

LOGICS

Sequent calculus- LK

1-Axioms

$$\frac{}{\perp \vdash \perp} \perp_\ell \quad \frac{}{A \vdash A} \mathbf{ax}$$

2-Structural rules

$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta} \mathbf{wkn}_\ell \quad \frac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta} \mathbf{wkn}_r$$

$$\frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta} \mathbf{contr}_\ell \quad \frac{\Gamma \vdash A, A, \Delta}{\Gamma \vdash A, \Delta} \mathbf{contr}_r$$

3-Connective rules

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \wedge_\ell \quad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta} \wedge_r$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta} \vee_\ell \quad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta} \vee_r$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \rightarrow B \vdash \Delta} \rightarrow_\ell \quad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \rightarrow B, \Delta} \rightarrow_r$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta} \neg_\ell \quad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta} \neg_r$$

4-Quantifier rules

$$\frac{\Gamma, A[x:=t] \vdash \Delta}{\Gamma, \forall x A \vdash \Delta} \forall_\ell \quad \frac{\Gamma \vdash A, \Delta}{\Gamma \vdash \forall x A, \Delta} \forall_r \text{ (if } x \notin \text{FV}(\Gamma, \Delta) \text{)}$$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma, \exists x A \vdash \Delta} \exists_\ell \text{ (if } x \notin \text{FV}(\Gamma, \Delta) \text{)} \quad \frac{\Gamma \vdash A[x:=t], \Delta}{\Gamma \vdash \exists x A, \Delta} \exists_r$$

5-Cut rule

$$\frac{\Gamma \vdash \Delta, A \quad A, \Gamma' \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'} \mathbf{cut}$$

Intuitionistic sequent calculus- LJ

1-Axioms

$$\frac{}{\perp \vdash \perp} \perp_g \quad \frac{}{A \vdash A} \text{ax}$$

2-Structural rules

$$\frac{\Gamma \vdash [C]}{\Gamma, A \vdash [C]} \text{wkn}_\ell \quad \frac{\Gamma \vdash \quad}{\Gamma \vdash A} \text{wkn}_r$$

$$\frac{\Gamma, A, A \vdash [C]}{\Gamma, A \vdash [C]} \text{contr}_\ell$$

3-Connective rules

$$\frac{\Gamma, A, B \vdash [C]}{\Gamma, A \wedge B \vdash [C]} \wedge_\ell \quad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge_r$$

$$\frac{\Gamma, A \vdash [C] \quad \Gamma, B \vdash [C]}{\Gamma, A \vee B \vdash [C]} \vee_\ell \quad \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee_r^1 \quad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee_r^2$$

$$\frac{\Gamma \vdash A, \quad \Gamma, B \vdash [C]}{\Gamma, A \rightarrow B \vdash [C]} \rightarrow_\ell \quad \frac{\Gamma, A \vdash \quad}{\Gamma \vdash A \rightarrow B} \rightarrow_r$$

$$\frac{\Gamma \vdash A}{\Gamma, \neg A \vdash \quad} \neg_\ell \quad \frac{\Gamma, A \vdash \quad}{\Gamma \vdash \neg A} \neg_r$$

4-Quantifier rules

$$\frac{\Gamma, A[x:=t] \vdash [C]}{\Gamma, \forall x A \vdash [C]} \forall_\ell \quad \frac{\Gamma \vdash A}{\Gamma \vdash \forall x A} \forall_r \text{ (if } x \notin \text{FV}(\Gamma))$$

$$\frac{\Gamma, A \vdash [C]}{\Gamma, \exists x A \vdash [C]} \exists_\ell \text{ (if } x \notin \text{FV}(\Gamma, [C])) \quad \frac{\Gamma \vdash A[x:=t]}{\Gamma \vdash \exists x A} \exists_r$$

5-Cut rule

$$\frac{\Gamma \vdash A \quad A, \Gamma' \vdash [C]}{\Gamma, \Gamma' \vdash [C]} \text{cut}$$