_R e^{-1})s_i$ belong to $F - K$, hence by minimality of $\ell(a)$ we must have

$$| a_R | + | s_R | = \ell(a) \leq \ell(a_i e s_R) \leq | a_i | + \ell(e) + | s_R |$$

hence

$$|| a_R | - | a_i || \leq \ell(e)$$

and by (12)

$$| R - i | \leq \kappa' \tag{13}$$

$$\ell(a_R^{-1} k a_R) = \ell((a_R^{-1} k a_i)(a_i^{-1} a_R)) \leq \ell(a_R^{-1} k a_i) + \ell(a_i^{-1} a_R). \tag{14}$$

By (11) (13) (14) we have :

$$\ell(a_R^{-1} k a_R) \leq 2\kappa'$$

hence

$$K^{a_R} \subseteq B(1, R).$$

Let us prove now that $\ell(a^{a_R}) < \ell(a)$.

Let us consider the word $\tilde{\bar{a}}$ (which represents a^{-1}).

As $a^{-1} \in F - K$, and $\tilde{\bar{a}}$ is a minimal path from 1 to a^{-1} in Γ, $\tilde{\bar{a}}$ has a decomposition, $\tilde{\bar{a}} = a'_R s'$ where $| a'_R | = R$. $a^{-1} \in E$, hence $a'_R \in \Delta E$, so that

$$\ell({a'_R}^{-1} a_R) \leq \kappa'. \tag{15}$$

As $\ell(a) \geq 2R$, a can be written :

$$a = a_R v {a'_R}^{-1} \text{ where } v \in Min((X \cup \bar{X})^*).$$

$$\begin{aligned}
\ell(a^{a_R}) - \ell(a) &= \ell(v {a'_R}^{-1} a_R) - \ell(a_R v {a'_R}^{-1}) \\
&\leq \ell(v) + \ell({a'_R}^{-1} a_R) - \ell(a_R v {a'_R}^{-1}) \\
&= \ell({a'_R}^{-1} a_R) - 2R \\
&\leq \kappa' - 2R \qquad (\text{ by } (15)),
\end{aligned}$$

hence $\ell(a^{a_R}) < \ell(a)$ $\square$

Proposition 4.4. Let Γ be the Cayley-graph on X of a group G and let us suppose that Γ is κ-triangulable. Then every finite subgroup $F \leq G$ has some conjugate inside the ball $\mathrm{B}(1, 252 \cdot \kappa + 168)$.

Proof. Let F be a finite subgroup of G.

By Lemma 4.2, $\exists g \in G, \exists H \leq F, [F : H] \leq 2, H^g \subseteq B(1, 42\kappa')$.

By Lemma 4.3, $\exists g' \in G,\ \ F^{g'} \subseteq B(1, 84\kappa')$ and by lemma 2.1, $\mathrm{B}(1, 84\kappa') \subseteq \mathrm{B}(1, 252 \cdot \kappa + 168)$. □

5. Accessibility length

5.1. Accessibility. Let us prove theorem 1.4.

Proof. By Linnel's theorem ([Lin83, theorem 2 p.40])

$$2 \cdot \mid X \mid \geq 1 + \sum_{e \in \mathcal{E}} 1/\sharp(G_e)$$

(where $\mathcal{E}$ is the set of edges of $\mathcal{G}$ and, for each $e \in \mathcal{E}$, G_e is the associated subgroup). Hence

$$N_e \leq (2 \cdot \mid X \mid -1) \max \{\sharp(G_e) \mid e \in \mathcal{E}\}$$

and by proposition 1.2 one gets

$$N_e \leq (2 \cdot \mid X \mid)^{12\kappa + 11}.$$

□

5.2. Application. As an application, we deduce from the above theorem 1.4 a proof of theorem 1.5.

Sketch of proof. Let G_1, G_2 be two groups defined by two p.d.a.'s $\mathcal{M}_1, \mathcal{M}_2$ recognizing their word-problem. Let us use the function T , from pairs of non-negative integers to non-negative integers defined by:

$$\forall m \geq 0, \forall n \geq 0, T(0, n) = n \text{ and } T(m + 1, n) = 2^{T(m,n)}$$

($n \rightarrow T(m, n)$ is a "tower of m exponentials"). By the arguments given in [Sén93, sketch of proof of theorem 17], one can decide whether G_1 is isomorphic to G_2 in time at most $T(5 \cdot (2 \cdot \mid X \mid)^{12\kappa+11}, n)$ (where n is the sum of the sizes of the p.d.a.'s). As κ is itself an exponential function of n , the whole process is primitive recursive. □

5.3. Complements. Let us mention (for comparison with theorem 1.5) the following known (or unknown) decision/complexity results.

1 We do not know at the moment whether the above problem (the isomorphism-problem for c.f. groups) is *elementary*.
2 The *equivalence-problem* for real-time d.p.d.a.'s is known to be *decidable* ([Oya87, Rom85]) but is not known to be *primitive recursive* ([Oya80]).
3 The *isomorphism-problem* for p.d.a.'s (i.e. for their associated computation graphs) is known to be *elementary* ([Sén89, Cau90]).
4 Theorem 1.5 was already proved in the case where G_1, G_2 are defined by outer actions of finite groups on finitely generated free groups ([Sén93]).
5 The problem to know if a given p.d.a. $\mathcal{M}$ is indeed defining some group (i.e. recognizes a formal languages which happens to be the word problem of some group) is *undecidable* for general p.d.a's but becomes *decidable* for non-ambiguous p.d.a's [GS] (hence, a fortiori, for deterministic p.d.a.'s).

6 Given an arbitrary real-time deterministic p.d.a. $\mathcal{M}$, the *automorphism-group* $G = \mathrm{Aut}(\mathcal{M})$ is always a context-free group ([Pel95]). Theorem 1.5 applies as well to pairs of groups (G_1, G_2) defined as $(\mathrm{Aut}(\mathcal{M}_1), \mathrm{Aut}(\mathcal{M}_2))$ for real-time d.p.d.a's $(\mathcal{M}_1, \mathcal{M}_2)$.

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