

Error-sensitive proof-labeling schemes

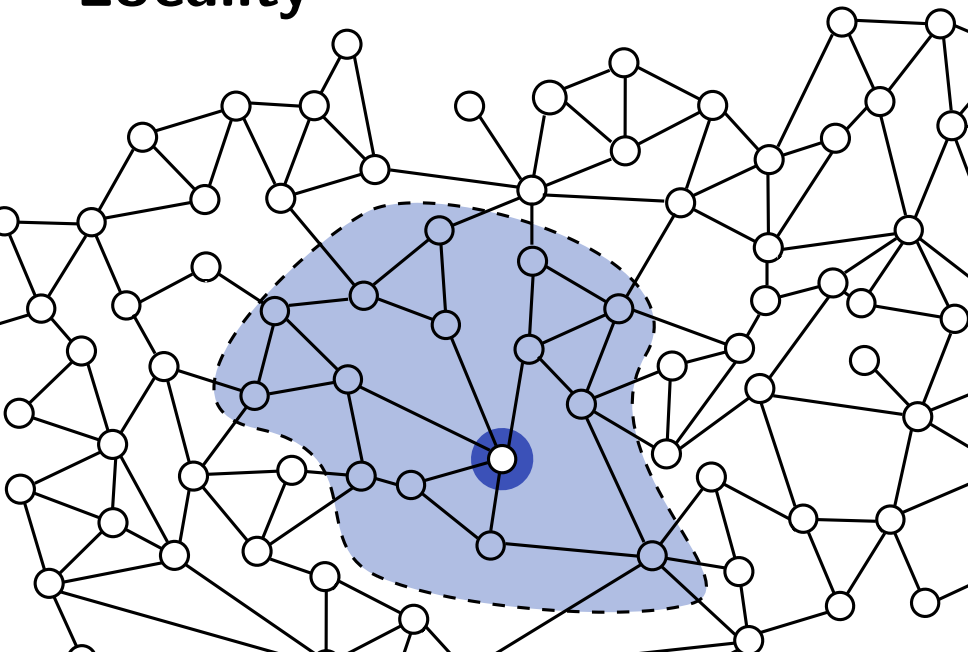
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Pierre Fraigniaud

Université Paris Diderot

ANR Descartes · October 2017

Locality



Decision problems

Decision problems

Context :

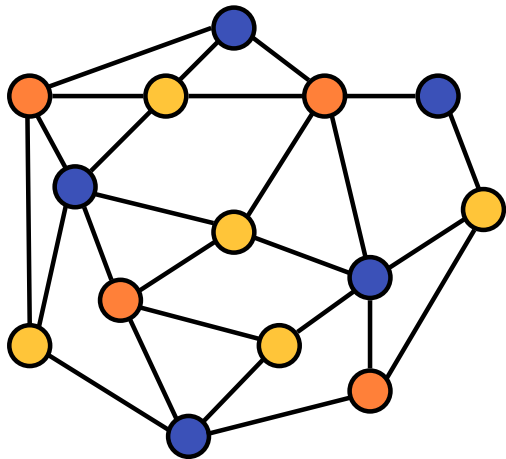
- ▶ Communication graph G
- ▶ Node inputs : $x : v \mapsto x(v)$

A language is

a set of configurations (G, x)

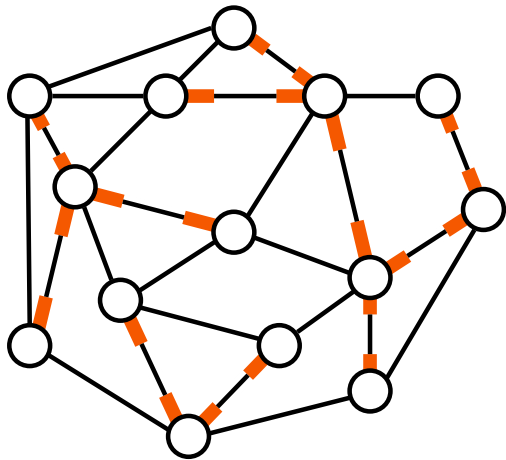
s.t. $\forall G, \exists x, (G, x) \in \mathcal{L}$

Well-colored graphs



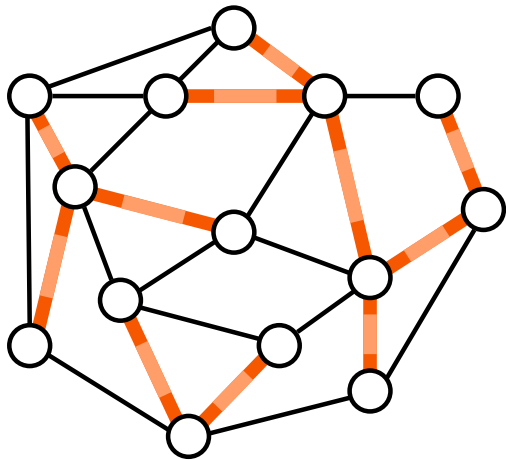
$$\mathcal{L} = \{(G, x) \text{ s.t. } x \text{ is a proper coloring of } G\}$$

Acyclic subgraphs



$\mathcal{L} = \{(G, x) \text{ s.t. } x \text{ describes an acyclic subgraph of } G\}$

Acyclic subgraphs

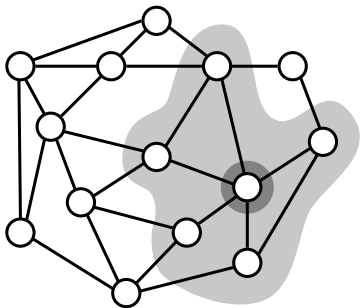


$\mathcal{L} = \{(G, x) \text{ s.t. } x \text{ describes an acyclic subgraph of } G\}$

Decision mechanism

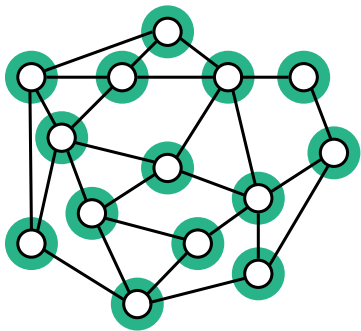
Every node :

- gathers its 1-neighbourhood
- outputs a local decision
accept or reject.



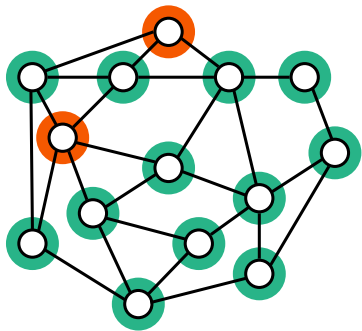
Decision mechanism

(G, x) is accepted
if all node accept.

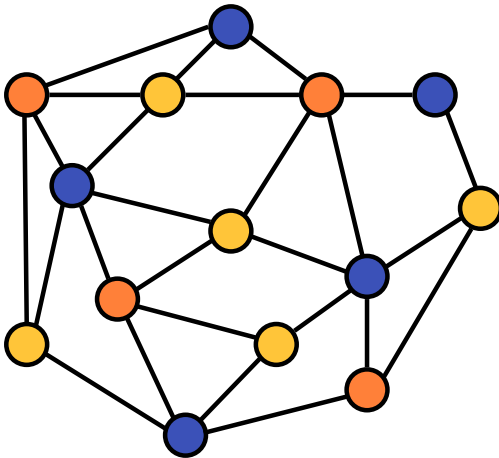


Decision mechanism

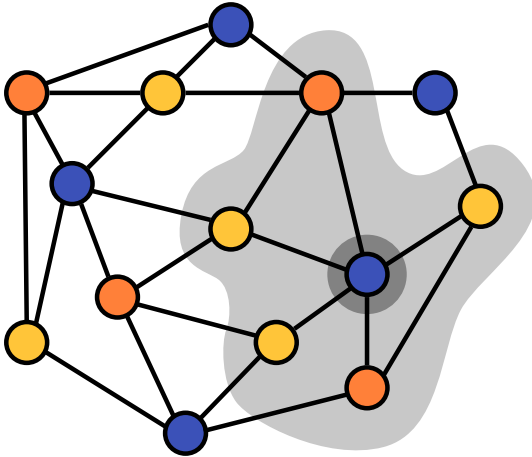
(G, x) is rejected
if at least one node
rejects.



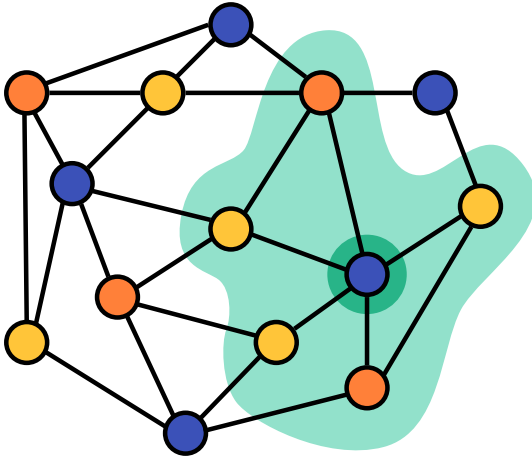
Well-colored graphs



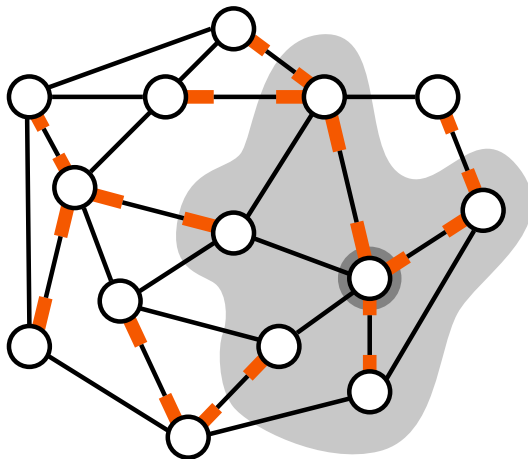
Well-colored graphs



Well-colored graphs



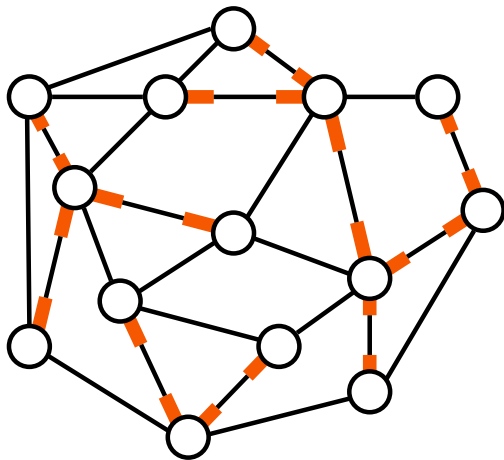
Acyclic subgraphs



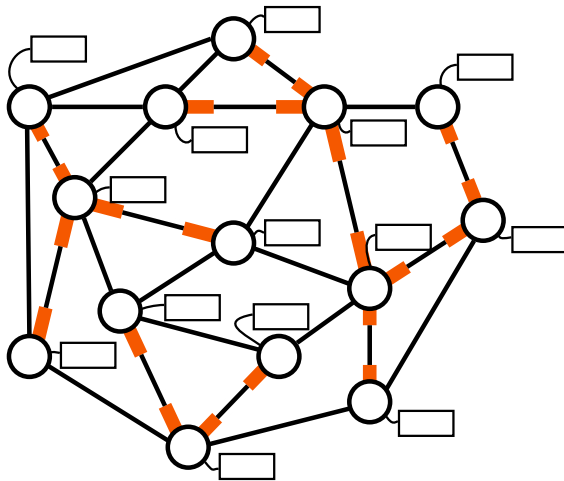
Proof-labeling schemes

**Distributed
non-determinism**

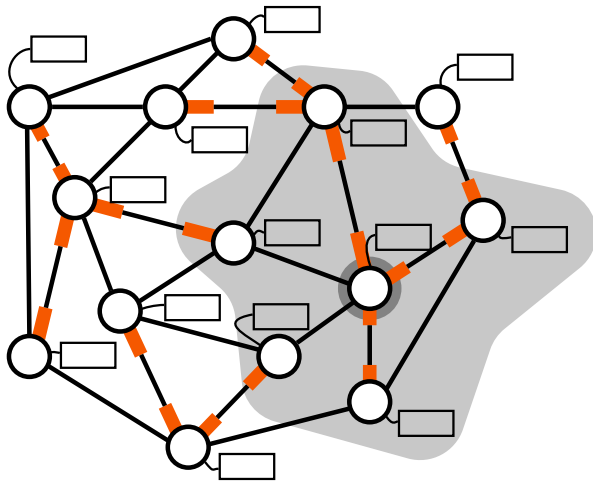
Proof-labeling schemes



Proof-labeling schemes



Proof-labeling schemes



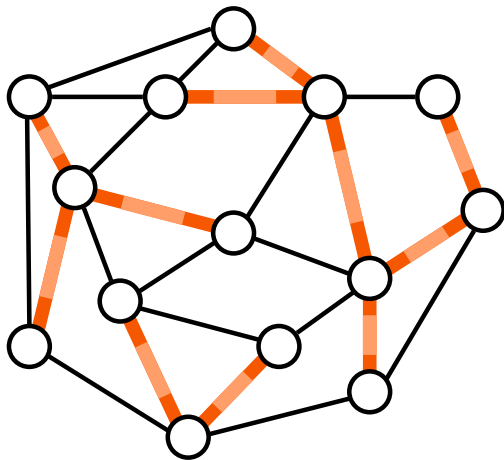
Proof-labeling schemes

Given a proof-labeling scheme for \mathcal{L} :

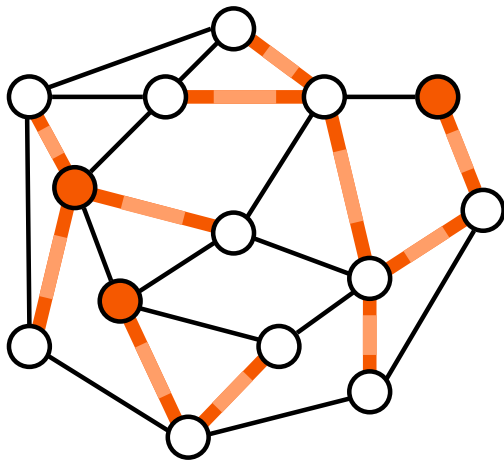
$\forall(G, x) :$

- ▶ If $(G, x) \in \mathcal{L}$, there exist certificates c ,
 (G, x, c) is accepted.
- ▶ If $(G, x) \notin \mathcal{L}$, for all certificates c ,
 (G, x, c) is rejected.

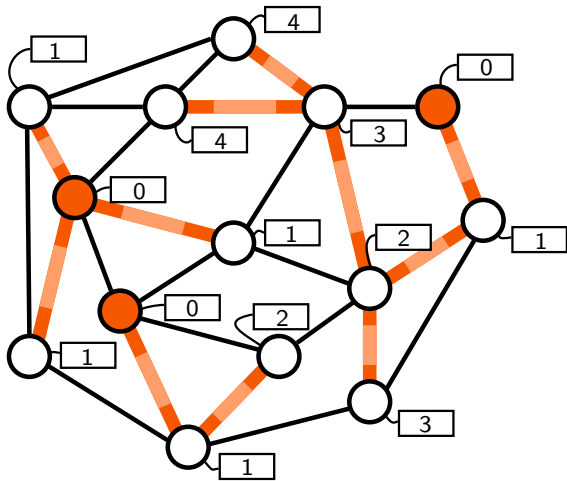
Proof-labeling schemes



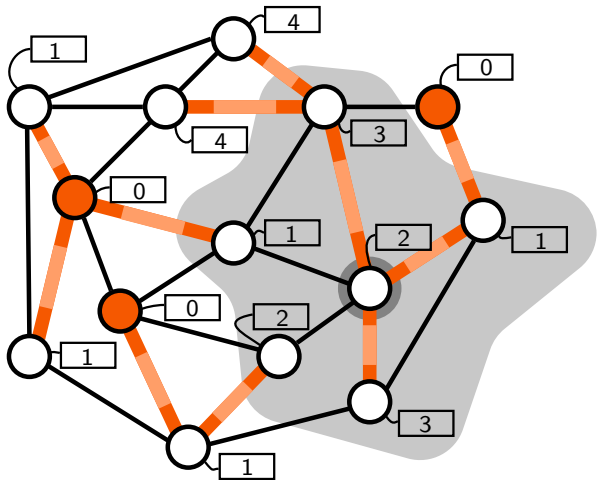
Proof-labeling schemes



Proof-labeling schemes

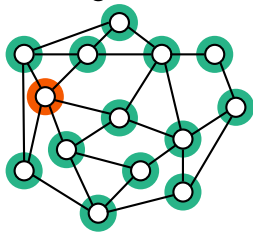
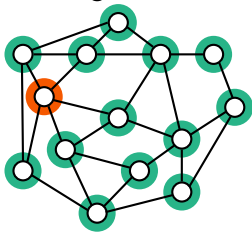
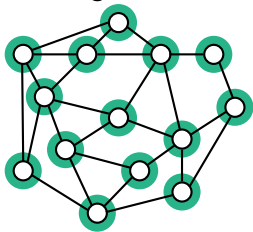
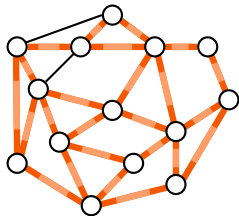
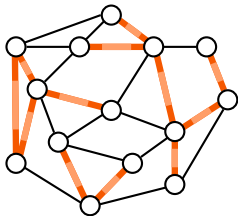
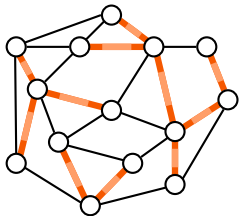


Proof-labeling schemes

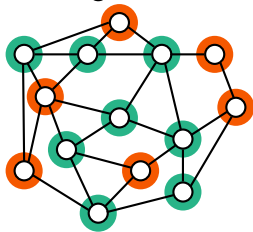
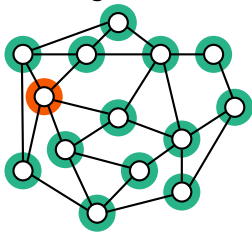
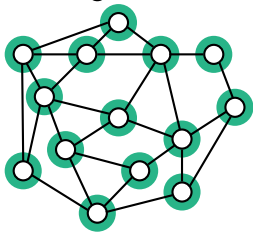
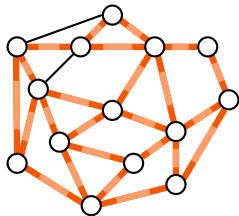
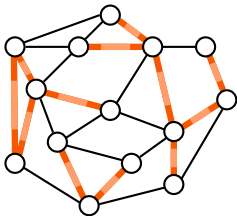
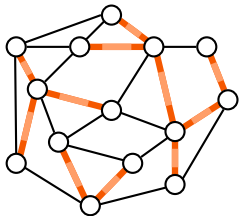


Error-sensitivity of proof-labeling schemes

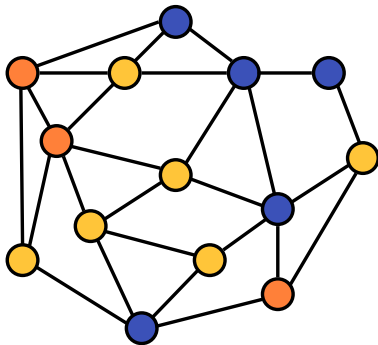
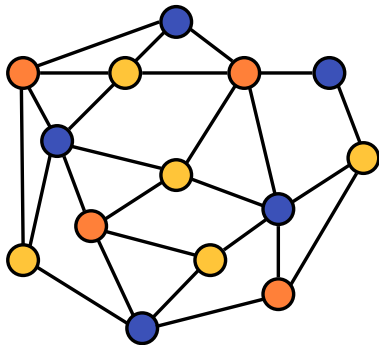
One node to reject



More nodes to reject

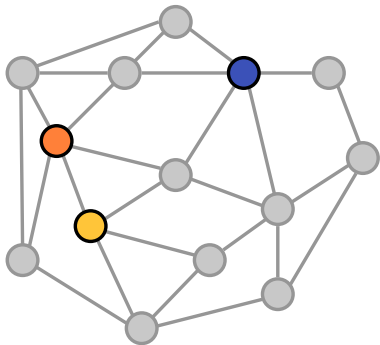
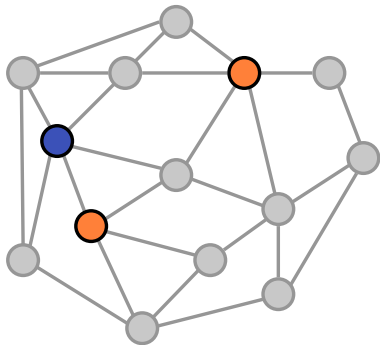


Distance



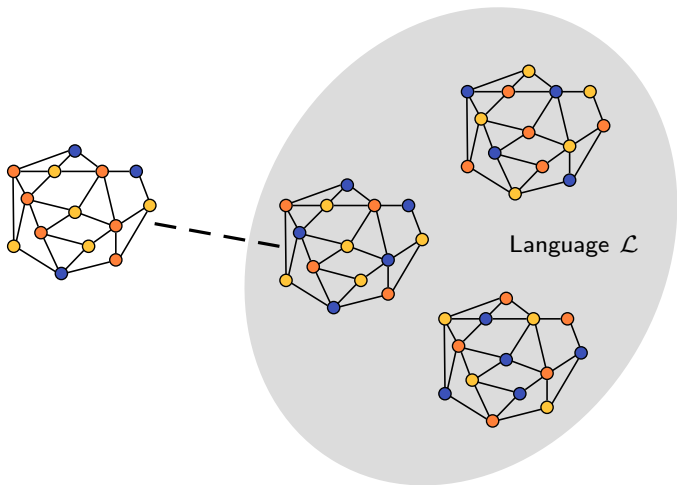
$$d((G, x_1), (G, x_2)) = \#\{v : x_1(v) \neq x_2(v)\}$$

Distance



$$d((G, x_1), (G, x_2)) = \#\{v : x_1(v) \neq x_2(v)\}$$

Distance



$$d((G, x), \mathcal{L}) = \min_{(G', x') \in \mathcal{L}} d((G, x), (G', x'))$$

Error-sensitivity

with words

A PLS is **error-sensitive** if,
the number of rejecting nodes
grows linearly with the distance.

Error-sensitivity

with a formula

A PLS is **error-sensitive** if,

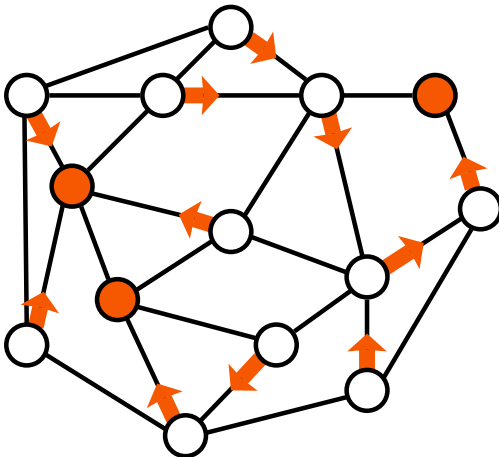
there exists $\alpha > 0$ s.t.,

for all (G, x) , for all certificate :

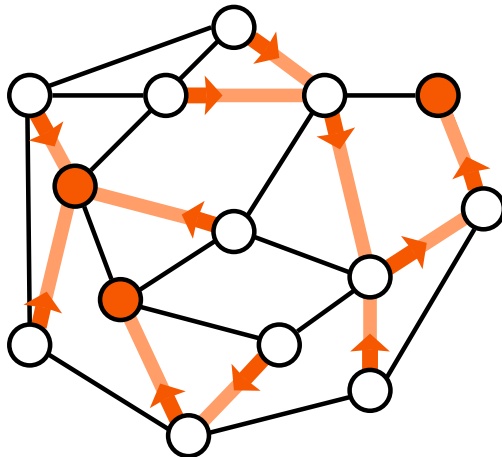
$$\#\{\text{Rejecting nodes}\} \geq \alpha \cdot d((G, x), \mathcal{L})$$

Basic examples

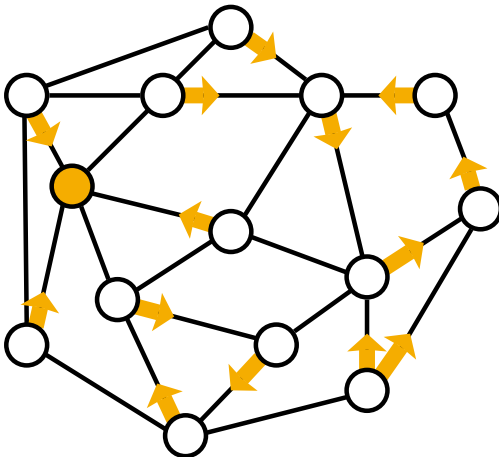
Acyclic subgraph with pointers



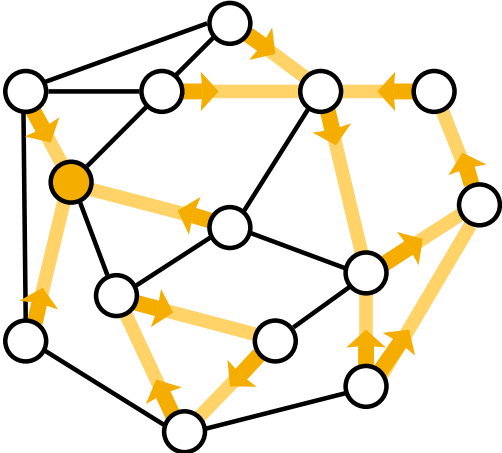
Acyclic subgraph with pointers



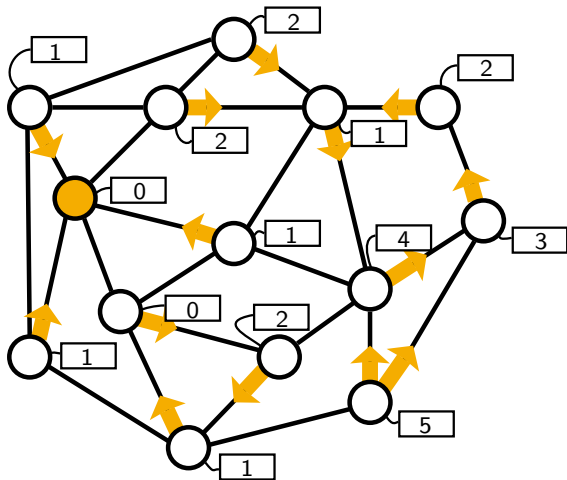
Acyclic subgraph with pointers



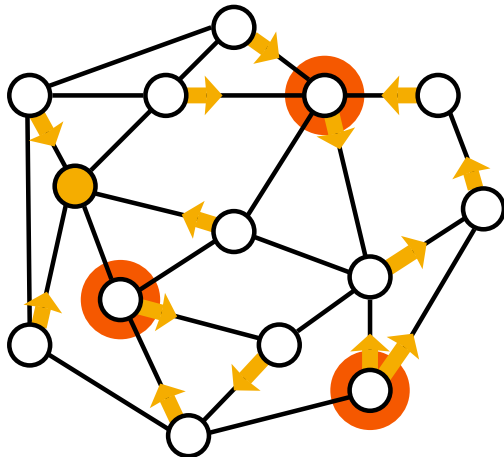
Acyclic subgraph with pointers



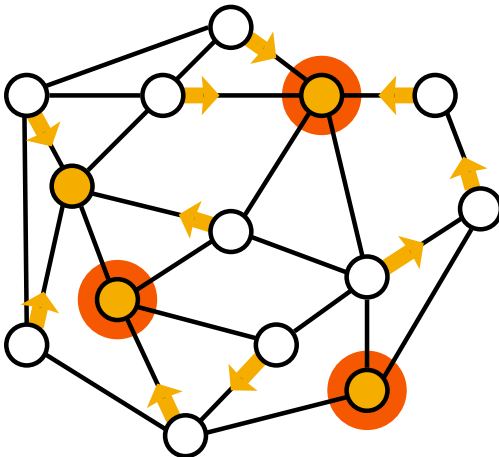
Acyclic subgraph with pointers



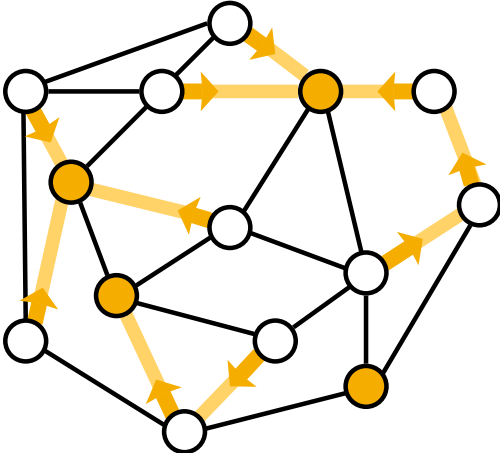
Acyclic subgraph with pointers



Acyclic subgraph with pointers



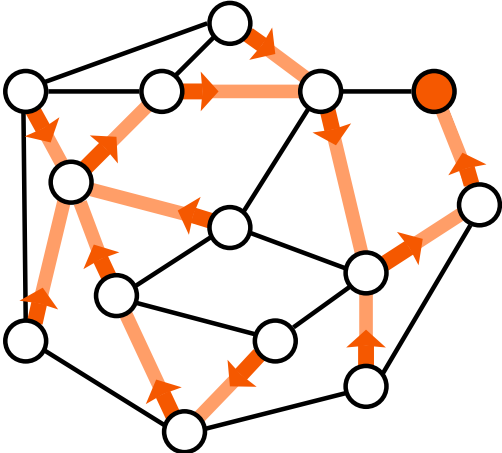
Acyclic subgraph with pointers



Acyclic subgraph with pointers

Acyclic subgraph with pointers
has an error-sensitive PLS,
(with $O(\log n)$ -size certificates)

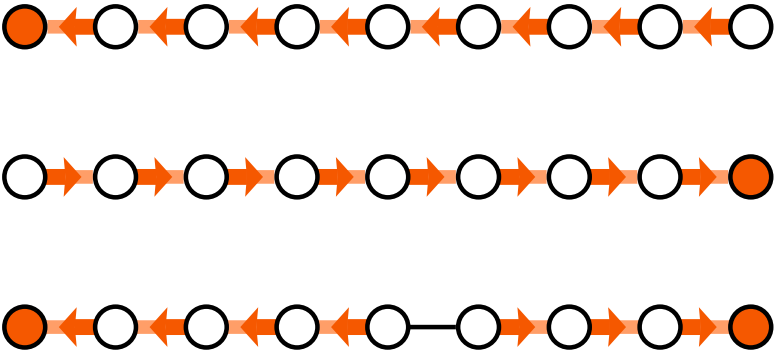
Spanning tree with pointers



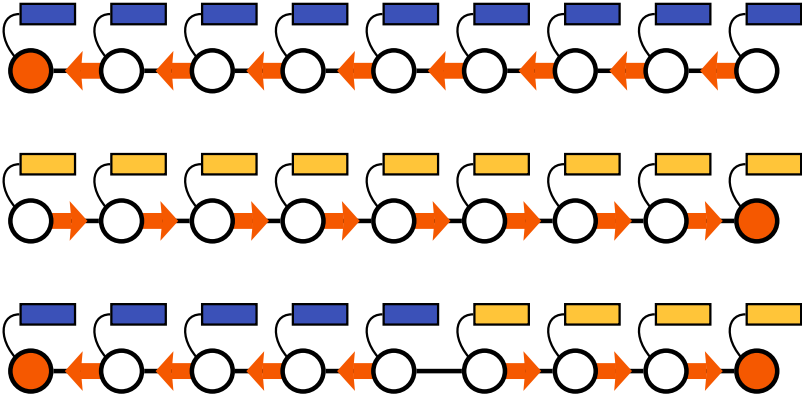
Spanning tree with pointers



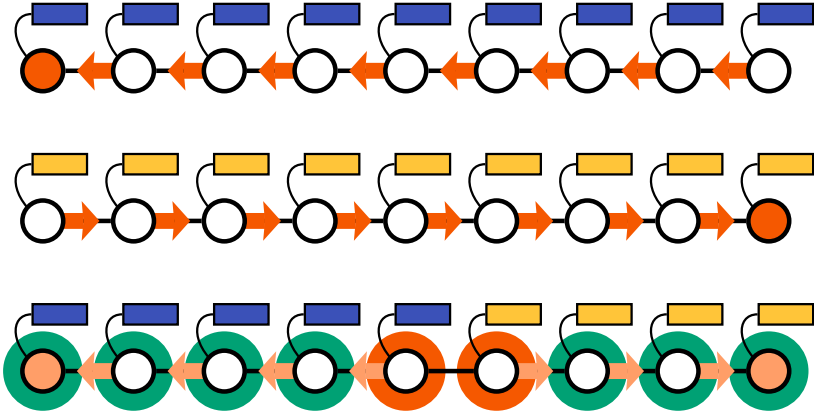
Spanning tree with pointers



Spanning tree with pointers



Spanning tree with pointers



Spanning tree with pointers

Spanning tree with pointers
has no error-sensitive PLS,
(for any certificates size)

Structural characterization

Theorem

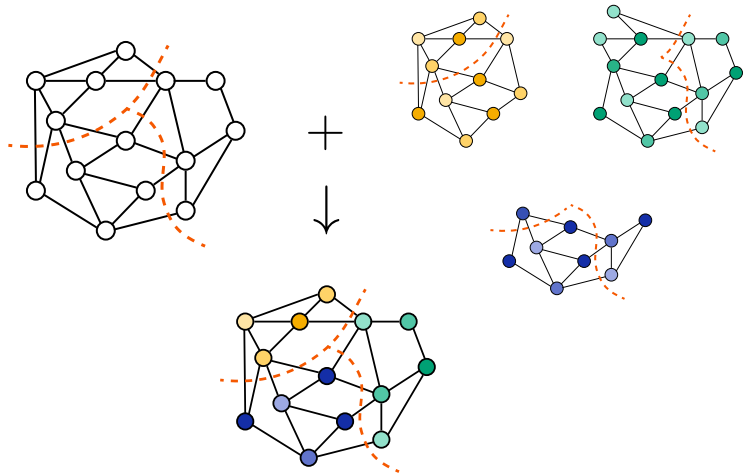
\mathcal{L} admits an error-sensitive PLS



\mathcal{L} is **locally stable**

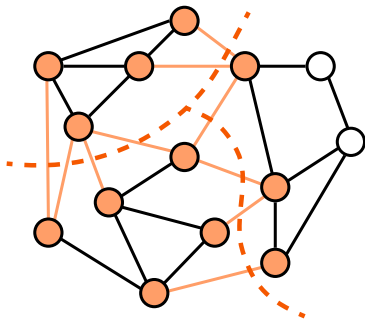
Local stability

Hybridization




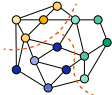
Local stability

Border nodes



Local stability

$\exists \beta, \forall G, \forall$ hybridization,

$$d(\text{graph}, \mathcal{L}) \leq \beta \cdot \#\{\text{graph}\}$$


$$d(\text{hybrid}, \mathcal{L}) \leq \beta \cdot \#\{\text{Border nodes}\}$$

Spanning tree with pointers

is not locally stable



But with adjacency lists...

Thm : Spanning tree and MST,
with *adjacency lists*,
are locally stable.

⇒ they have error-sensitive PLS.

Compact schemes

Compact PLS

(Old) Theorem :

- ▶ ST has a $O(\log n)$ -PLS ;
- ▶ MST has a $O(\log^2 n)$ -PLS.

Compact PLS

New Theorem :

- ▶ ST has a $O(\log n)$ -ESPLS ;
- ▶ MST has a $O(\log^2 n)$ -ESPLS.