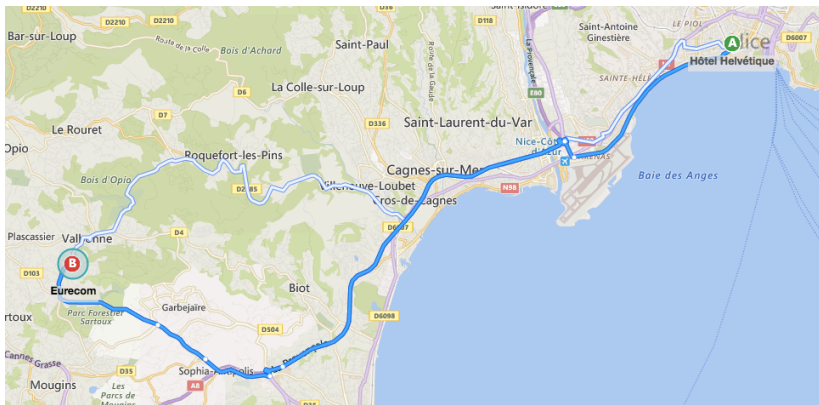


Hardness of exact distance queries in sparse graphs through hub labeling

Adrian Kosowski, Przemysław Uznański and Laurent Viennot

Inria - University of Wrocław - Irif (Paris Univ.)

Shortest-path oracle



What is the **shortest** path from A to B?

Distance oracle

What is the **distance** from A to B ?

Trade-off data-structure **size** vs query **time**.

Fastest oracles in **road networks** use **hub** labeling
[Abraham, Delling, Fiat, Goldberg, Werneck 2016]

Huge gap between lower and upper bounds for **sparse** graphs.

This talk : better understand why.

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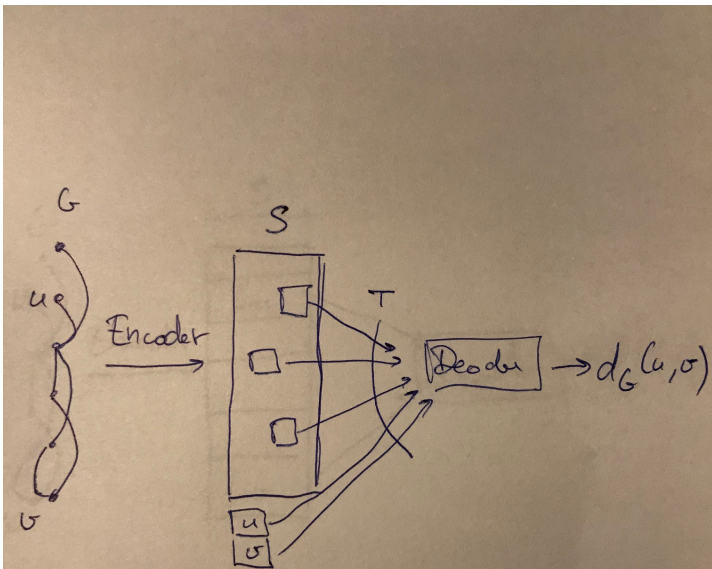
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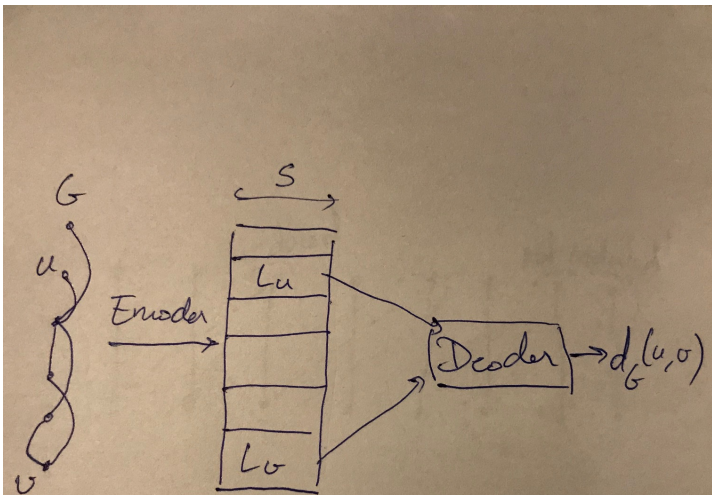
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Distance oracles



Distance labelings



Motivation : understand gaps for sparse graphs

Sparse : $m = O(n)$ or $\Delta = O(1)$

$$1/ \Omega(\sqrt{n}) \leq \text{DistLab}(n) \leq O\left(n \frac{\log \log n}{\log n}\right)$$

$$2/ \Omega\left(\frac{\sqrt{n}}{\log n}\right) \leq \text{HubLab}(n) \leq O\left(\frac{n}{\log n}\right)$$

[Gavoille, Peleg, Pérennes, Raz 2004]

[Alstrup, Dahlgaard, Bæk, Knudsen 2016]

[Gawrychowski, Kosovski, Uznański 2016]

Our results ($\Delta = O(1)$)

$$1/ \frac{1}{2^{O(\sqrt{\log n})}} \text{SumIndex}(n) \leq \text{DistLab}(n)$$

$$\Omega(\sqrt{n}) \leq \text{SumIndex}(n) \leq \tilde{O}\left(\frac{n}{2^{\sqrt{\log n}}}\right)$$

[Nisan, Wigderson 1993]

[Babai, Gal, Kimmel, Lokan 1995, 2003]

[Pudlak, Rodl, Sgall 1997]

$$2/ \frac{n}{2^{O(\sqrt{\log n})}} \leq \text{HubLab}(n) \leq O\left(\frac{n}{\text{RS}(n)^{1/7}}\right)$$

$$2^{\Omega \log^* n} \leq \text{RS}(n) \leq 2^{O(\sqrt{\log n})}$$

[Ruzsa, Szemerédi 1978]

[Behrand 1946] [Elkin 2010] [Fox 2011]

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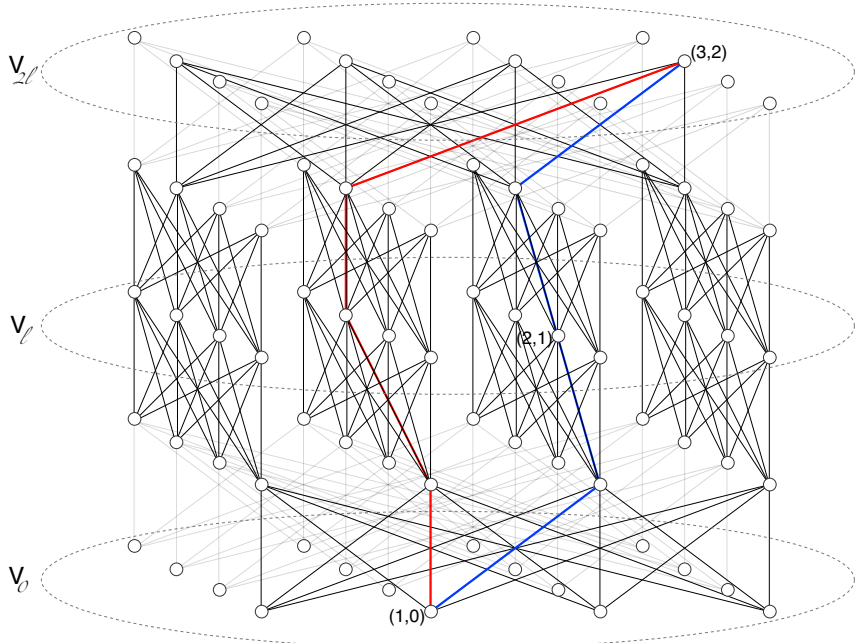
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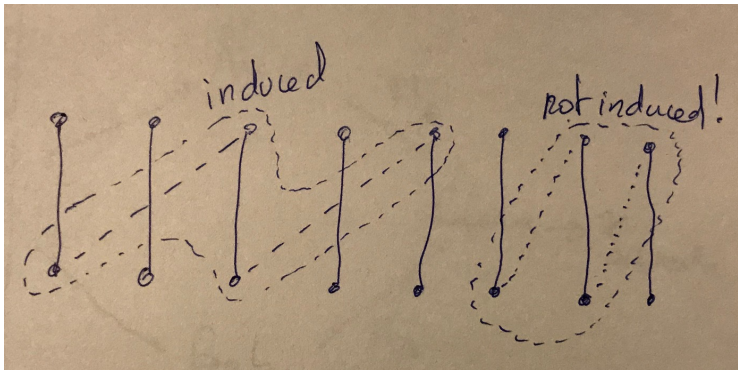
[Ruzsa, Szemerédi 1978]

[Behrand 1946] [Elkin 2010] [Fox 2011]

A hard instance : $2\ell + 1$ grids of dim. $\ell = \sqrt{\log n}$

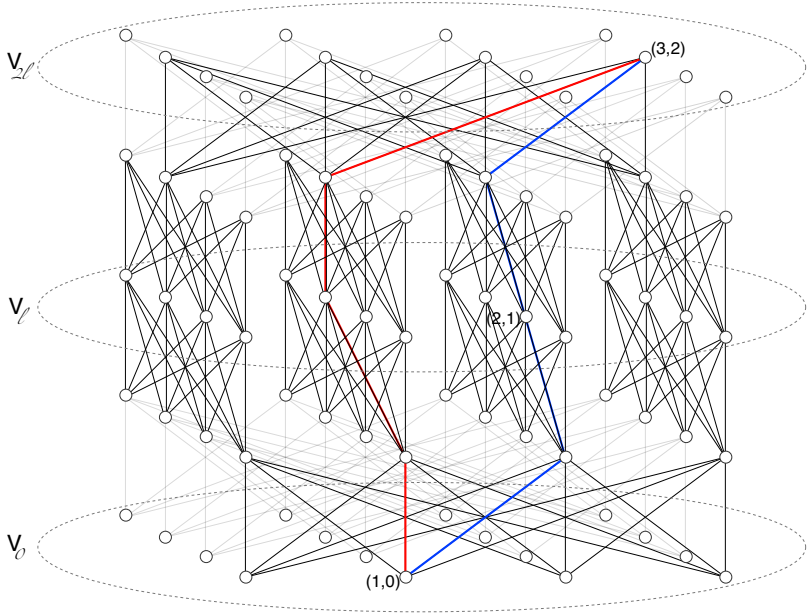


Connection with Ruzsa-Szemerédi



RS-graph : can be decomposed into n induced matchings.

$\frac{n^2}{RS(n)}$ is the maximum number of edges in an RS-graph.



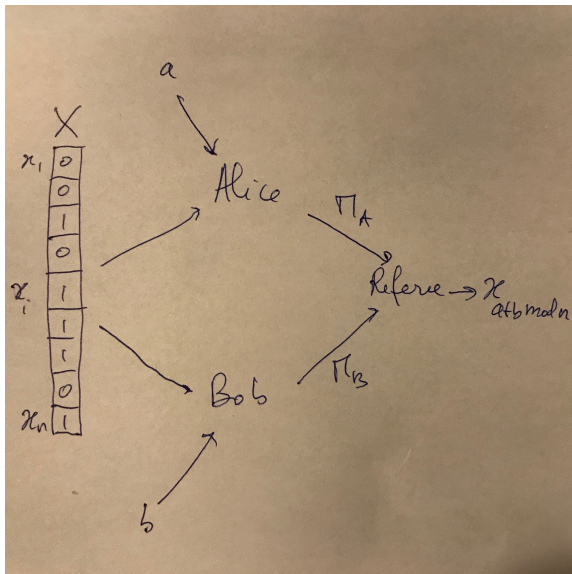
$$G_Y^D = \{x_0 z_{2l} \mid y = \frac{x+z}{2} \text{ and } d_G(x, z) = D\} \quad \exists D \text{ s.t. } |\cup_Y G_Y^D| \geq \frac{n^2}{2^{O(\sqrt{\log n})}}$$

Converse

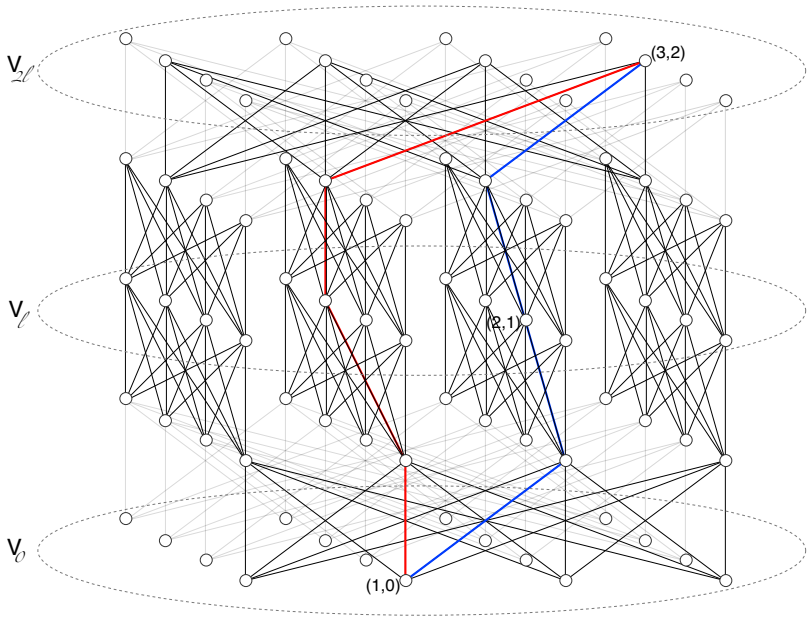
Any cst. deg. graph G has **hub sets** of av. size $O\left(\frac{n}{RS(n)^{1/7}}\right)$.

Idea : use a **vertex cover** of each G_y^D ($VC \leq 2MM$).

Connection with SumIndex



$$\text{SumIndex}(n) = \min_{\text{Encoder}} \max_X |M_A| + |M_B|$$



$G_X = G \setminus \{y_\ell \mid X_y = 0\}$, send $x = 2a, L_{x_0}, z = 2b, L_{z_{2\ell}}$, check $d(x_0, z_{2\ell})$.

Thanks