

LOGICS, J1IN7M12

Test on 17/10/2013

All documents allowed ; duration : 1h 30.

The four exercises are independant one of each other.

Exercise 1 (5 pts)

Let us recall the rule \forall_ℓ of LK :

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}$$

Is this rule *reversible*? i.e. does LK admit the two following derived rules? :

$$\frac{\Gamma, A \vee B \vdash \Delta}{\Gamma, A \vdash \Delta} \text{LK} \quad \frac{\Gamma, A \vee B \vdash \Delta}{\Gamma, B \vdash \Delta} \text{LK}$$

Exercise 2 (5 pts)

1- Give a proof, within system LK, of the sequents :

$$\neg(A \wedge (B \vee C)) \vdash (\neg A) \vee ((\neg B) \wedge \neg C)$$
$$\neg(A \wedge \exists x Q(x)) \vdash (\neg A) \vee \forall x \neg Q(x)$$

2- Give a proof, within system LJ, of the sequent :

$$(\forall x P(x)) \wedge (\exists y Q(y)) \vdash \exists y (P(y) \wedge Q(y))$$

Exercise 3 (5 pts)

Let us recall that MO is a set of axioms for the theory of monoids (see the special sheet about sets of axioms). Let us consider the sequent S :

$$\text{MO} \vdash \forall x \forall y \exists z x = y * z$$

- 1- Build a model for this sequent.
- 2- Build a counter-model for this sequent.
- 3- Is the sequent S provable within LK? within LJ?

Exercise 4 (5 pts)

Let us consider the following Kripke structure $\mathcal{K} := (K, \leq, \Vdash_0)$, over the propositional signature consisting of two predicate symbols A, B of arity 0 :

$K := \{0, 1, 2\}$, the partial ordering over K is defined by $0 \leq 1, 0 \leq 2$ and the initial forcing relation is $\Vdash_0 := \{(0, A), (1, A), (1, B), (2, A), (2, B)\}$.

1- Determine whether, in this structure :

$0 \Vdash A \vee \neg A$?

$0 \Vdash (\neg\neg B) \rightarrow B$?

2- Determine whether :

$\vdash_{\text{LJ}} A \vee \neg A$?

$\vdash_{\text{LJ}} (\neg\neg B) \rightarrow B$?