

## LOGICS

Some first-order theories

**EG** (Equality theory)

**REF** :  $\forall x \ x = x$

**SYM** :  $\forall x, y \ (x = y \rightarrow y = x)$

**TRANS** :  $\forall x, y, z \ (x = y \wedge y = z) \rightarrow x = z)$

**COMPF** :  $\forall \vec{x}, \vec{y} \ (\vec{x} = \vec{y} \rightarrow f(\vec{x}) = f(\vec{y}))$

**COMPR** :  $\forall \vec{x}, \vec{y} \ (\vec{x} = \vec{y} \rightarrow R(\vec{x}) \rightarrow R(\vec{y}))$

**P0** (Elementary arithmetics)

All the axioms from EG ;

**A1** :  $\forall x \ \neg S(x) = 0$

**A2** :  $\forall x \ (x = 0 \vee \exists y, x = S(y))$

**A3** :  $\forall x, y \ (S(x) = S(y) \rightarrow x = y)$

**A4** :  $\forall x \ (x + 0 = x)$

**A5** :  $\forall x, y \ (x + S(y) = S(x + y))$

**A6** :  $\forall x \ (x \times 0 = 0)$

**A7** :  $\forall x, y \ (x \times S(y) = x \times y + x)$

**PA** (Peano arithmetics)

All the axioms from P<sub>0</sub> ;

**REC<sub>Φ</sub>** :  $(\Phi(0) \wedge (\forall x(\Phi(x) \rightarrow \Phi(S(x)))) \rightarrow \forall x\Phi(x)$

for every formula  $\Phi(x)$  ;

**MO** (Monoid theory)

All the axioms from EG ;

**ASS** :  $\forall x, y, z \ x * (y * z) = (x * y) * z$

**NE** :  $\forall x \ (x * e = x \ \wedge \ e * x = x)$

**GR** (Group theory)

All the axioms from MO ;

**INV** :  $\forall x \ (x * I(x) = e \ \wedge \ I(x) * x = e)$