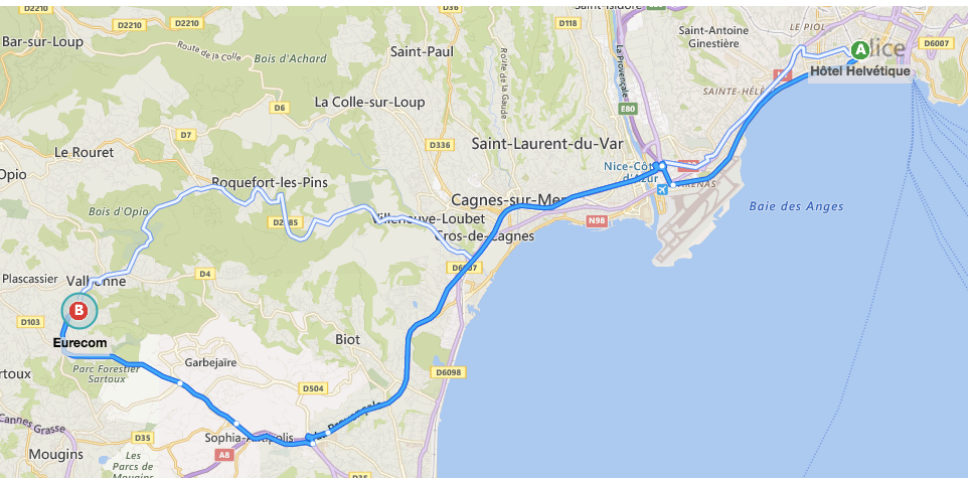


Beyond highway dimension : small distance labels using tree skeletons

Adrian Kosowski and Laurent Viennot

Inria - Univ. Paris Diderot (Irif lab.)

Context : shortest path queries



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Recent progress, in particular for transportation networks.

Two ingredients :

- Tool : precompute small "hub sets".
- Graph property : small "hub sets" do exist, e.g. in road networks.

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- Make any usefull pre-computation.
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Trivial solution : pre-compute for all s, t ...
but $O(n^2)$ space !!! (1 Po for $n = 20m$)

Recent progress, e.g. in road networks (20m nodes) :

- Dijkstra (1959) : 4s
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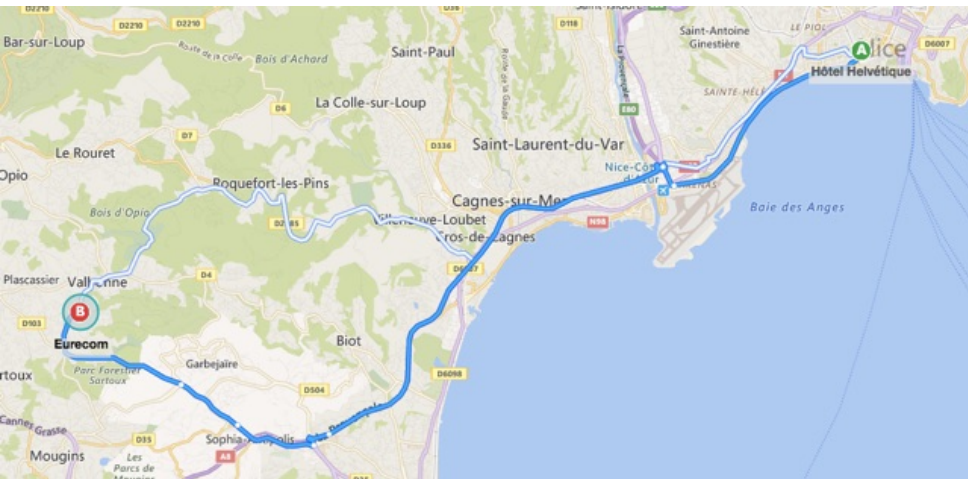
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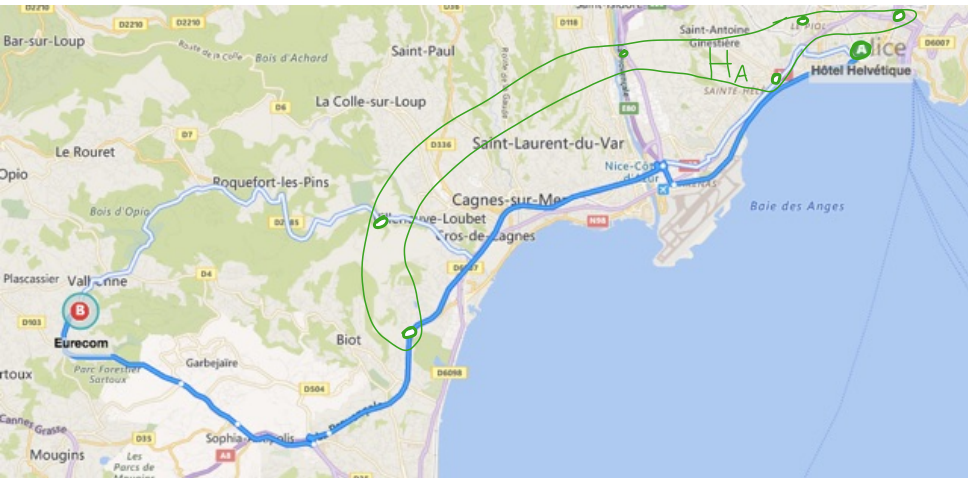
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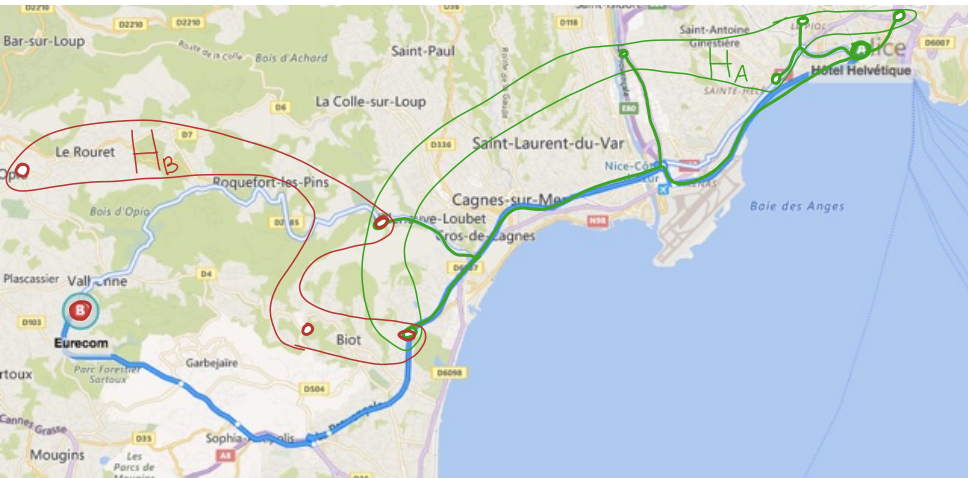
Hubs



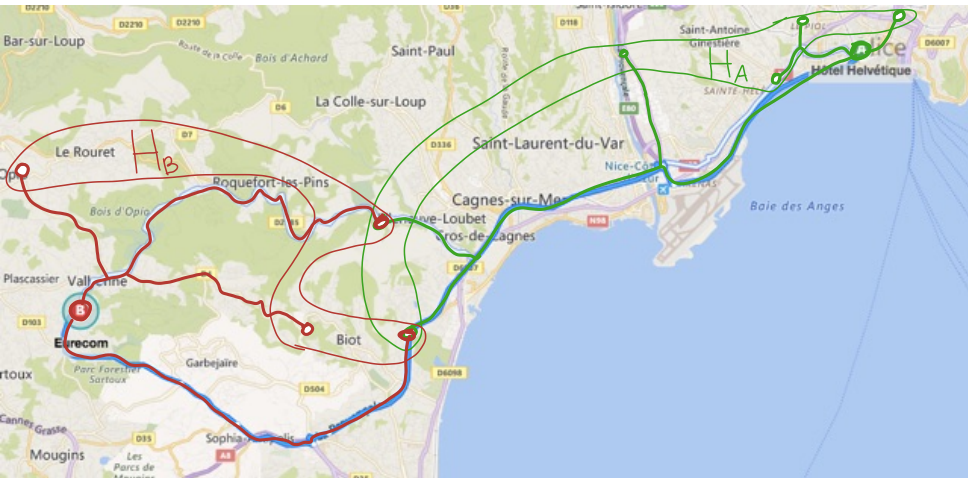
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Distance from sorted hub sets

H_A	2	5	6	8	11	22
D_A	42	12	13	70	8	19

H_B	3	4	8	15	18	22	31
D_B	50	47	31	7	3	80	1002

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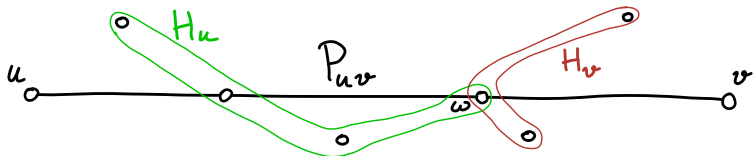
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Hub sets

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Given a graph G , assign a **hub set** $H_u \subseteq V(G)$ to each node u , s.t. for all u, v there exists $w \in H_u \cap H_v$ with $w \in P_{uv}$.



Application : Distance labels : $L_u = \{(w, d(u, w)) : w \in H_u\}$

Distance query : $\text{Dist}(L_u, L_v) = \min_{w \in H_u \cap H_v} d(u, w) + d(w, v)$

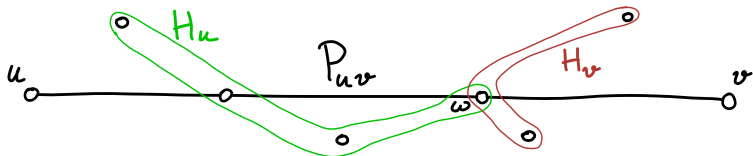
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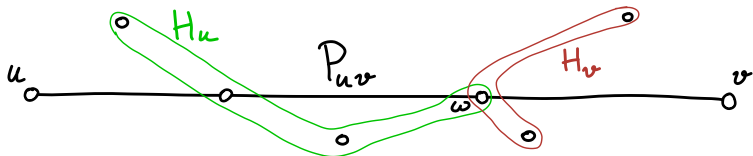
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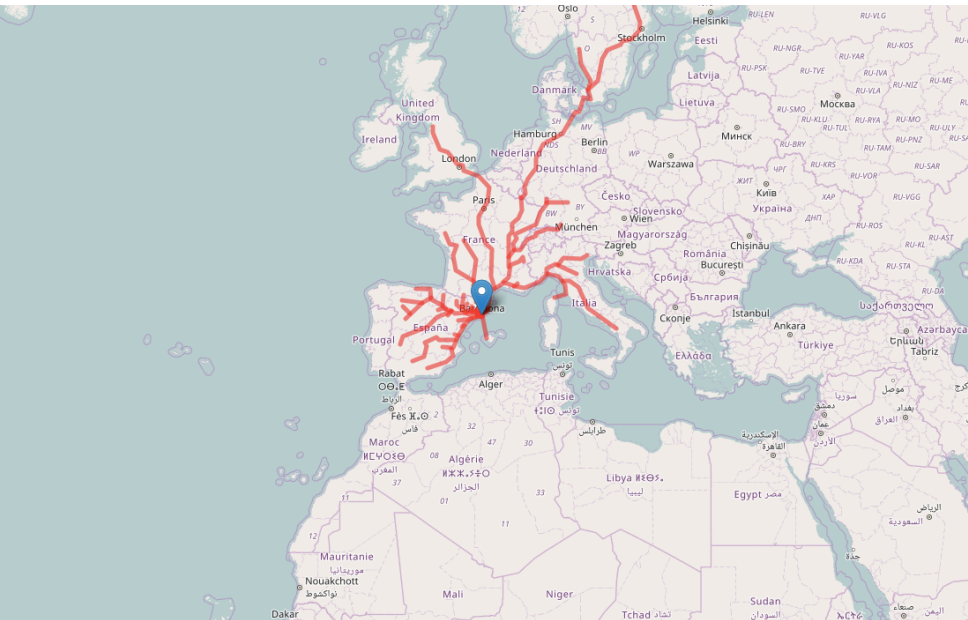
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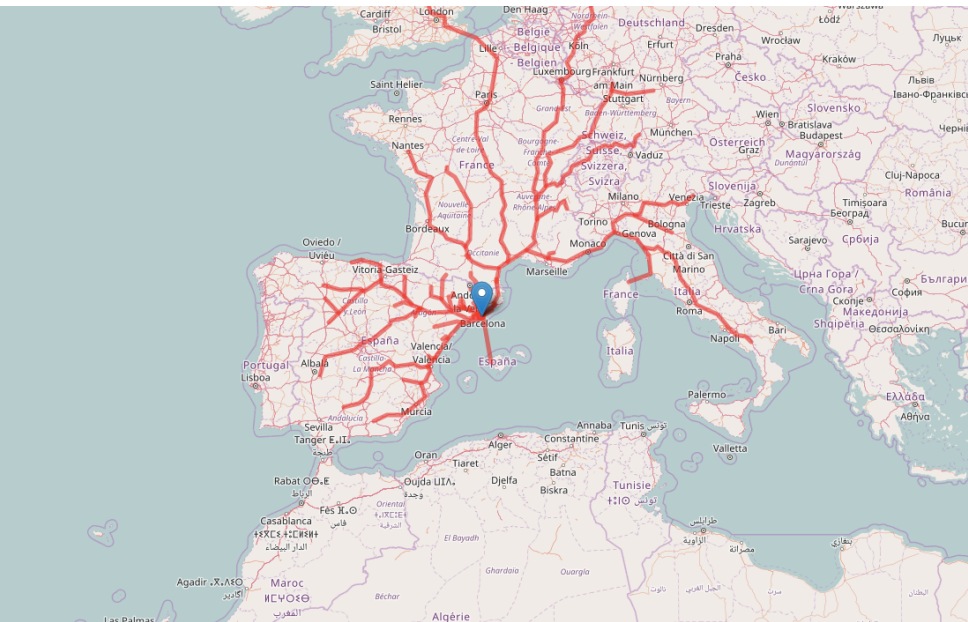
Skeleton dimension

The **skeleton dimension k** of G is the maximum “width” of a “pruned” shortest path tree.

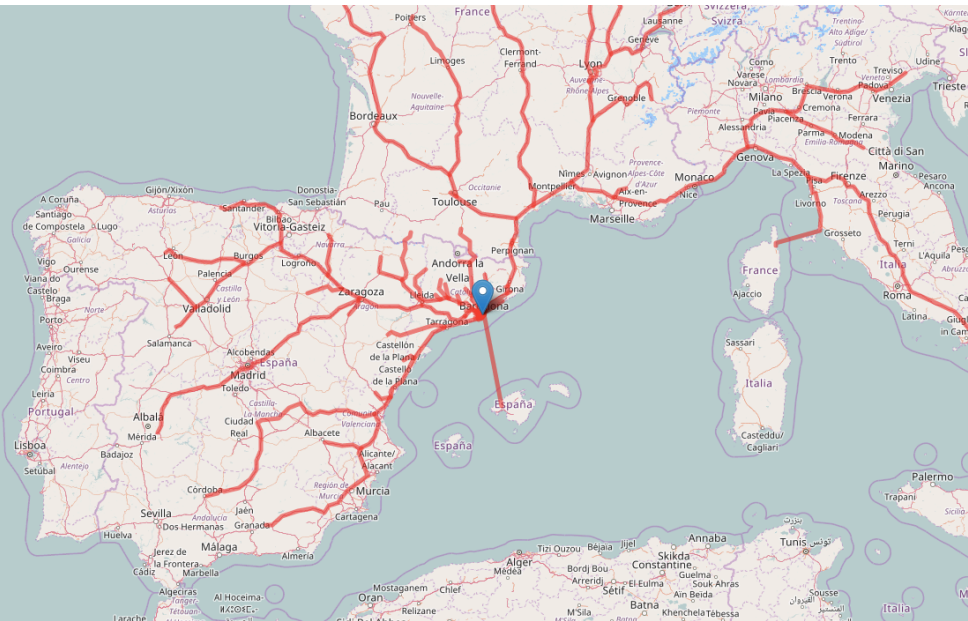
Barcelona tree skeleton : prune last third



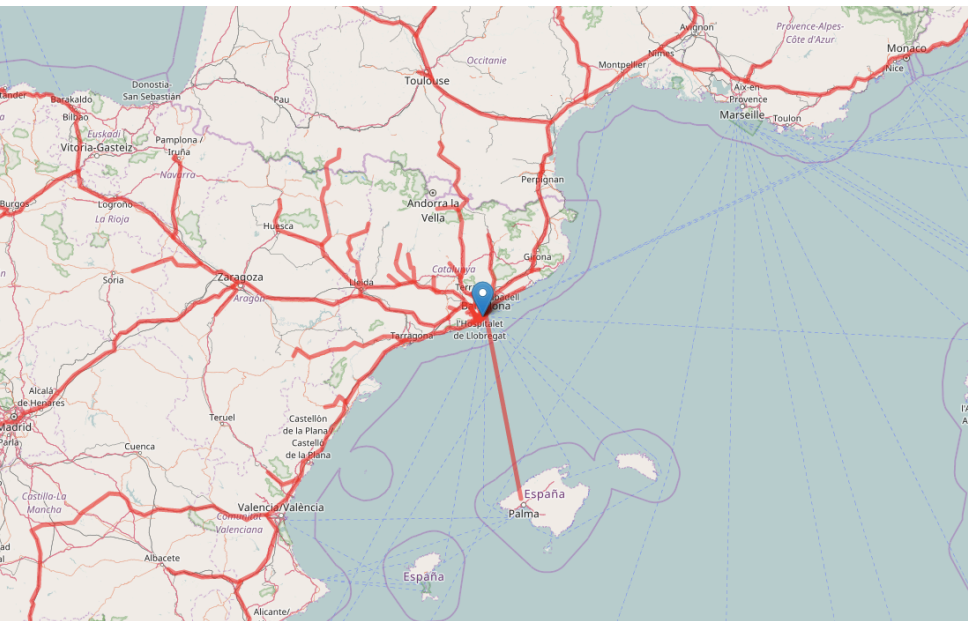
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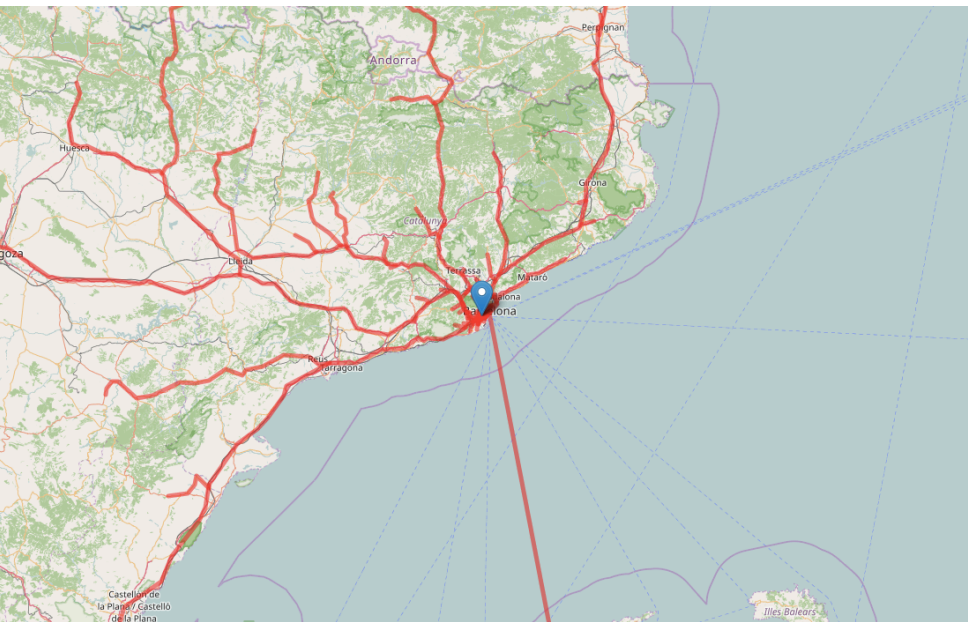
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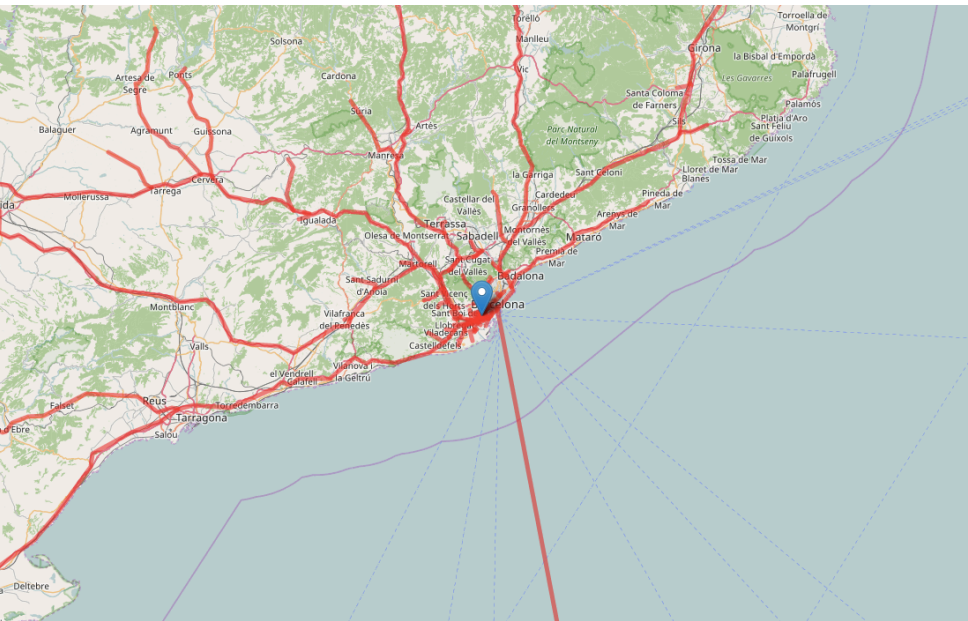
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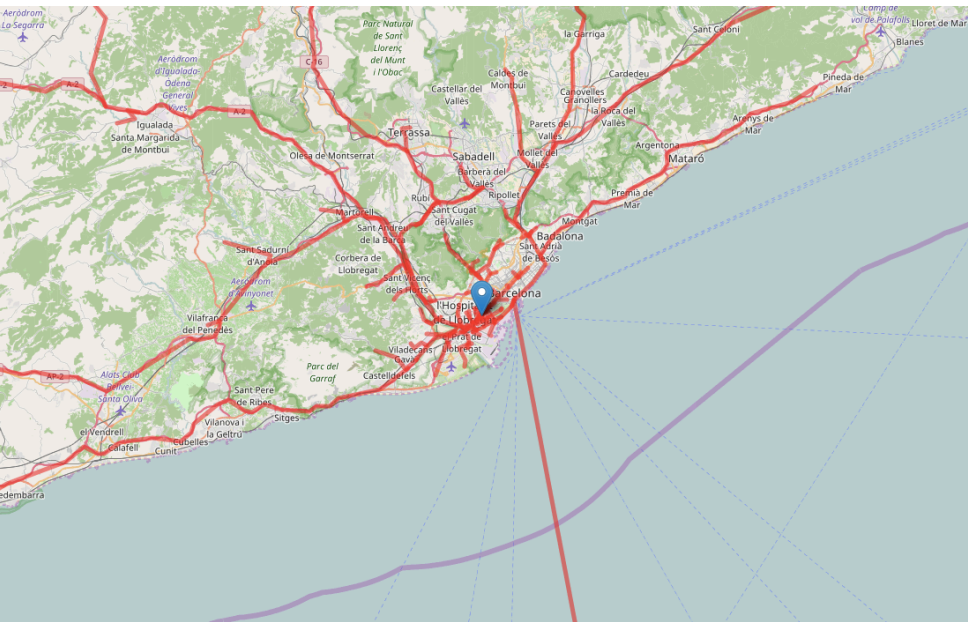
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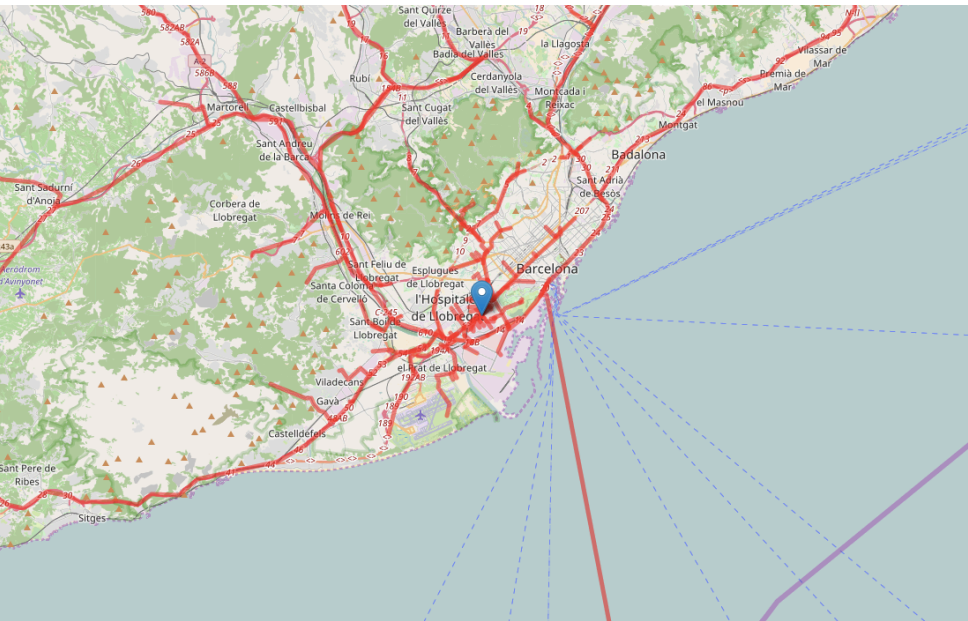
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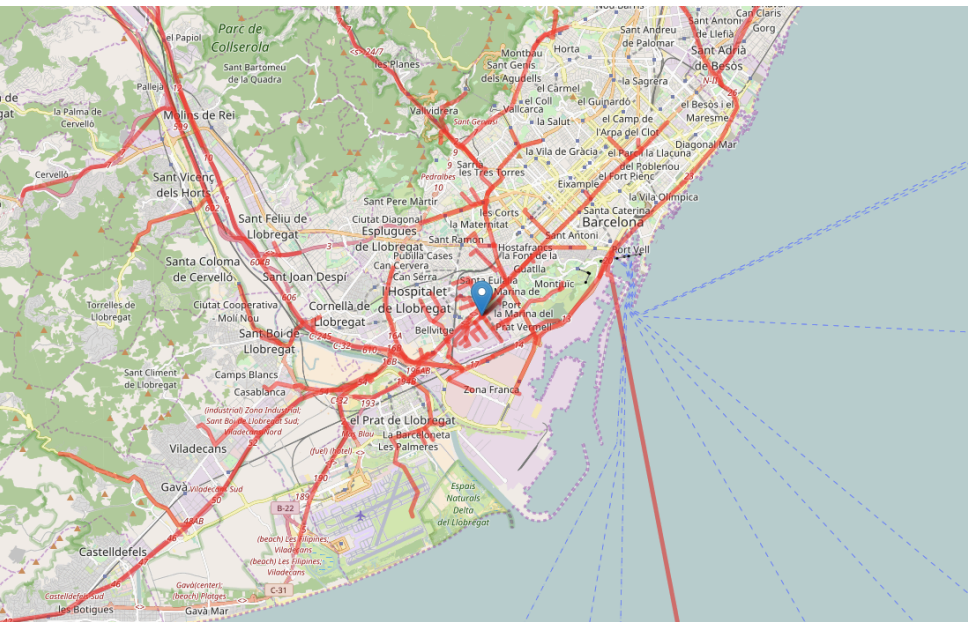
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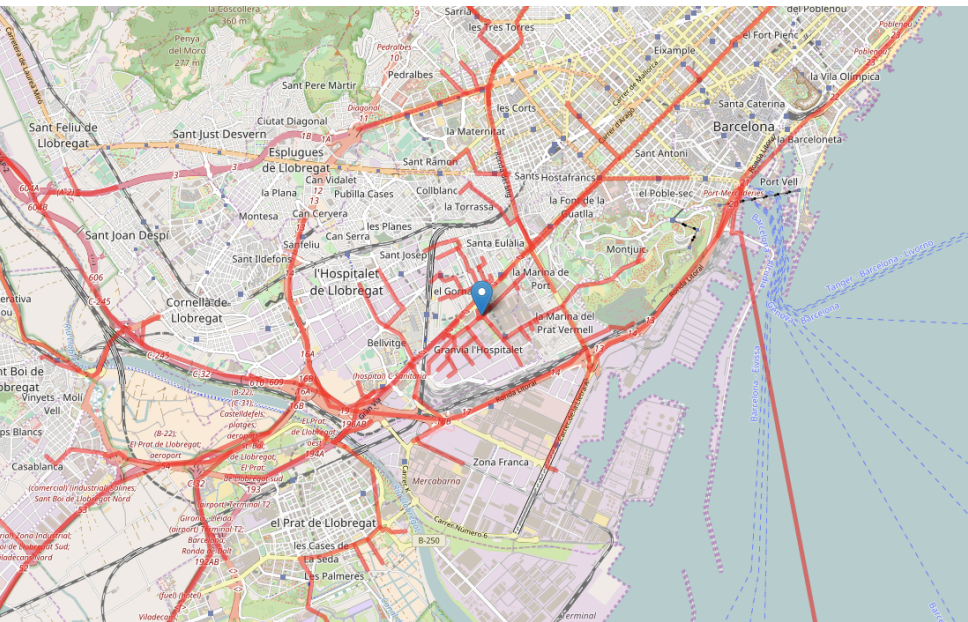
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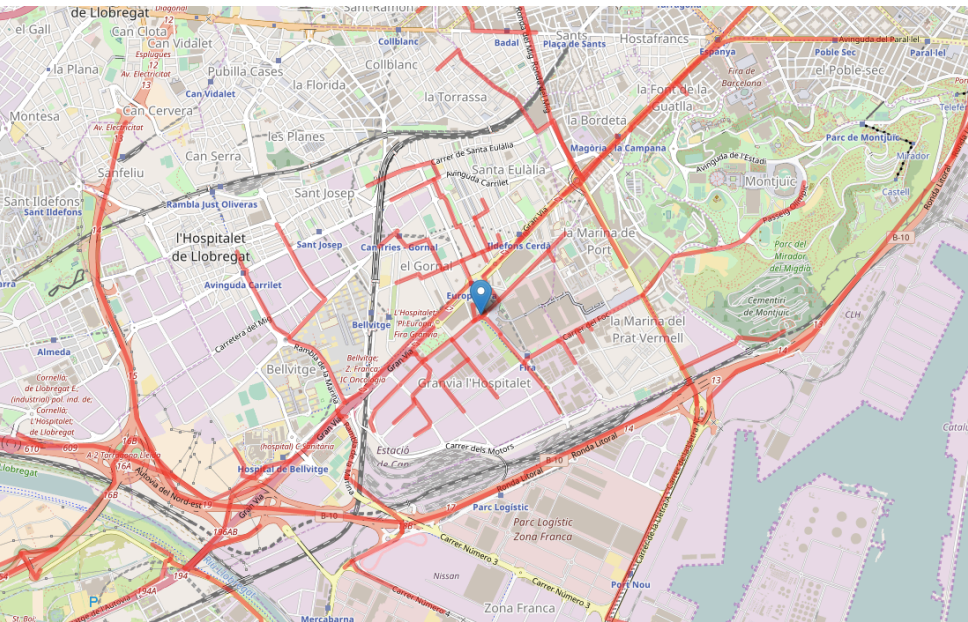
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Assumptions

A directed graph G with

unique shortest paths

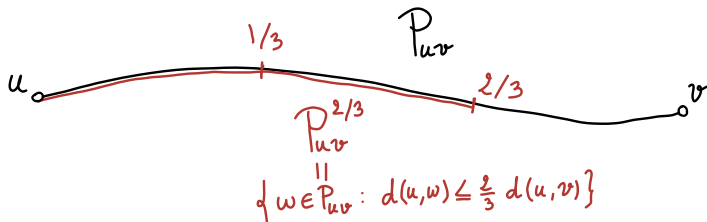
and integer edge lengths (aspect ratio is $O(D)$).

In the presentation : unweighted undirected graph G .

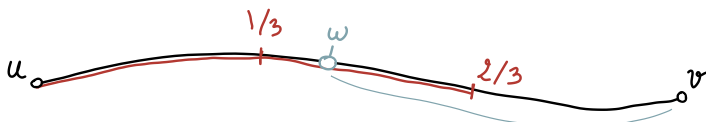
Tree skeleton



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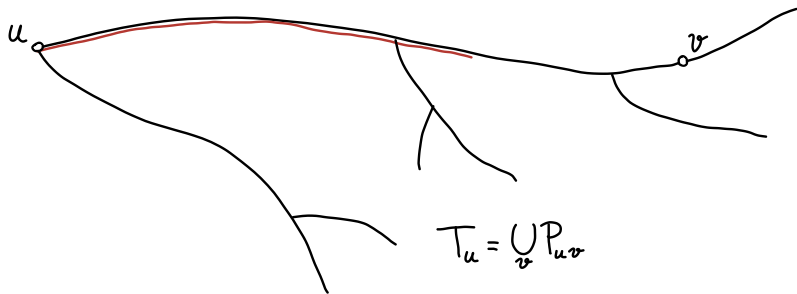


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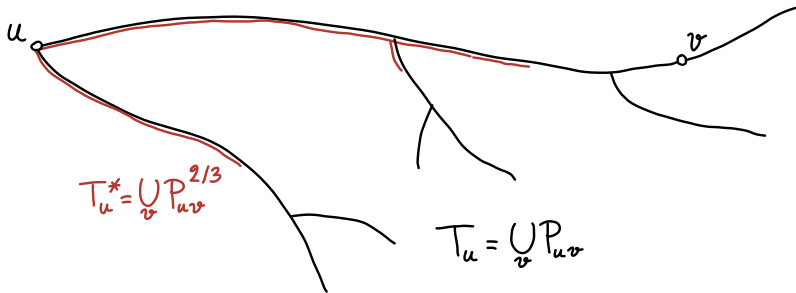


$$\text{Reach}_{P_{uv}}(w) \geq \frac{d(u,w)}{2}$$

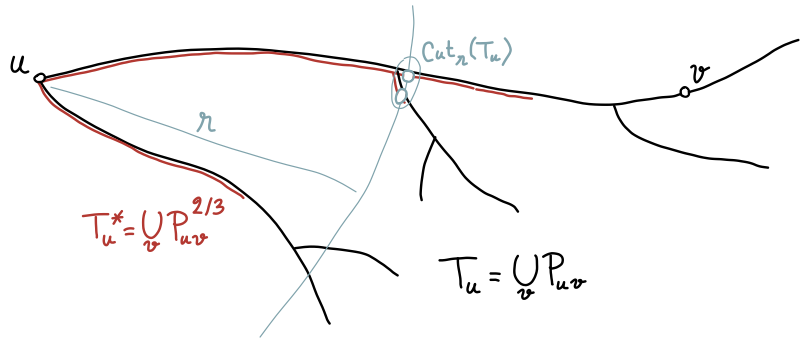
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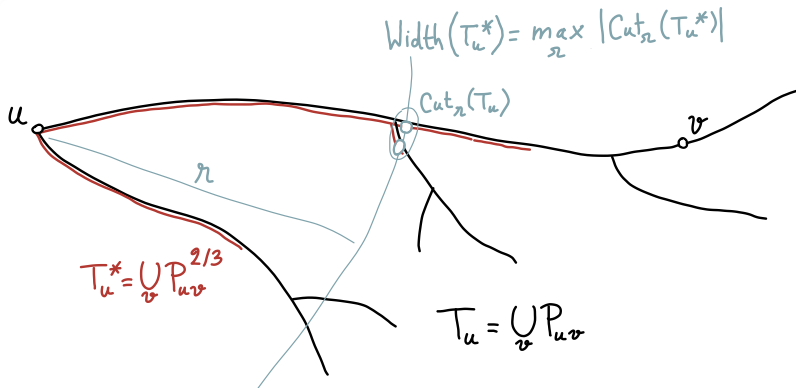
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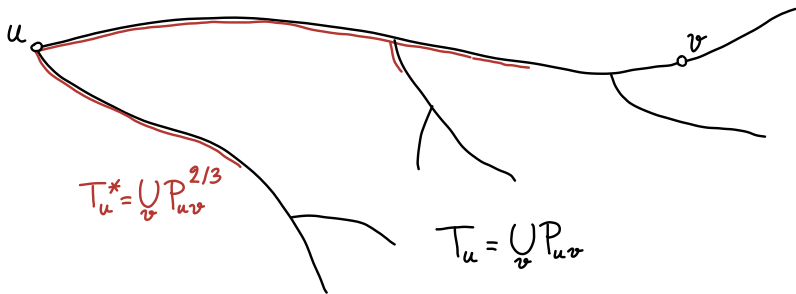


Tree skeleton



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skel. dim. $k = \max_u \text{Width}(T_u^*)$



Main result

Theorem

Given a graph G with **skeleton dimension k** and diameter D , a simple random sampling technique allows to find in polynomial time **hub sets** with size $O(k \log D)$ on average and maximum size $O(k \log \log k \log D)$ with high probability.

Comparison with highway dimension h :

- more general : $k \leq h$
(some graphs have $h = \Omega(\sqrt{n})$ and $k = O(\log n)$),
- naturally extends to directed graphs,
- shorter : $O(k \log \log k \log D)$ vs $O(h \log h \log D)$
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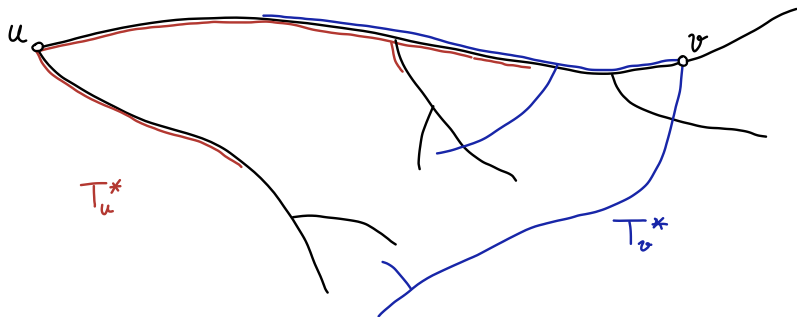
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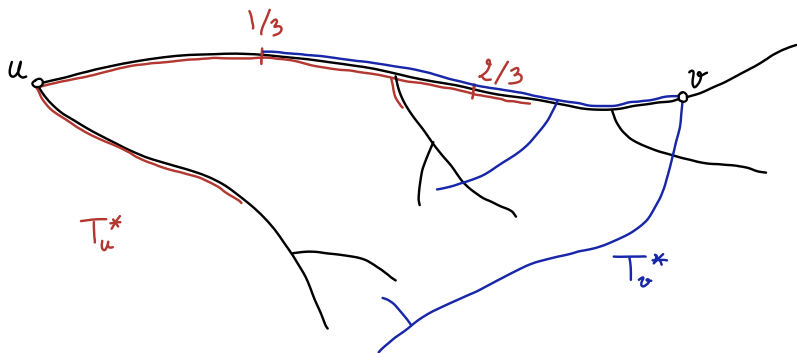
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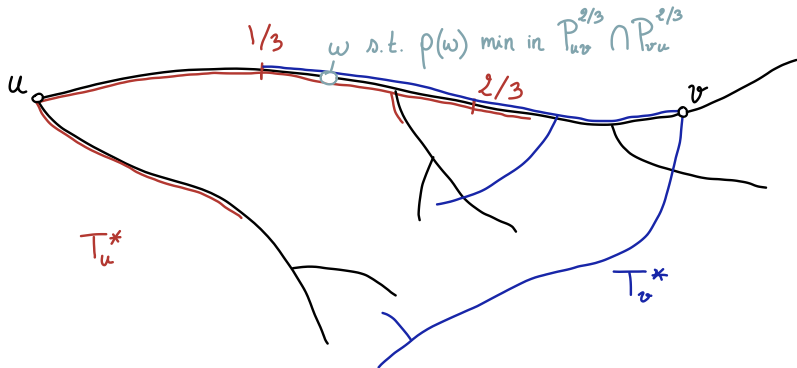
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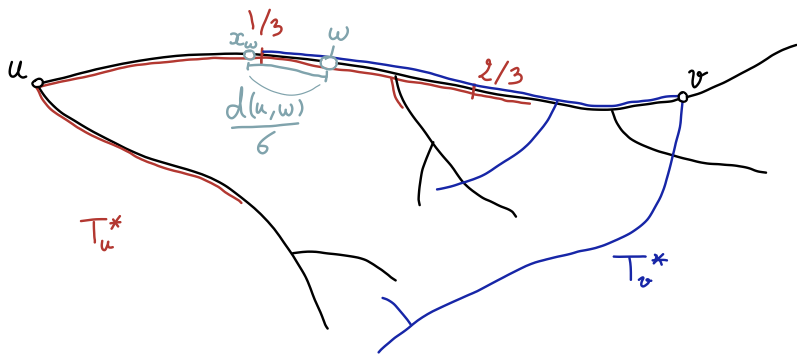
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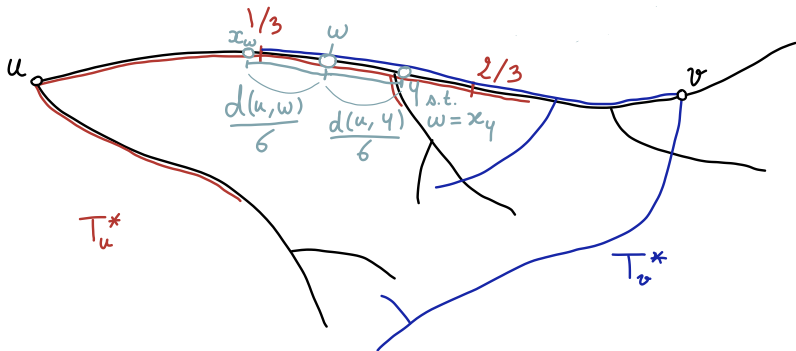
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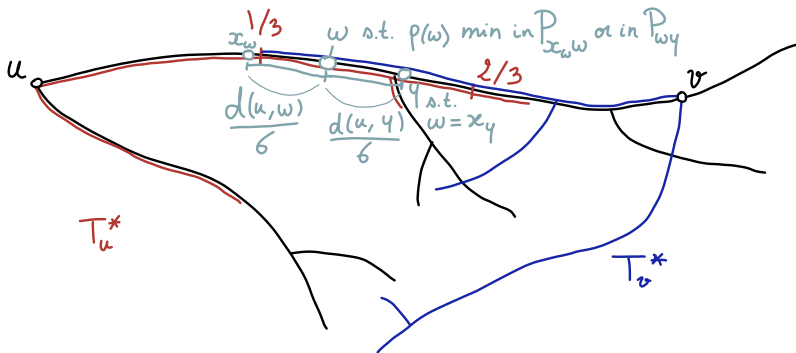
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(Can be computed in $\tilde{O}(n + m)$ separately for each node with shared randomness.)

A sub-path $P_{x_y y}$ has length $\frac{d(u,y)}{6}$ and generates a hub in H_u with probability at most $\frac{12}{d(u,y)}$.

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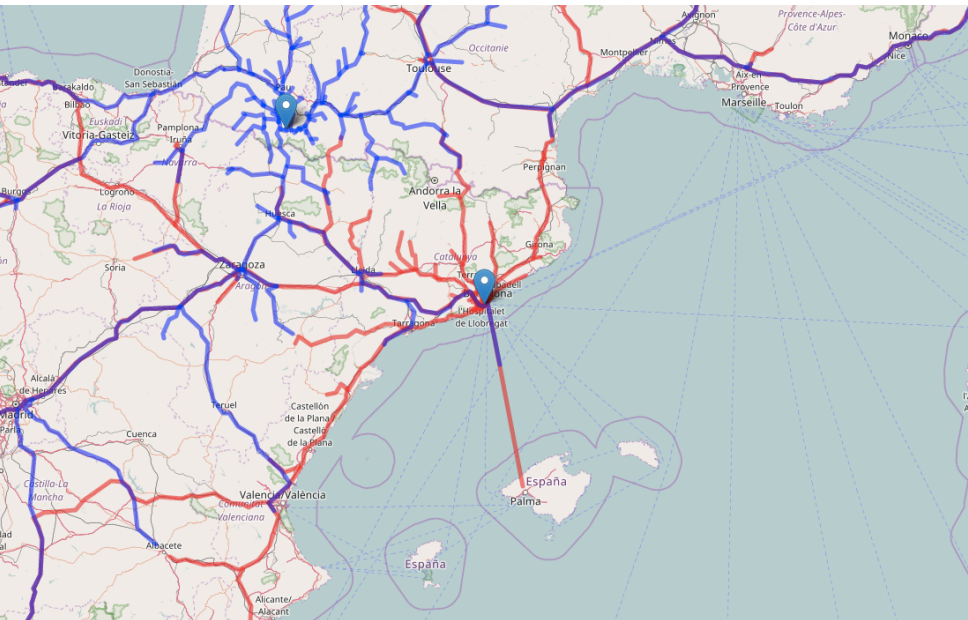
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Road networks : two tree skeletons



Technicalities

Branching introduces non-trivial correlations between sub-paths.

An edge of length ℓ is virtually subdivided into an unweighted path of length 12ℓ .

We construct *edge* hub sets.

Naturally extends to directed graphs.

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Highway dimension

A graph G has small **highway dimension h** if “long” paths in a given region go through “few” transit nodes.

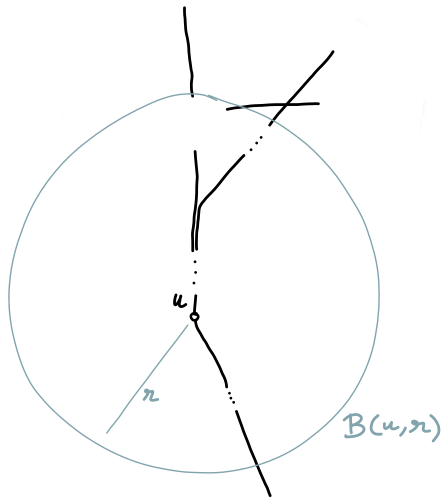
Highway dimension \geq skeleton dimension

$$\mathcal{P}_{ur} = \{P \mid |P| > \frac{r}{2} \text{ and } P \cap B(u, r) \neq \emptyset\}$$

H hits \mathcal{P}_{ur} if $H \cap P \neq \emptyset$ for all $P \in \mathcal{P}_{ur}$

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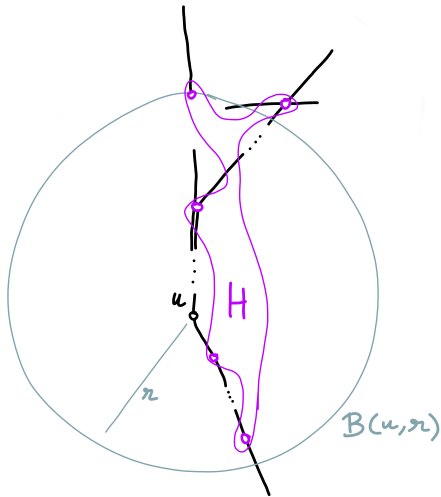
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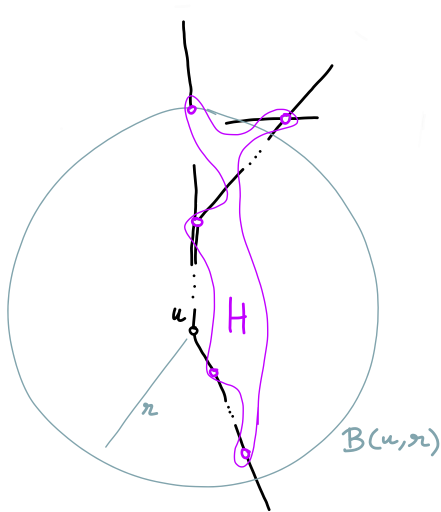
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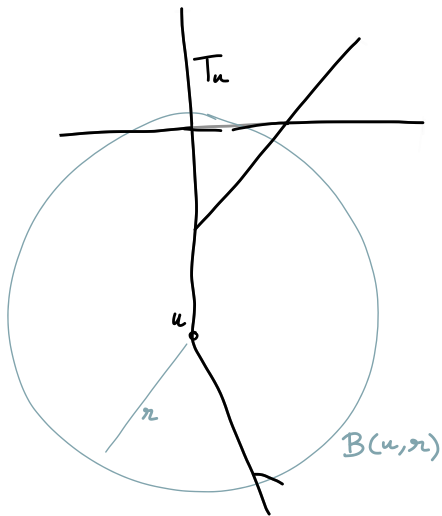
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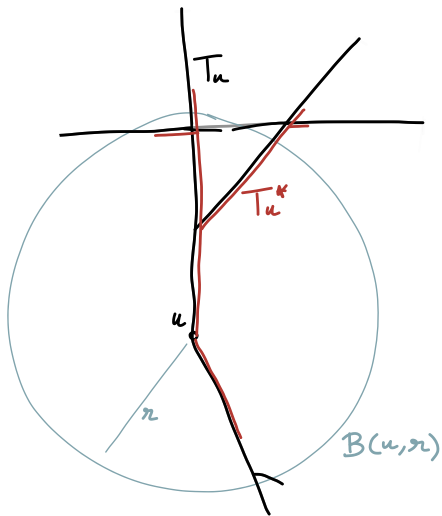
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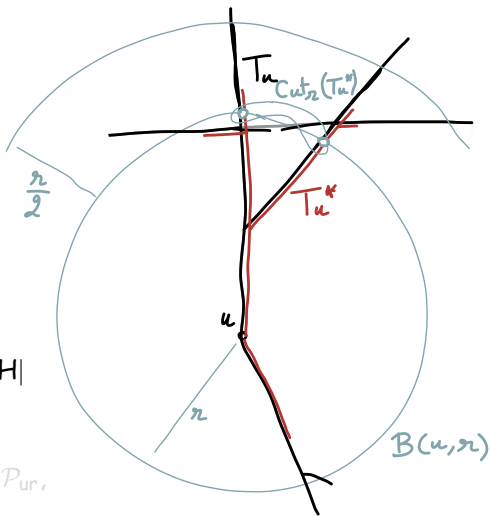
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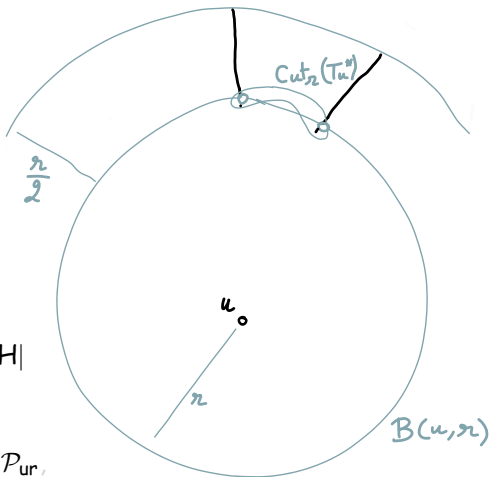
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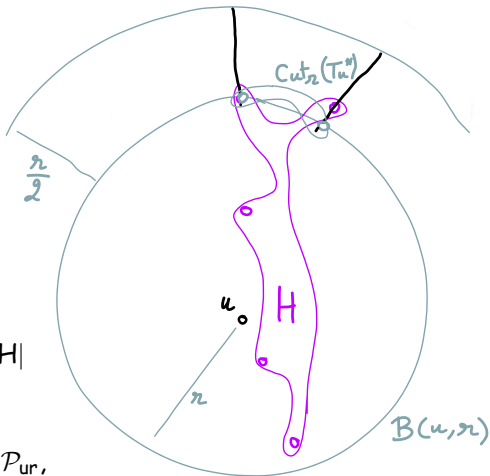
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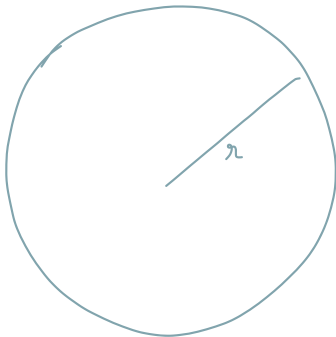
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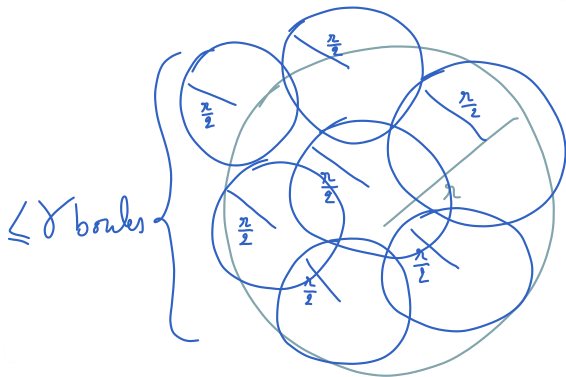
Highway vs skeleton w.r.t. doubling property

A graph G is γ -doubling if any ball $B(u, r)$ can be covered by at most γ balls with radius $\frac{r}{2}$: $\exists H$ s.t. $B(u, r) \subseteq \cup_{v \in H} B(v, \frac{r}{2})$ and $|H| \leq \gamma$.



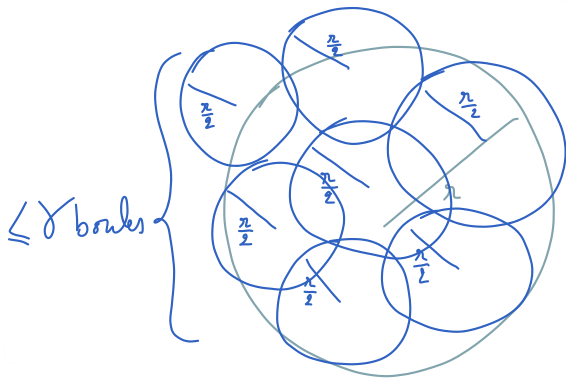
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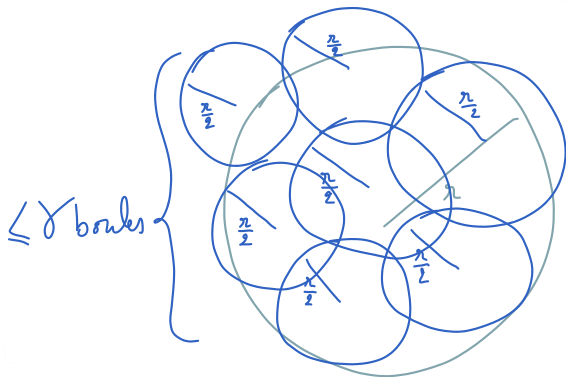
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$B(u, r) \subseteq \gamma^2$ balls with radius $\frac{r}{4}$.

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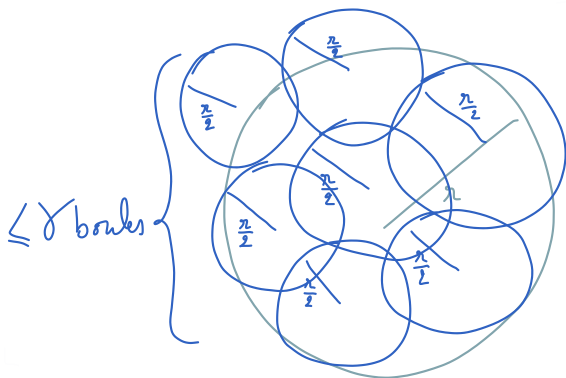
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$B(u, r) \subseteq \gamma^{1+\log r}$ balls with radius $\frac{1}{2}$.

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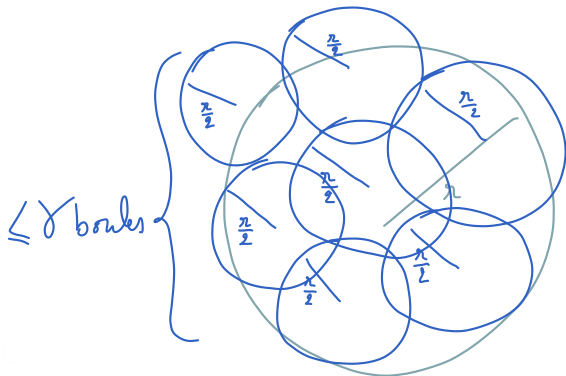
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$$|B(u, r)| \leq \gamma^{1+\log r}$$

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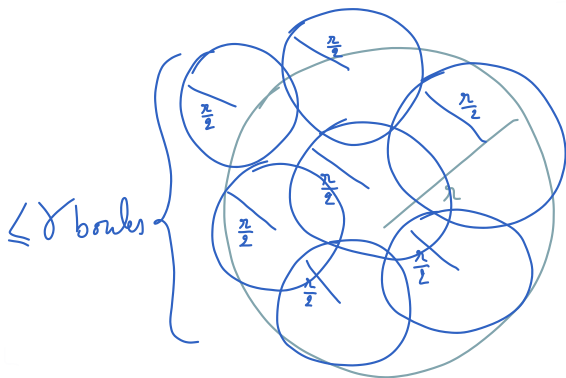
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$$n = |B(u, D)| \leq \gamma^{1+\log D}$$

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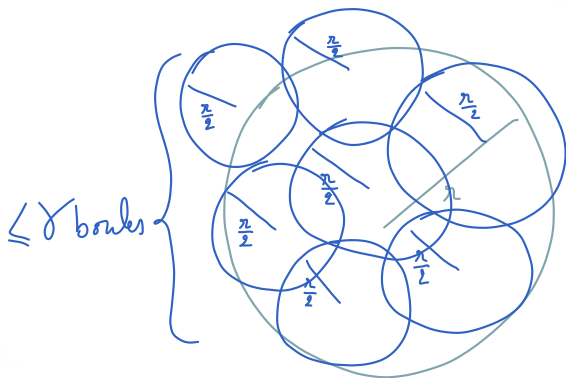
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$$\log n = O(\log D \log \gamma)$$

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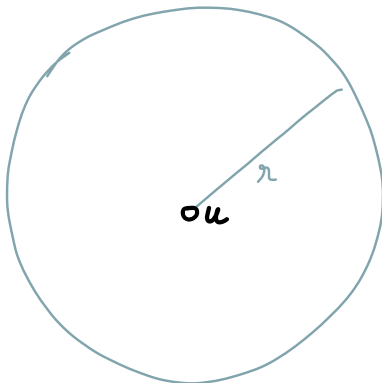


$\log n = O(\log D \log k)$ when $\gamma = O(k)$

Highway vs skeleton w.r.t. doubling property

Proposition

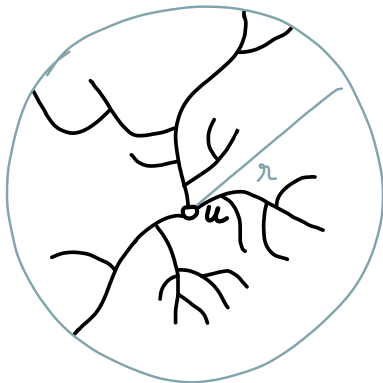
Any graph with highway dimension h and skeleton dimension k is $\min\{h + 1, 2k + 1\}$ -doubling.



Highway vs skeleton w.r.t. doubling property

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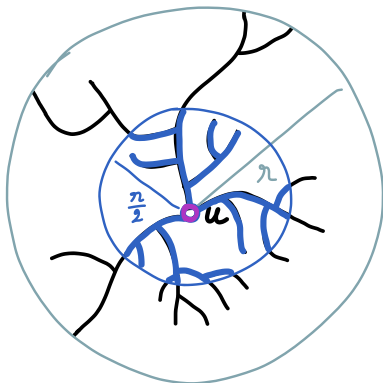
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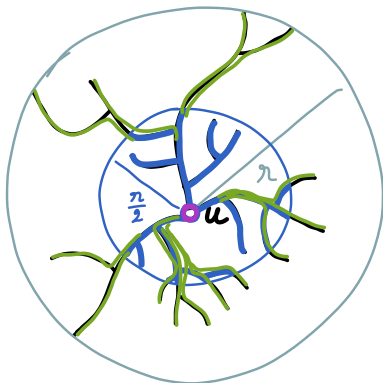
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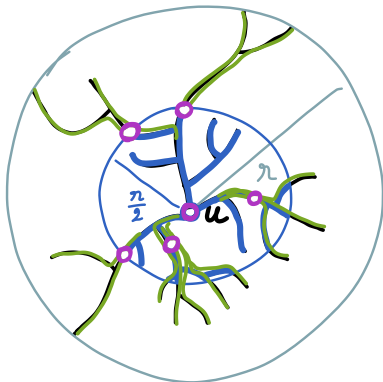
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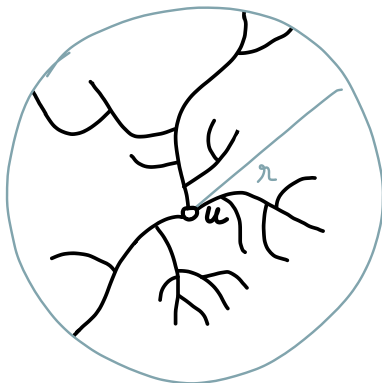
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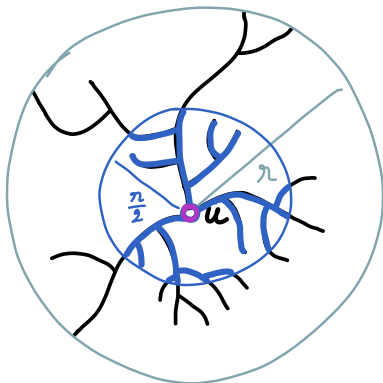
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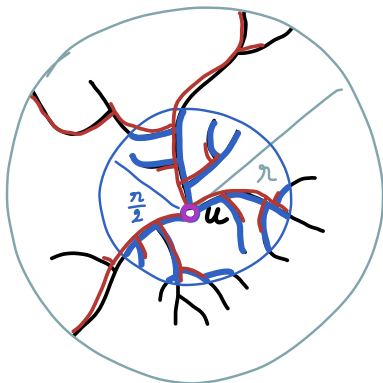
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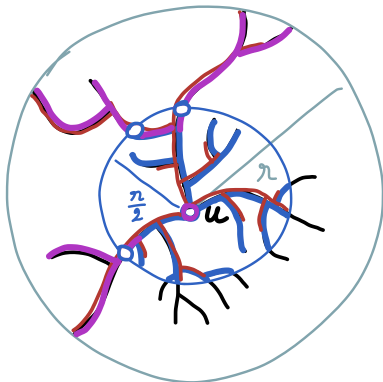
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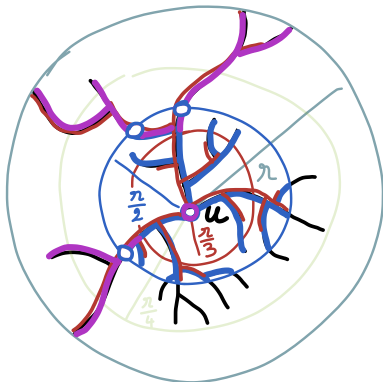
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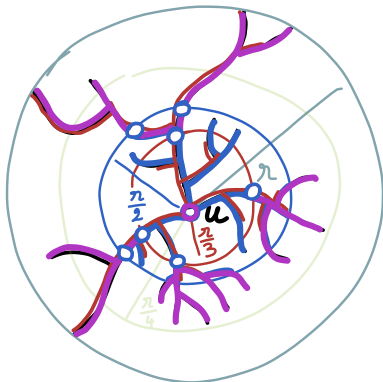
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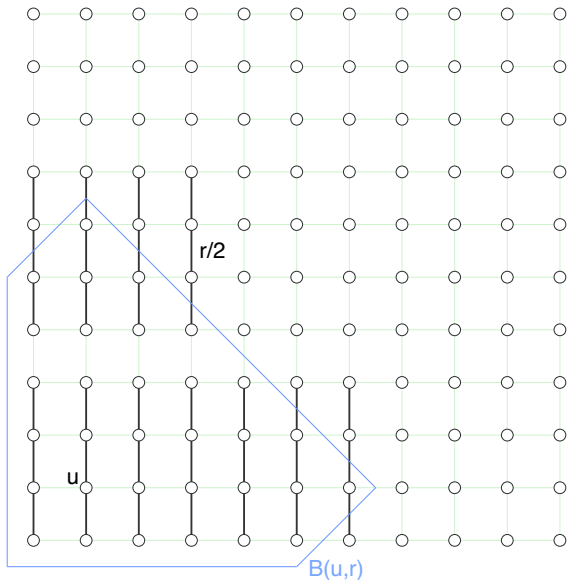
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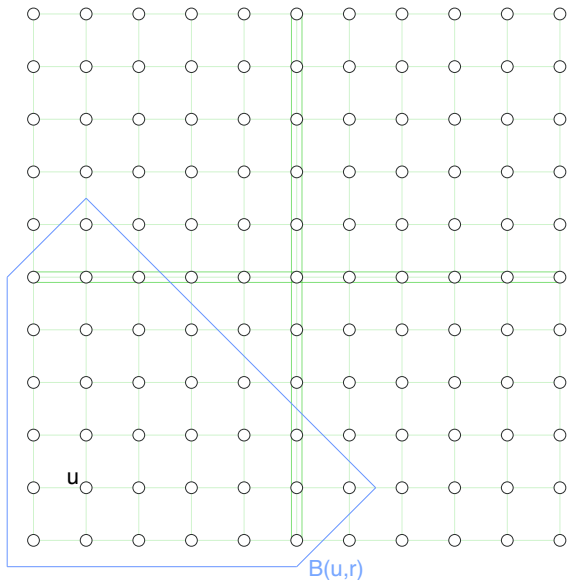
Highway vs skeleton in grids



$$h = \Theta(\sqrt{n})$$

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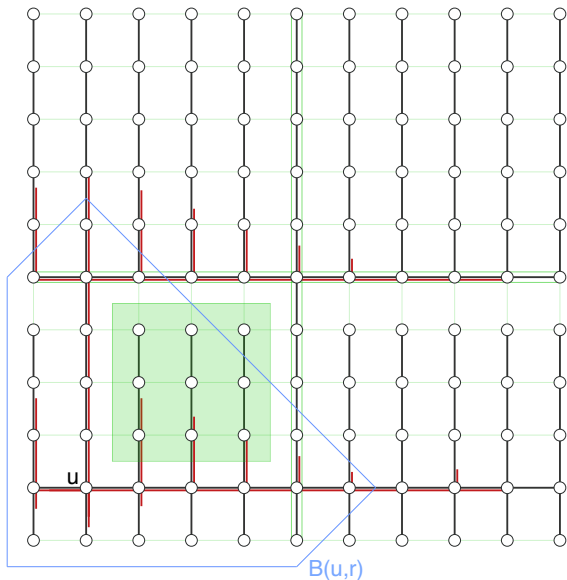
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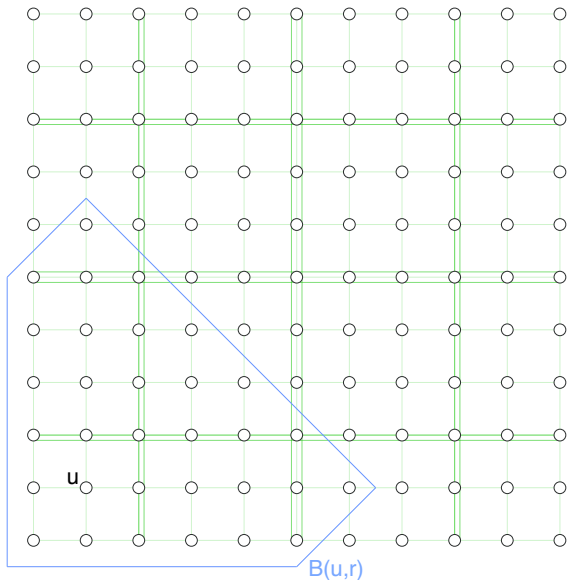
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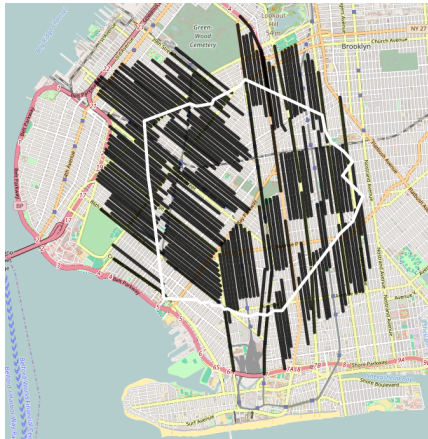
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Highway vs skeleton in Brooklyn



Packing of 172 paths



Skeleton width 48

Summary

Theorem : Hub sets of size $O(k \log D \max \left\{ 1, \log \frac{\log n}{\log D} \right\})$ can be constructed in randomized polynomial time for skeleton-dimension- k graphs.

Bonus : improvement of δ -preserving distance labeling in unweighted graphs (building block for $o(n)$ distance labeling in sparse graphs [Alstrup et al. 2010]) :

For $r \geq \delta$, we have $|\text{Cut}_r(\mathbb{T}_u^*)| = O(\frac{n}{r})$, and we obtain hub sets of size $O(\sum_{r \geq \delta} \frac{n}{r^2}) = O(\frac{n}{\delta})$.

Perspectives

Other types of transportation networks?

Skeleton dimension of random spatial networks? [Aldous 2014]

Beyond skeleton dimension?

- Small hub sets imply intersecting sub-streets with few leaves.
- Fast computation : additional property (low treewidth, small reach?).

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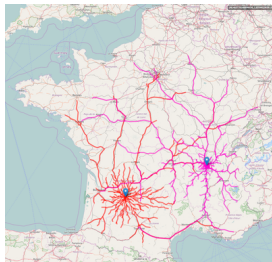
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Thanks.



More to see at gang.inria.fr/road/ :