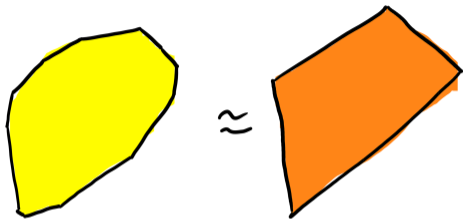


QUANTUM SPEEDUPS FOR LPs VIA IPMs



Simon Apers
(CNRS & IRIF, Paris)

with **Sander Gribling** (Tilburg University)

arXiv:2311.03215

ANR QuDATA workshop, Bordeaux, January '24

LPs and IPMs

Approximate Hessian

Approximate gradient

Quantum LP solver

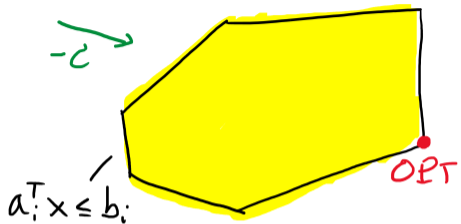
LPs and IPMs

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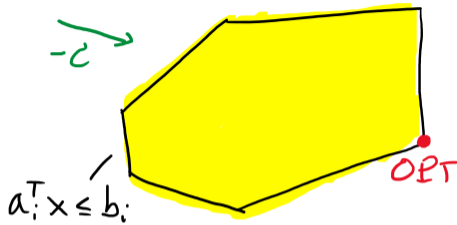
Approximate gradient

Quantum LP solver

Linear program (LP)

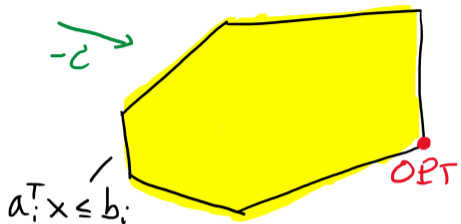


Linear program (LP)



$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & Ax \geq b \end{aligned}$$

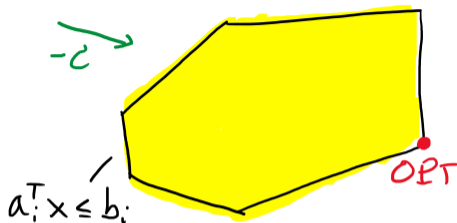
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d dimensions $\ll n$ constraints ($x \in \mathbb{R}^d, A \in \mathbb{R}^{n \times d}$)

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= (constrained) convex optimization

Interior point method (IPM)

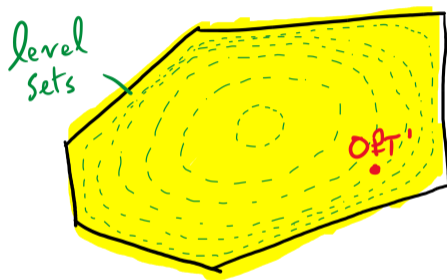
barrier f :

$$f(x) \rightarrow \infty \text{ when } a_i^T x \rightarrow b_i$$

Interior point method (IPM)

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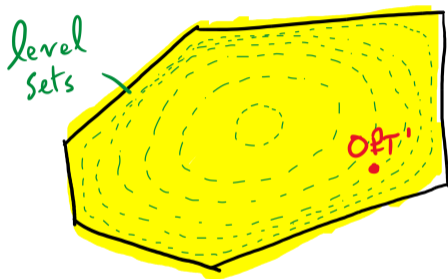
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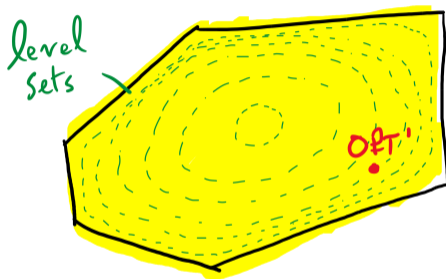


e.g., logarithmic barrier: $f(x) = -\sum_i \log(a_i^T x - b_i)$

Interior point method (IPM)

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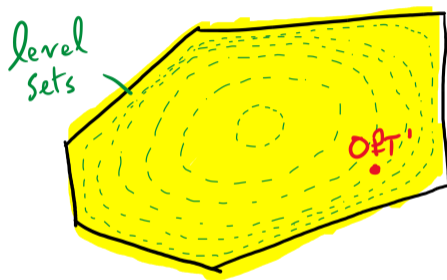
e.g., logarithmic barrier: $f(x) = -\sum_i \log(a_i^T x - b_i)$

$$\min_x f(x) + c^T x$$

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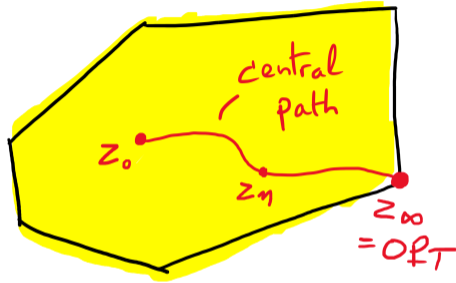


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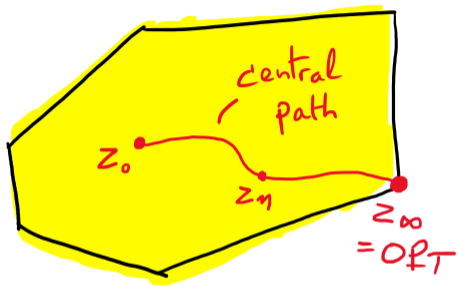
= *unconstrained* convex optimization

Interior point method (IPM)



$$f_\eta(x) = f(x) + \eta \cdot c^T x$$

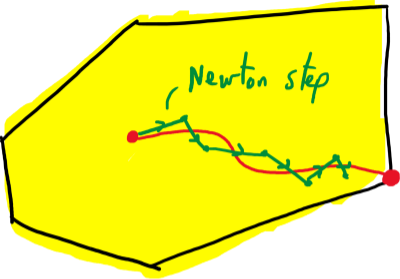
Interior point method (IPM)



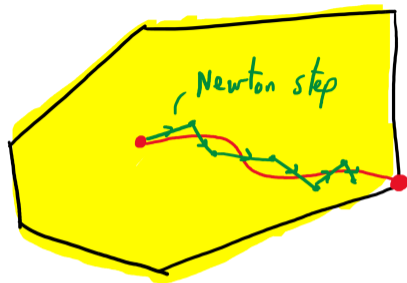
$$f_\eta(x) = f(x) + \eta \cdot c^T x$$

central path $\{z_\eta = \operatorname{argmin}_x f_\eta(x)\}_{\eta \geq 0}$

Path following



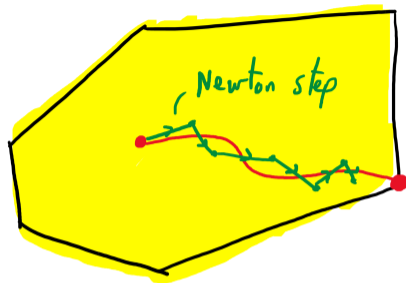
Path following



1. increase η :

$$\eta' = (1 + \gamma)\eta$$

Path following



1. increase η :

$$\eta' = (1 + \gamma)\eta$$

2. Newton step:

$$x' = x - H(x)^{-1}g(x)$$

(Hessian $H(x) = \nabla^2 f_\eta(x)$, gradient $g(x) = \nabla f_\eta(x)$)

Path following



e.g., logarithmic barrier:

$$f(x) = - \sum_i \log(\underbrace{a_i^T x - b_i}_{\text{slack } s_i})$$

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$$H = \begin{bmatrix} \frac{1}{s_i} a_i & B^T \end{bmatrix} B$$

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$$(B^T)_{\cdot i} = \frac{1}{s_i} a_i$$

Path following



$$H = \begin{bmatrix} \frac{1}{s_i} a_i & B^T \end{bmatrix} B$$

$$g = - \begin{bmatrix} B^T \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

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$$(B^T)_{\cdot i} = \frac{1}{s_i} a_i$$

gradient

$$g(x) = -B^T \vec{1}$$

Runtime IPM



number of steps

\sim number of increases η



number of steps

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logarithmic: \sqrt{n} [Renegar '88]

volumetric: \sqrt{nd} [Vaidya '89]

Lewis weight: \sqrt{d} [Lee-Sidford '15]



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single step

= computation (inverse) Hessian and gradient of barrier



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single step

= computation (inverse) Hessian and gradient of barrier

logarithmic: matrix inversion

volumetric: matrix inversion + leverage scores

Lewis weight: matrix inversion + Lewis weights

Solving Tall Dense Linear Programs in Nearly Linear Time

'20

Jan van den Brand
KTH Royal Institute of Technology, Sweden
janvdb@kth.se

Aaron Sidford
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runtime $nd + d^3$
(GOAT for $n \gg d$: linear in input size)

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IPM + clever use of dynamic data structures

Prior work (quantum)



quantum speedup
for **Newton step**
 $x' = x - H(x)^{-1}g(x)$

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A Quantum Interior Point Method for LPs and SDPs*

'18

IORANIS KERENIDIS and ANUPAM PRAKASH, CNRS, IRIF, Université Paris Diderot

quantum linear system solving + tomography

* and follow-up works: [Augustino-Nannicini-Terlaky-Zuluaga '21,'23], [Huang-Jiang-Song-Tao-Zhang '22], [Dalzell-Clader-Salton-Berta-Lin-Bader-Stamatopoulos-Schuetz-Brandão-Katzgraber-Zeng '22], ...

** non-IPM: multiplicative weights [Brandão-Svore '17], [van Apeldoorn-Gilyén-Gribling-de Wolf-Brandão-Kalev-Li-Lin-Svore-Wu '17], simplex method [Nannicini '19], ...

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! dependence on condition number $\kappa(H)$
($\rightarrow \infty$ as $x \rightarrow \text{OPT}$)

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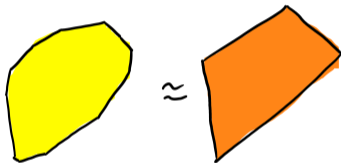
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This work



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This work



quantum speedup
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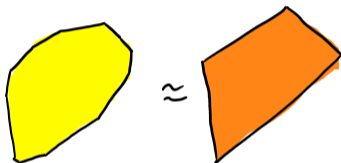
- (i) approximate Hessian $H' \approx H$
- (ii) approximate gradient $g' \approx g$
(using H' as preconditioner)

This work



quantum speedup
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runtime $\sqrt{n} \cdot \text{poly}(d) \cdot \log(1/\epsilon)$

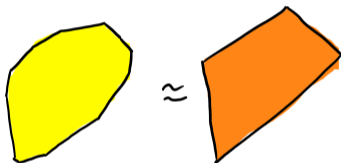
(no condition number, sublinear for $n \gg d$)

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GOAT: $\Omega(\sqrt{nd})$ row queries

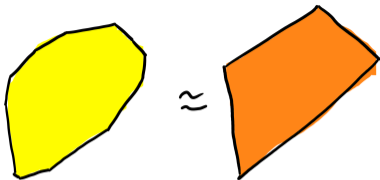
LPs and IPMs

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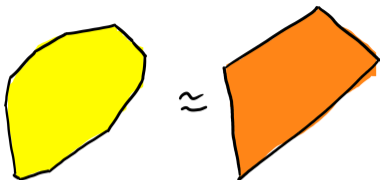
Approximate gradient

Quantum LP solver

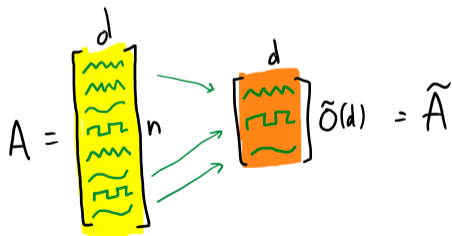
Spectral approximation



Spectral approximation



via constraint sampling



Spectral approximation

approximate Newton step

$$x' = x - \tilde{H}^{-1}g$$

needs *spectral approximation*

$$H = \begin{bmatrix} A^T \\ A \end{bmatrix} \approx^* \begin{bmatrix} \tilde{A}^T \\ \tilde{A} \end{bmatrix} = \tilde{H}$$

Spectral approximation

approximate Newton step

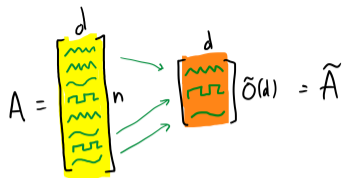
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$$* H \approx \tilde{H} \Leftrightarrow \forall y: y^T H y = (1 \pm 0.1) y^T \tilde{H} y \Leftrightarrow 0.9 \tilde{H} \preceq H \preceq 1.1 \tilde{H}$$

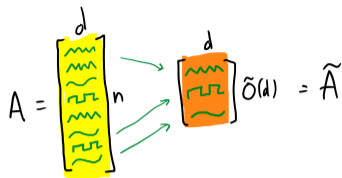
Spectral approximation



sampling via
“statistical leverage scores”

$$\sigma_i = a_i^T (A^T A)^{-1} a_i$$

Spectral approximation

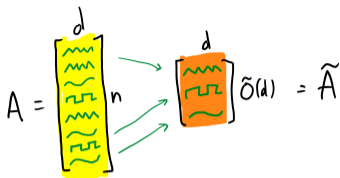


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! chicken-and-egg

Spectral approximation



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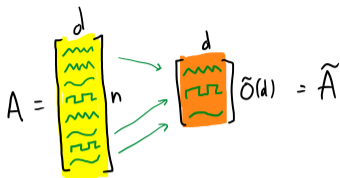
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use

uniform subsampling + bootstrapping scheme

[Cohen-Lee-Musco-Musco-Peng-Sidford '14]

Spectral approximation



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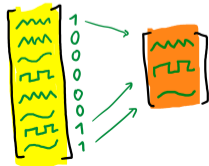
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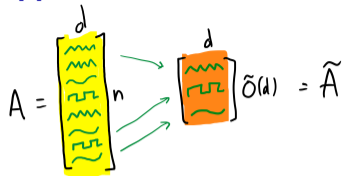
[Cohen-Lee-Musco-Musco-Peng-Sidford '14]

+

Grover search



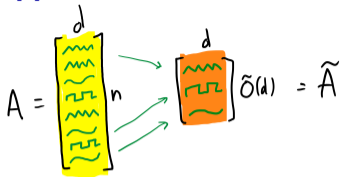
Spectral approximation



quantum algorithm:

- returns \tilde{A} with $\tilde{O}(d)$ rows,
 \tilde{A} spectral approximation of A
- makes $\tilde{O}(\sqrt{nd})$ row queries

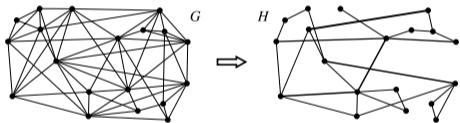
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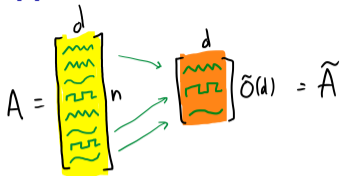
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– generalizes graph sparsification



[Apers-de Wolf '19]

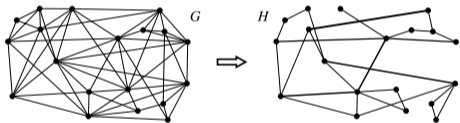
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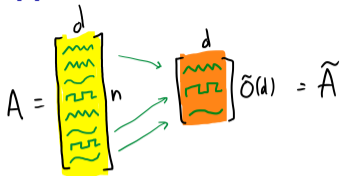
- applications in regression

[Submitted on 24 Nov 2023]

Revisiting Quantum Algorithms for Linear Regressions: Quadratic Speedups without Data-Dependent Parameters

Zhao Song, Junze Yin, Ruizhe Zhang

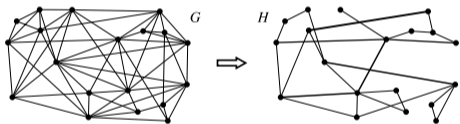
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Revisiting Quantum Algorithms for Linear Regressions: Quadratic Speedups without Data-Dependent Parameters

Zhao Song, Junze Yin, Ruizhe Zhang

- IPMs: need additional work
(e.g., Lee-Sidford barrier uses “Lewis weights”)

LPs and IPMs

Approximate Hessian

Approximate gradient

Quantum LP solver

Approximate gradient

typical gradient:

$$g = \begin{bmatrix} a_i \\ A^T \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = n \cdot \frac{1}{i} \begin{bmatrix} a_i \\ 1 \end{bmatrix}$$

Approximate gradient

typical gradient:

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→ quantum multivariate mean estimation:

[Cornelissen-Hamoudi-Jerbi '22]

Approximate gradient

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→ quantum multivariate mean estimation:

[Cornelissen-Hamoudi-Jerbi '22]

approximate $g = \mathbb{E}[X]$ with sample complexity

$$\sqrt{d \operatorname{Tr}(\Sigma_X)}$$

Preconditioning

$$\left. \begin{array}{l} \text{gradient } g = \mathbb{E}[X] = \mathbb{E}_i[n a_i] \\ \text{covariance matrix } \Sigma_X \preceq \mathbb{E}[XX^T] = nA^T A \end{array} \right\} \Rightarrow \sqrt{dn \text{Tr}(A^T A)} \text{ samples}$$

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! $\text{Tr}(A^T A)$ introduces **condition number** :(

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precondition with spectral approximation:

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s.t.

$$g = (\tilde{A}^T \tilde{A})^{1/2} \cdot \mathbb{E}[Y] \quad \text{and} \quad \Sigma_Y \preceq 1.1 n I_d$$

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$$\Rightarrow O(d\sqrt{n}) \text{ samples}$$

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Approximate Hessian

Approximate gradient

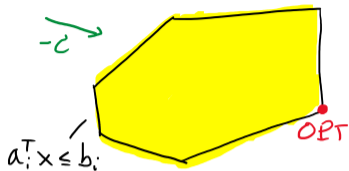
Quantum LP solver

Quantum LP solver

explicitly returns \tilde{x} satisfying

$$c^T \tilde{x} \leq c^T \text{OPT} + \varepsilon$$

$$\text{and } A\tilde{x} \leq b$$

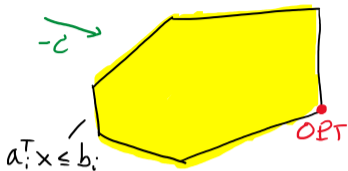
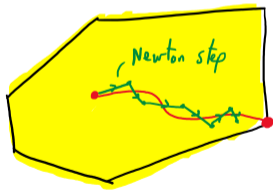


Quantum LP solver

explicitly returns \tilde{x} satisfying

$$c^T \tilde{x} \leq c^T \text{OPT} + \varepsilon$$

$$\text{and } A\tilde{x} \leq b$$



row queries*:

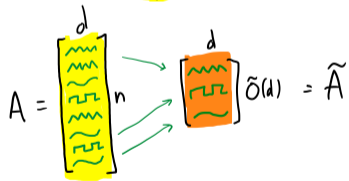
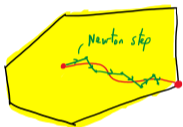
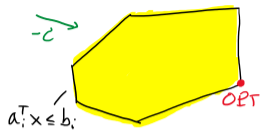
$$\begin{aligned} & (\# \text{ steps}) \times (\# \text{ cost Newton step}) \\ & = \sqrt{d} \log(1/\varepsilon) \times \sqrt{nd}^{2.5} \in \tilde{O}(\sqrt{nd}^3) \end{aligned}$$

time complexity: $\sqrt{n} \log(1/\varepsilon) \text{poly}(d)$

* using Lewis weight barrier.

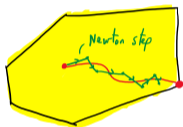
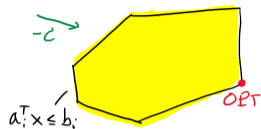
log-barrier: $\sqrt{n} \log(1/\varepsilon) \times \sqrt{nd}$, volumetric barrier: $(nd)^{1/4} \log(1/\varepsilon) \times \sqrt{nd}^2$

Summary and open questions



$$g = \begin{bmatrix} a_i \\ \vdots \\ A^T \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = n \cdot \mathbb{E}_i \begin{bmatrix} a_i \\ \vdots \\ 1 \end{bmatrix}$$

Summary and open questions

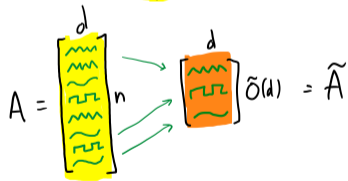
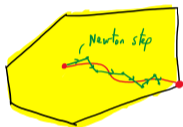
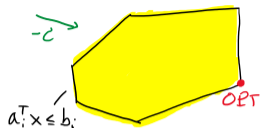


$$A = \begin{matrix} d \\ \text{[wavy lines]} \\ n \end{matrix} \rightarrow \begin{matrix} d \\ \text{[wavy lines]} \\ \end{matrix} \tilde{O}(d) = \tilde{A}$$

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– **quantum IPM for solving LPs**
(without condition numbers)

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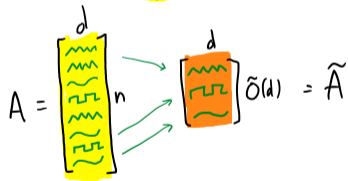
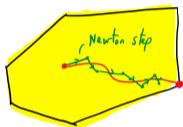
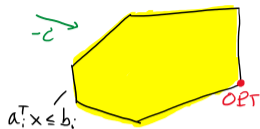
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(without condition numbers)

– **runtime** ($n \gg d$)

$$\sqrt{n} \cdot \text{poly}(d) \cdot \log(1/\varepsilon)$$

versus $n \cdot d \cdot \log(1/\varepsilon)$ classical

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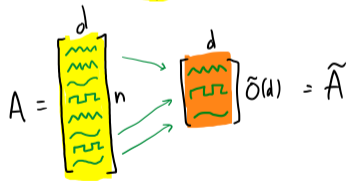
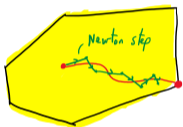
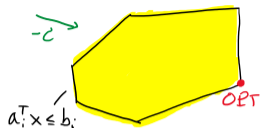
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– **new tools**

spectral approximation (Grover)

approximate matrix-vector (mean estimation)

Summary and open questions



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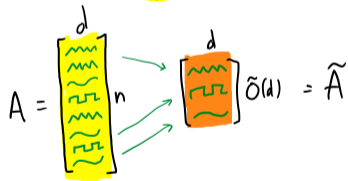
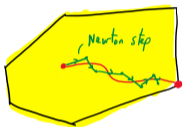
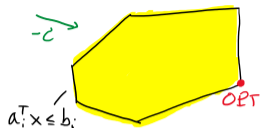
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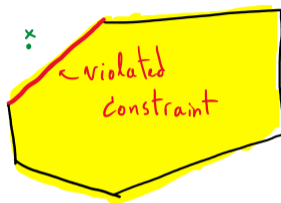
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– **thanks!**

Extra slide: cutting plane & lower bound

- *separation query*:
given x , return violated constraint (if any)



- *fastest cutting plane method* [Lee-Sidford-Wong '15]:
solve LP with d separation queries
 - *using Grover*:
answer separation query with \sqrt{n} row queries
solve LP with $d \cdot \sqrt{n}$ row queries
 - *quantum lower bound*:
 $\sqrt{d \cdot n}$ row queries