# Silent Self-Stabilizing Scheme for Spanning-Tree-like Constructions

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Meeting DESCARTES, March 28 2018, Paris





# Self-Stabilization

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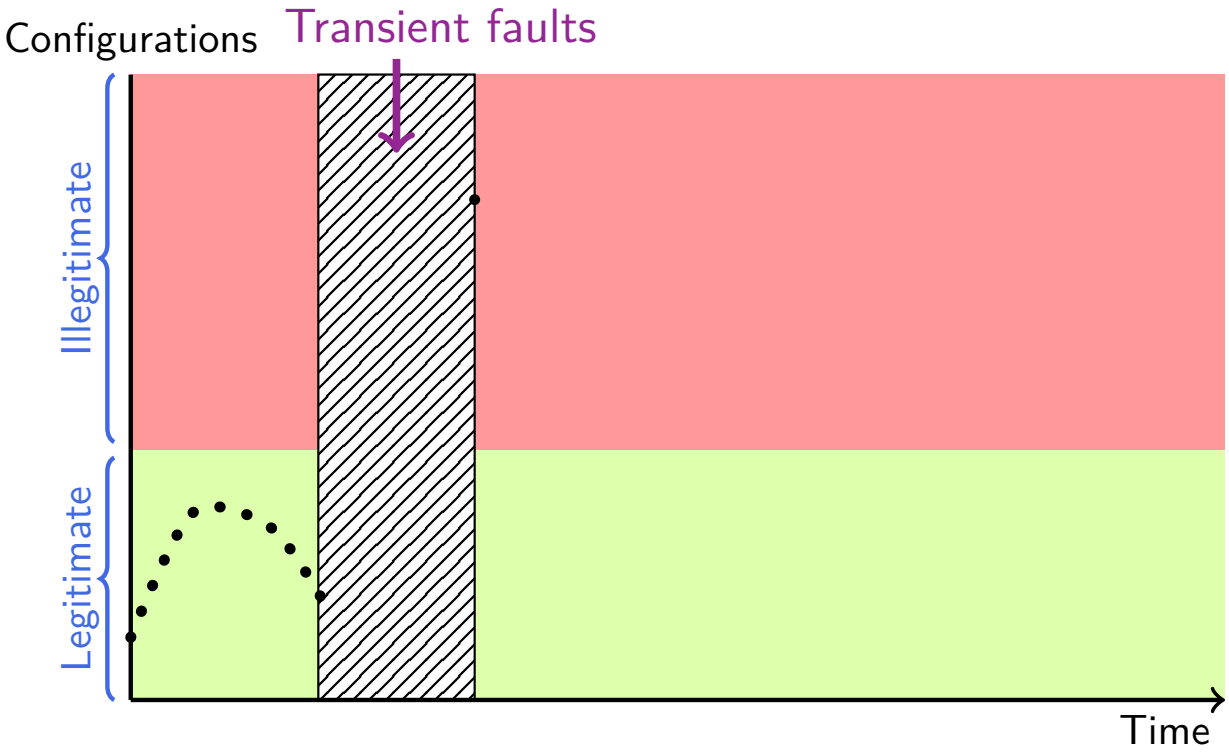
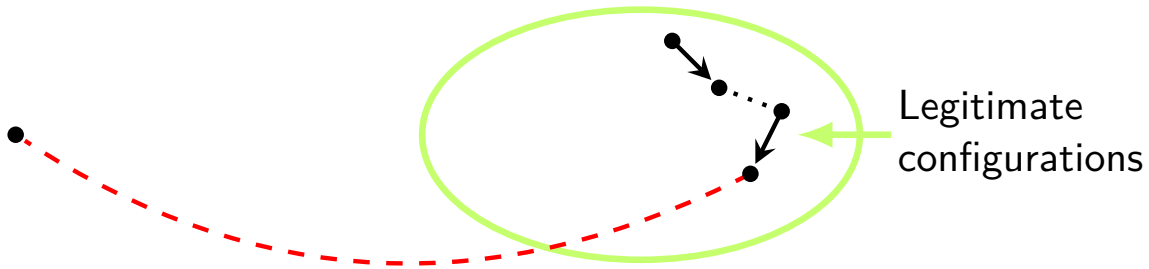
## [Dijkstra, ACM Com., 74]



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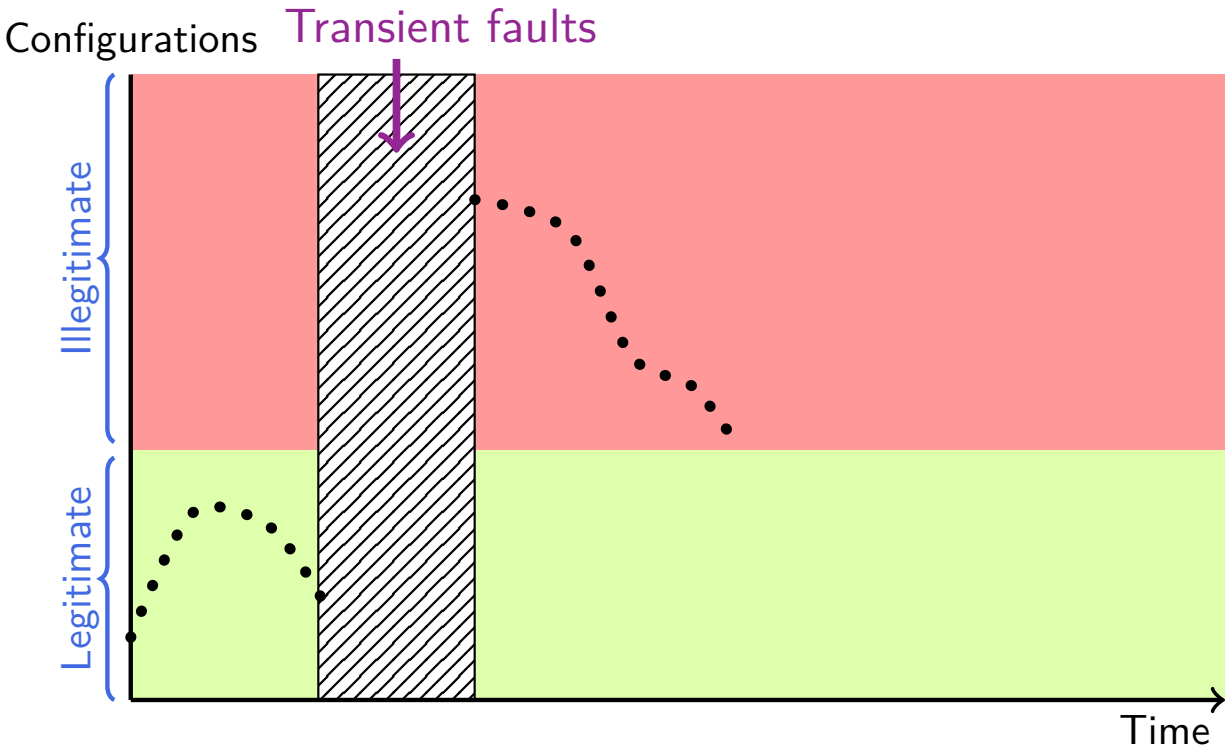


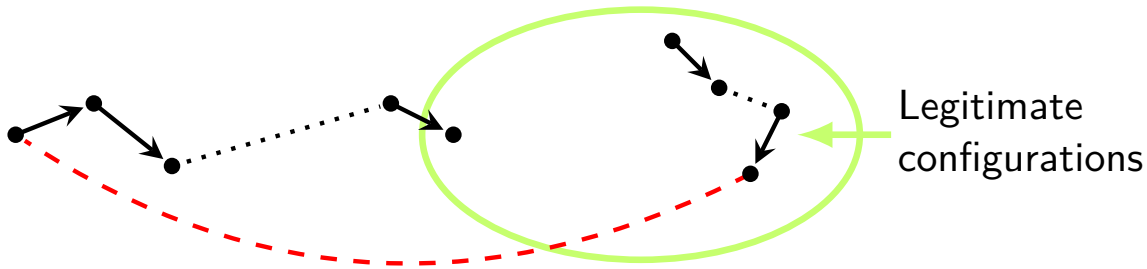

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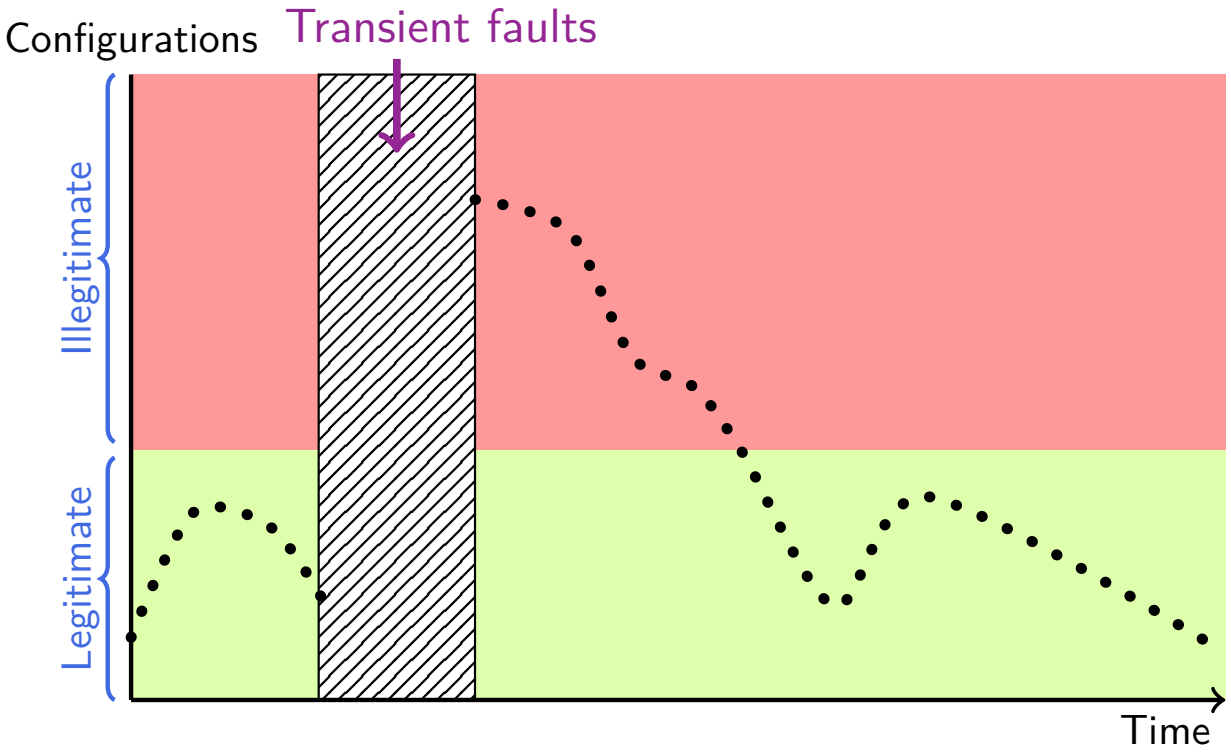


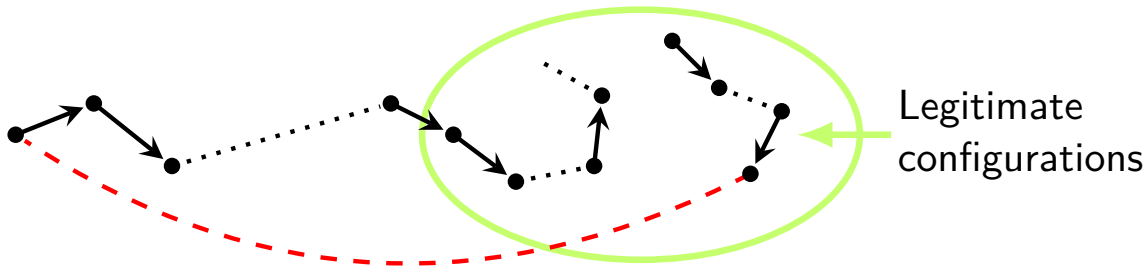

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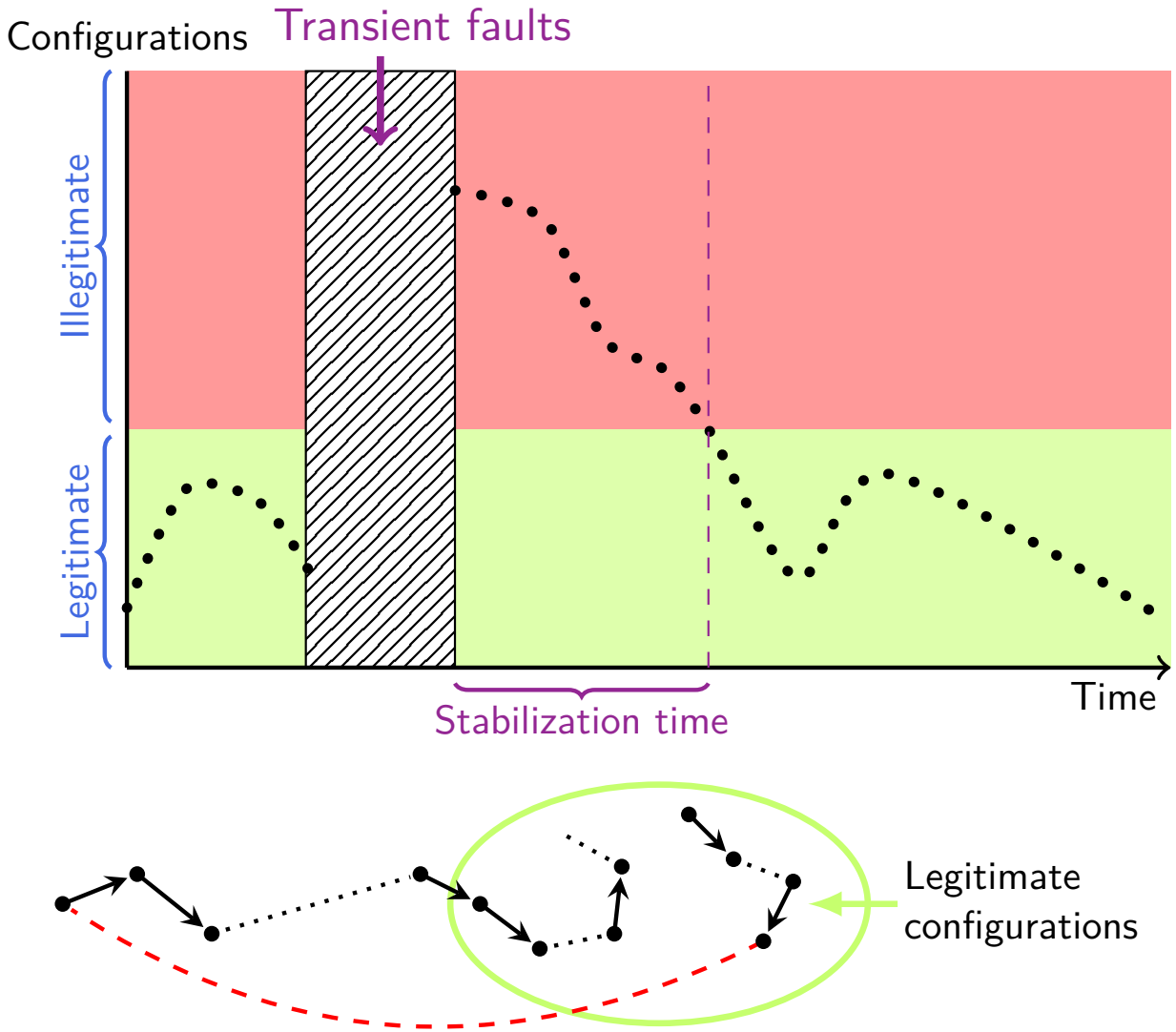
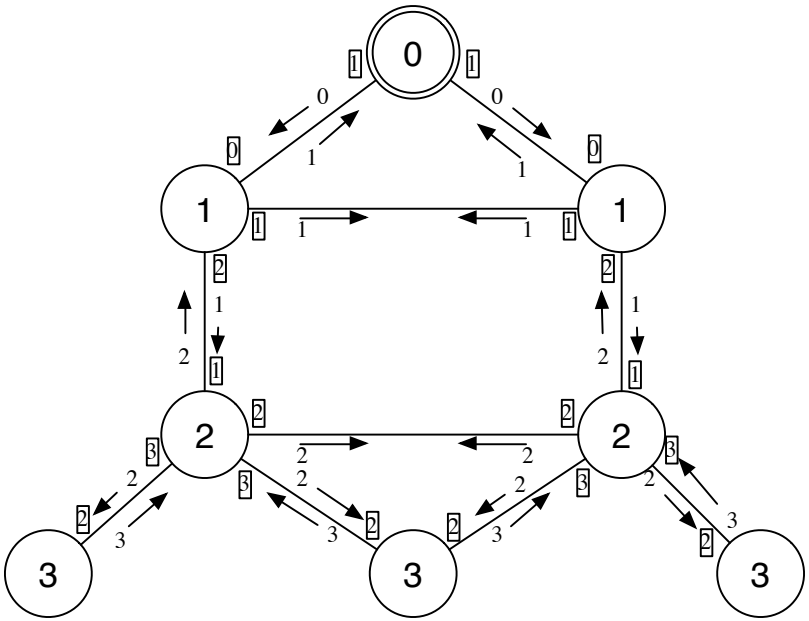


# Silent Algorithm

## [Dolev *et al*, Acta Informatica, 96]

A silent self-stabilizing algorithm converges within finite time to a configuration from which the values of the registers used by the algorithm remain fixed.



# Silent Algorithm

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A silent self-stabilizing algorithm converges within finite time to a configuration from which the values of the registers used by the algorithm remain fixed.

Advantages:

- Silence implies **more simplicity** in the algorithm design (classically used in compositions).
- A silent algorithm may utilize **less communication operations and communication bandwidth**.
- Well-suited to compute **distributed data structures** such as spanning trees.

# Model

# Locally Shared Memory Model with Composite Atomicity

Abstraction of the message-passing model

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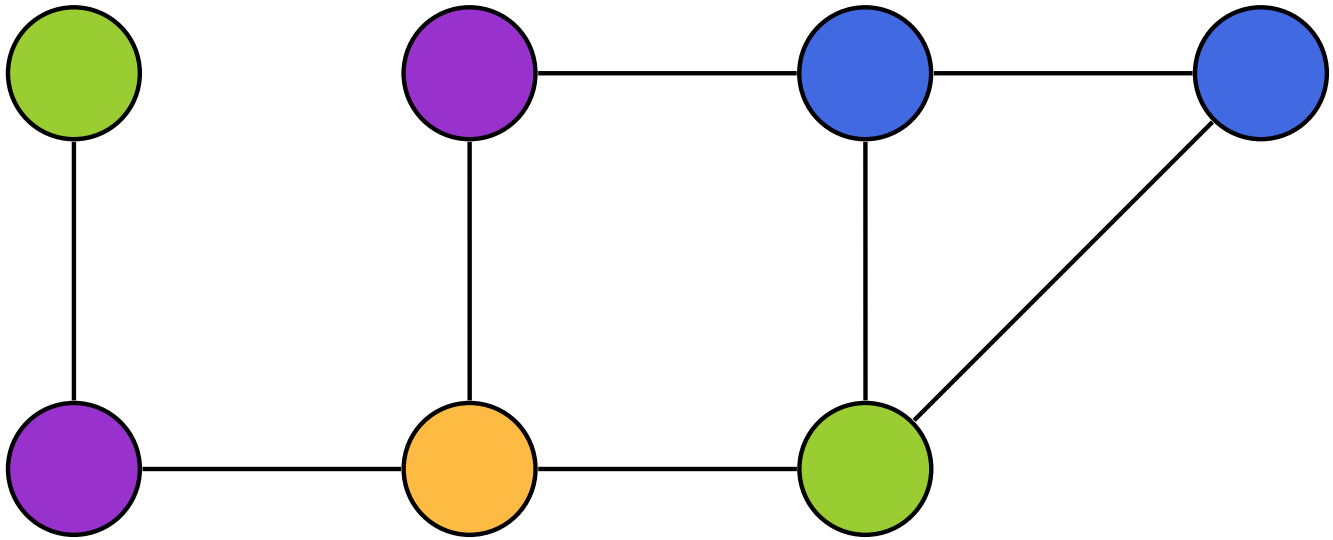
Abstraction of the message-passing model

Locally shared registers (variables) instead of communication links

**A process can only read its variables and that of its neighbors.**

# Locally Shared Memory Model with Composite Atomicity

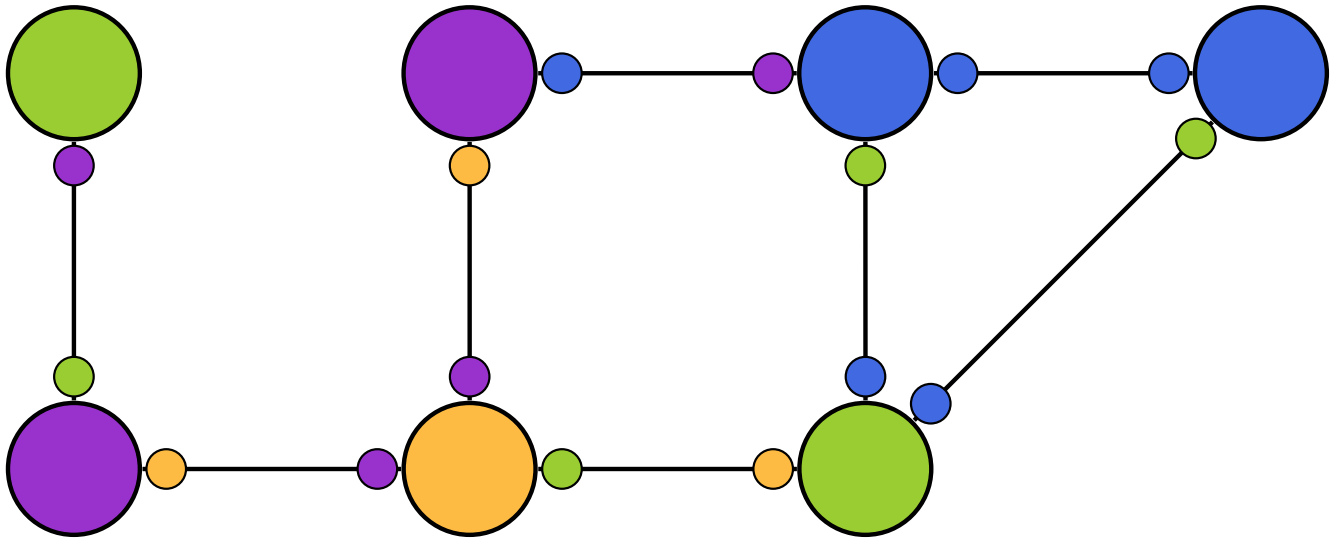
## Configuration



# Locally Shared Memory Model with Composite Atomicity

## Atomic Step

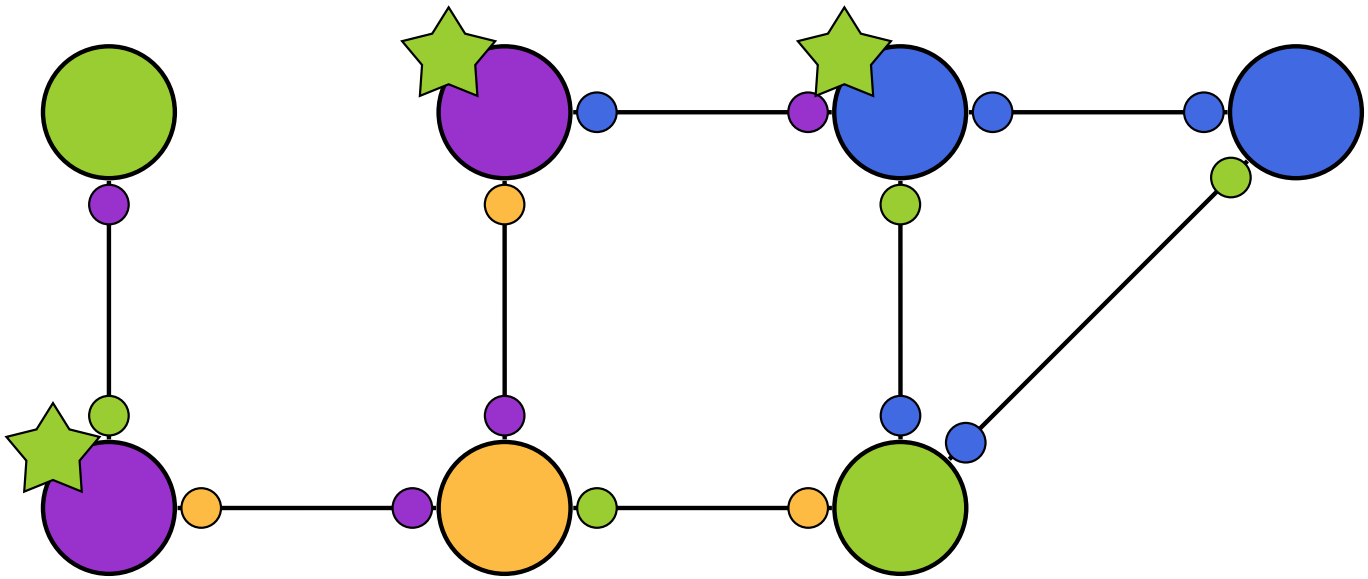
- Reading of the variables of the neighbors



# Locally Shared Memory Model with Composite Atomicity

## Atomic Step

- Reading of the variables of the neighbors
- Enabled nodes



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## Atomic Step

- Reading of the variables of the neighbors
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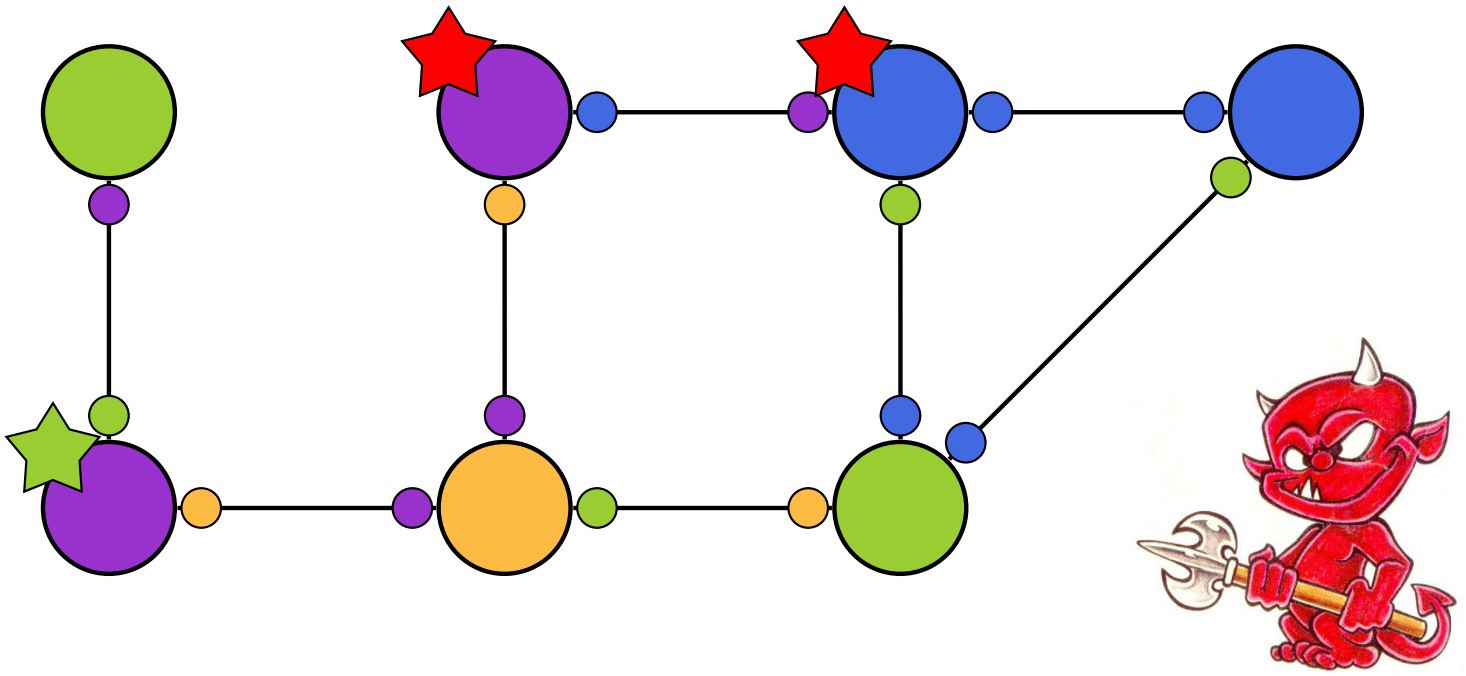
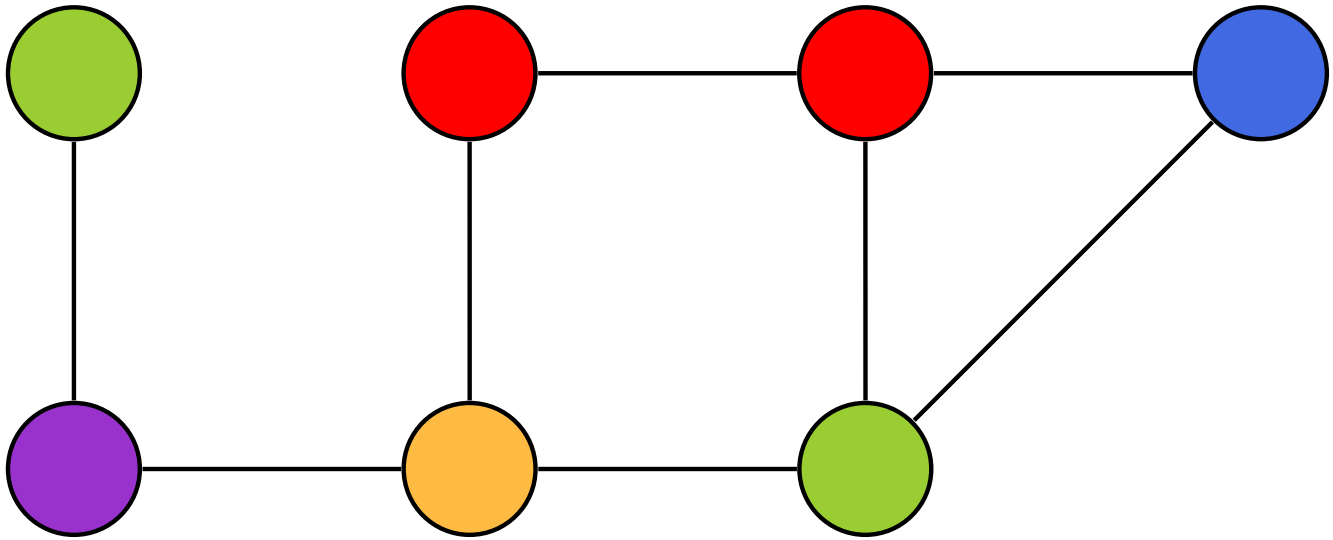
- Daemon election: models the asynchronism



# Locally Shared Memory Model with Composite Atomicity

## Atomic Step

- Reading of the variables of the neighbors
- Enabled nodes
- Daemon election: models the asynchronism
- Update of the local states



# Daemons

- Synchronous
- Central / Distributed
- Fairness : Strongly Fair, Weakly Fair, Unfair

Distributed unfair daemon: no constraint, except progress!

# Complexity

## Space

Memory requirement in bits.

## Time

(mainly stabilization time)

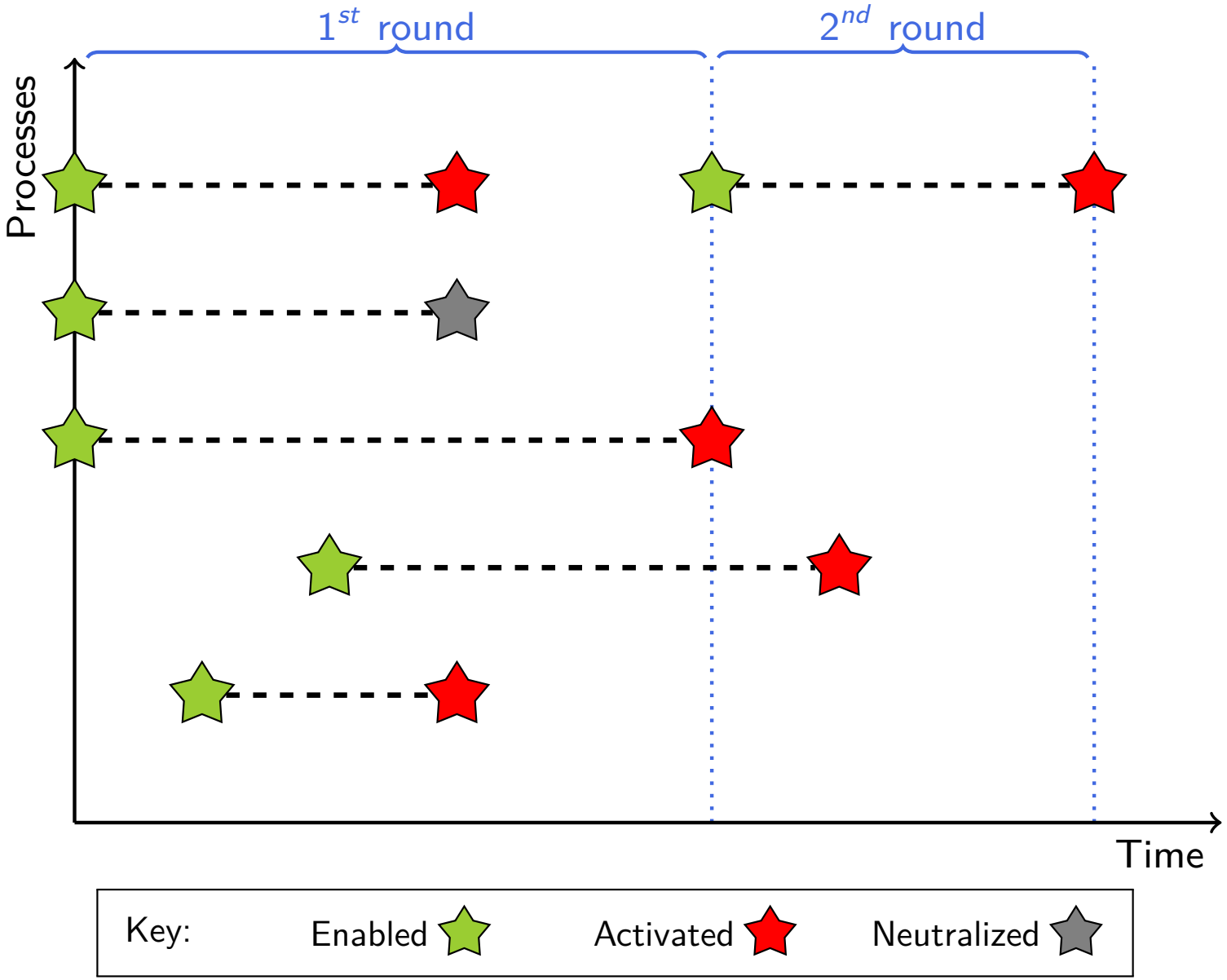
Rounds: execution time **according to the slowest process**.

Essentially similar to the notion of (asynchronous) rounds in message-passing models.

Moves: local state updates.

Rather unusual.

# Rounds


1st round
2nd round
Processes
Time
Key:
Enabled
Activated
Neutralized

# Complexity in moves: "a measure of energy"

**The stabilization time in moves**

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This means that there are processes whose moves do not make the system progress in the convergence: **these processes waste computation power and so energy.**

# Complexity in moves: unusual

Several ***a posteriori*** analyses show that (classical) self-stabilizing algorithms that work under a distributed unfair daemon have an exponential stabilization time in moves in the worst case.

- BFS spanning tree construction of Huang and Chen **[Devismes and Johnen, JPDC 2016]**
- Leader election of Datta, Larmore, Vemula **[Durand *et al*, Inf. & Comp. 2017]**
- ...

# General Schemes for Self-Stabilization

# Related Work

- The general transformer of **[Katz & Perry, Dist. Comp. 93]**: not efficient, the purpose is only to demonstrate the feasability of the transformation (characterization).

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- Proof labeling scheme **[Korman *et al*, Dist. Comp. 2010]**: restricted class of self-stabilizing algorithms (silent algorithms), stabilization time linear in $n$. No move complexity analysis.
- **[Devismes *et al*, TAAS 2009]**: restricted class of self-stabilizing algorithms (wave algorithms), stabilization time linear in $n$ and polynomial in moves.

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Here, we restrict our study to silent spanning-tree-like data structure (*i.e.*, trees or forests).

# Our contribution

# Algorithm Scheme: a general scheme to compute spanning-tree-like data structures

## Theorem 1

Scheme *is silent and self-stabilizing under the distributed unfair daemon in any bidirectional weighted networks of arbitrary topology.*[*]

[*] *n.b.*, the topologies are not necessarily connected. Disconnection may be due to a transient fault.

# Algorithm Scheme: a general scheme to compute spanning-tree-like data structures

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*The stabilization time in rounds of* Scheme *is at most* $4n_{maxCC}$, *where* $n_{maxCC}$ *is the maximum number of processes in a connected component.*

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## Theorem 3

*When all weights are strictly positive integers bounded by* $W_{\max}$, *the stabilization time of* Scheme *in moves is at most* $(W_{\max}(n_{maxCC}-1)^2+5)(n_{maxCC}+1)n$.

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# Results on particular instances of Algorithm Scheme

- In an identified network, leader election in each connected component (+ a spanning tree rooted at each leader): $O(n_{\texttt{maxCC}}{}^2 n)$ **moves**
  $\approx$ the best known move complexity **[Durand *et al*, Inf & Comp 2017]**

[†] Every process in a connected component that does not contain the root should eventually take a special state notifying that it detects the absence of a root.

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- Given an input, spanning forest with non-rooted components detection[†]: $O(n_{\texttt{maxCC}} n)$ **moves**
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- In assuming a rooted network, shortest-path spanning tree with non-rooted components detection: $O(\texttt{W}_{\max} n_{\texttt{maxCC}}{}^3 n)$ **moves** ($\texttt{W}_{\max}$ is the maximum weight of an edge)
  $\approx$ the best known move complexity **[Devismes *et al*, OPODIS 2016]**

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# The problem

# Inputs (constants)

$canBeRoot_u$: true if $u$ is candidate to be root.

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In a terminal configuration, every tree root satisfies *canBeRoot*, but the converse is not necessarily true.

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For every connected component $GC$, if there is at least one candidate $u \in GC$, then at least one process of $GC$ should be a tree root in a terminal configuration.

# Inputs (constants)

$canBeRoot_u$: true if $u$ is candidate to be root.

If there is no candidate in a connected component, all processes of the component should converge to a particular terminal state notifying that it detects the absence of candidate.
**(non-rooted components detection)**

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$pname_u \in IDs$, where $IDs = \mathbb{N} \cup \{\perp\}$ is totally ordered by $<$ and $\min_<(IDs) = \perp$.

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$pname_u$: the name of $u$.

Two considered cases:

- $\forall v \in V, pname_v = \perp$.
- $\forall u, v \in V, pname_u \neq \perp \wedge (u \neq v \Rightarrow pname_u \neq pname_v)$

# Weights

- $\omega_u(v) \in \mathit{DistSet}$ denotes the weight of the arc $(u, v)$
- $(\mathit{DistSet}, \oplus, \prec)$ is an ordered magma:
  - $\oplus$ is a closed binary operation on $\mathit{DistSet}$
  - $\prec$ is a total order on this set
  - $\forall (u, v), \forall d \in \mathit{DistSet},\ d \prec d \oplus \omega_u(v)$
- $\mathit{distRoot}(u)$: the distance value of $u$ is $u$ is a root
- $\mathit{P\_nodeImp}(u)$ is a local predicate which is true is $u$ should move to improve the solution

# Variables

$st_u \in \{I, C, EB, EF\}$

$parent_u \in \{\perp\} \cup Lbl$: parent in the tree

$d_u \in DistSet$: distance to the root

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Normal behavior

$I$ : *Isolated*

$C$ : *Correct* (belong to a tree)

$parent_u \in \{\perp\} \cup Lbl$: parent in the tree

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# Variables

$st_u \in \{I, C, EB, EF\}$

In a terminal configuration, if $V_u$ contains a candidate, then $st_u = C$, otherwise $st_u = I$.

$parent_u \in \{\perp\} \cup Lbl$: parent in the tree

$d_u \in DistSet$: distance to the root

# Variables

$st_u \in \{I, C, EB, EF\}$

Correction mechanism

$EB$: Error Broadcast

$EF$: Error Feedback

$parent_u \in \{\perp\} \cup Lbl$: parent in the tree

$d_u \in DistSet$: distance to the root

# Instances

# Leader Election

Inputs:

- $canBeRoot_u$ is true for any process,
- $pname_u$ is the identifier of $u$ ($n.b.$, $pname_u \in \mathbb{N}$)
- $\omega_u(v) = (\perp, 1)$ for every $v \in \Gamma(u)$

Ordered Magma:

- $DistSet = IDs \times \mathbb{N}$
  for every $d = (a, b) \in DistSet$, we let $d.id = a$ and $d.h = b$.
- $(id1, i1) \oplus (id2, i2) = (id1, i1 + i2)$;
- $(id1, i1) \prec (id2, i2) \equiv$
  $(id1 < id2) \vee [(id1 = id2) \wedge (i1 < i2)]$
- $distRoot(u) = (pname_u, 0)$

Predicate:

- $P\_nodeImp(u) \equiv ((\exists v \in \Gamma(u) \mid st_v = C \wedge$
  $d_v.id < d_u.id)) \vee (canBeRoot_u \wedge distRoot(u) \prec d_u)$

# Shortest-Path Spanning Tree

Inputs:

- $canBeRoot_u$ is false for any process except for $u = r$,
- $pname_u$ is $\perp$, and
- $\omega_u(v) = \omega_v(u) \in \mathbb{N}^*$, for every $v \in \Gamma(u)$.

Ordered Magma:

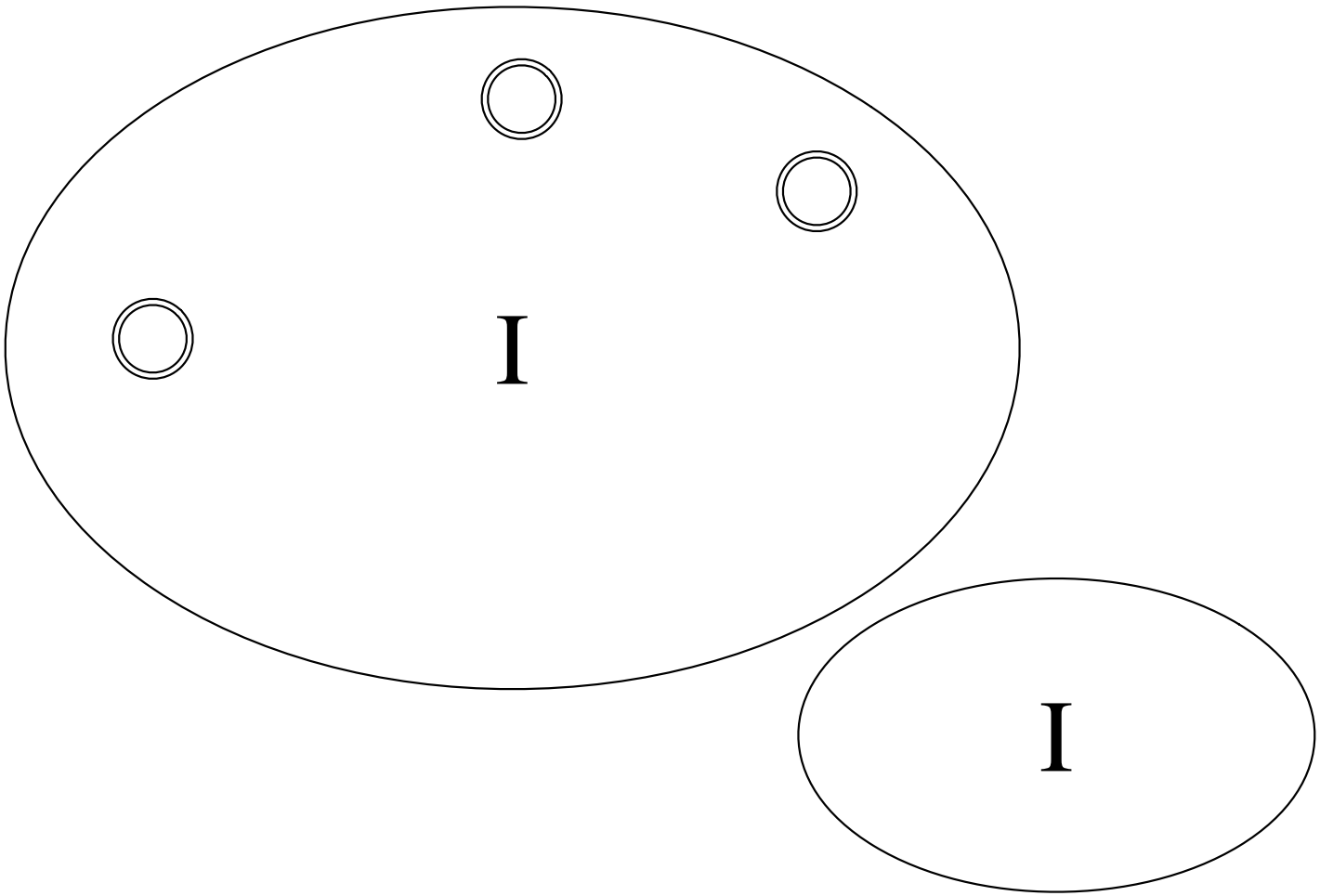
- $DistSet = \mathbb{N}$,
- $i1 \oplus i2 = i1 + i2$,
- $i1 \prec i2 \equiv i1 < i2$, and
- $distRoot(u) = 0$.

Predicate:

- $P\_nodeImp(u) \equiv$
  $(\exists v \in \Gamma(u) \mid st_v = C \wedge d_v \oplus \omega_u(v) \prec d_u)$
  $\vee$
  $canBeRoot_u \wedge distRoot(u) \prec d_u$

# Our solution in a nutshell

# Typical Execution



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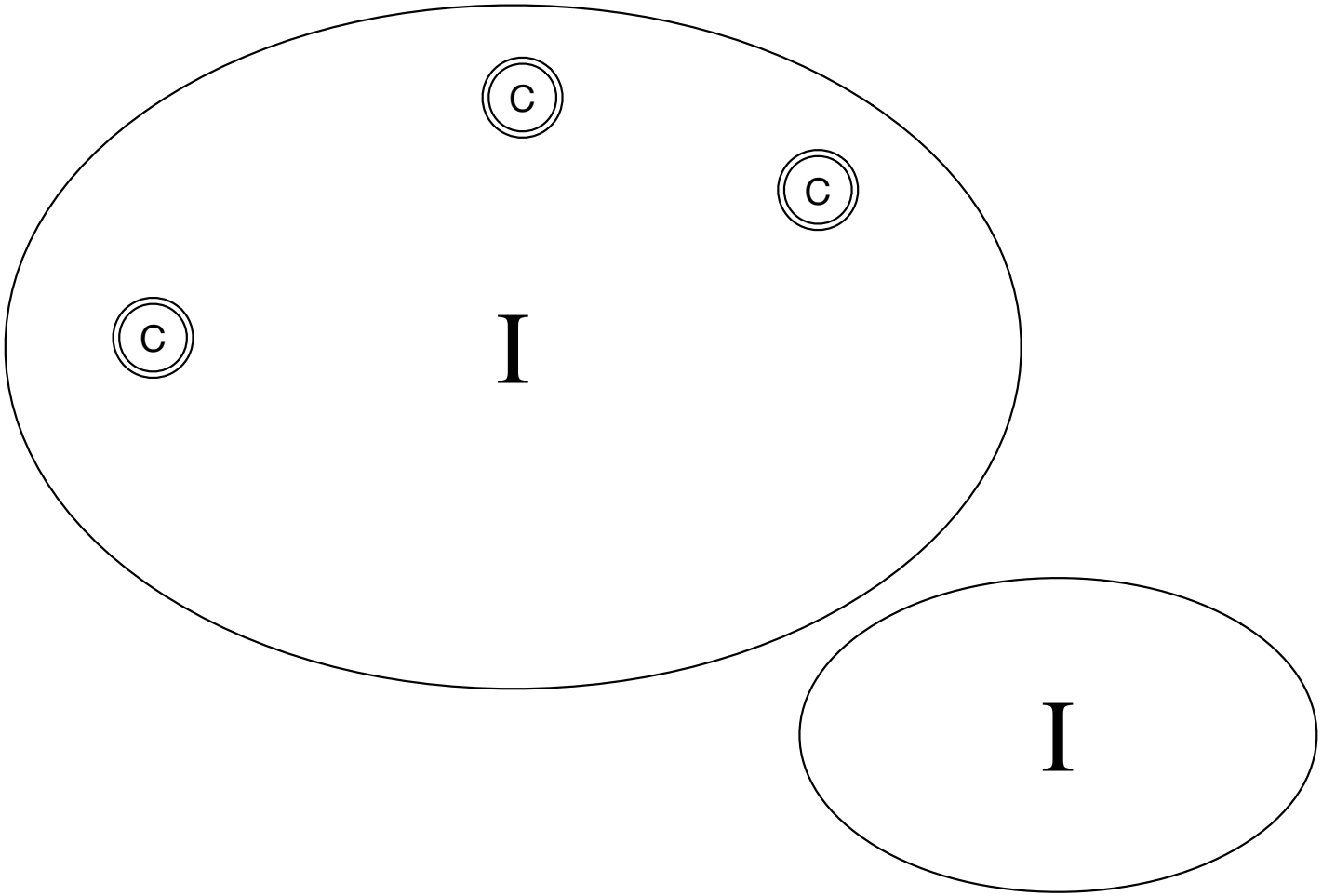


Any candidate $u$ executes $\mathbf{R_R}$: $st_u \leftarrow C$, $d_u \leftarrow distRoot(u)$, $parent_u \leftarrow \perp$

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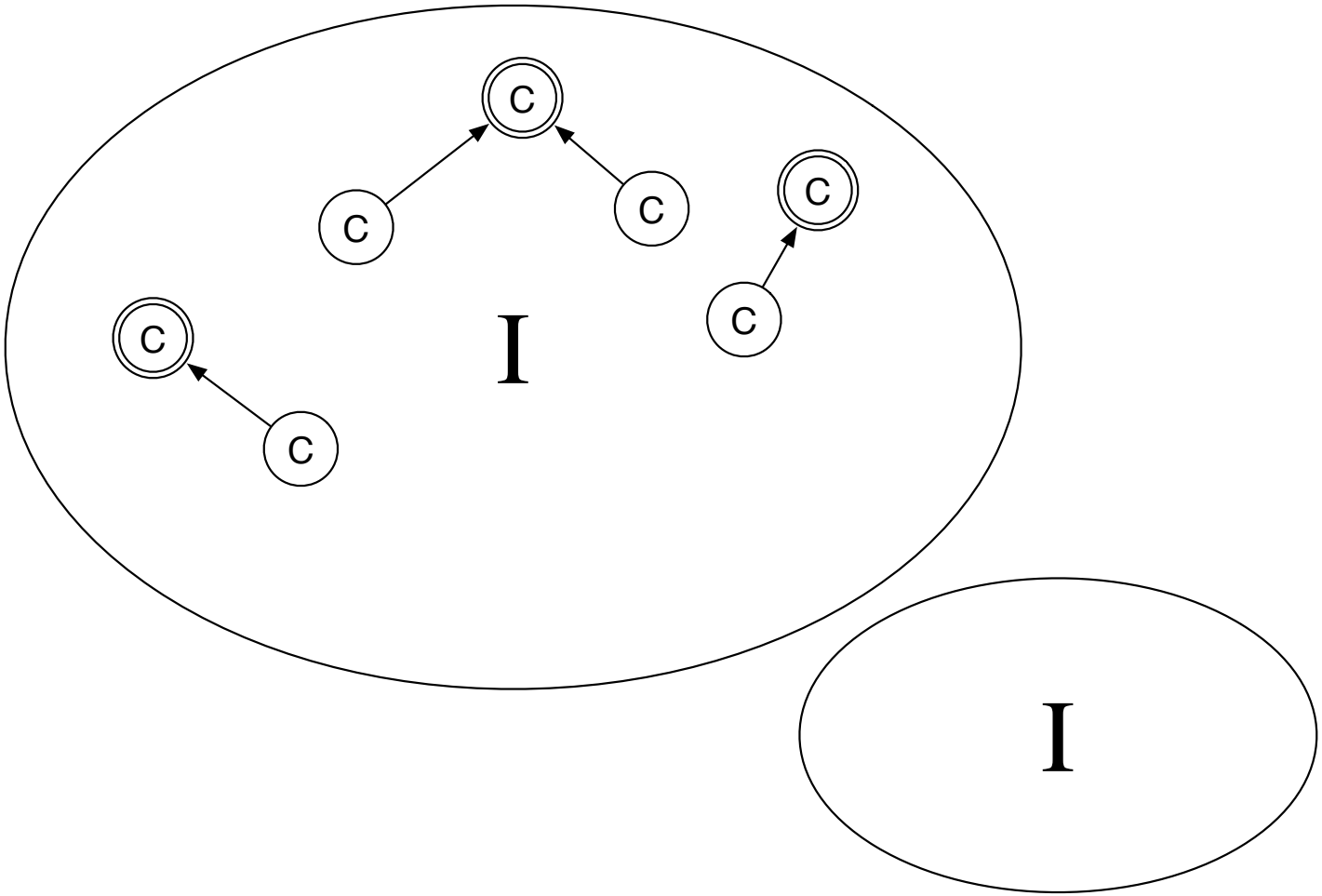


Any non-candidate $v$ executes $\mathbf{R_R}$ when it finds a neighbor with status $C$: $st_u \leftarrow C$, select a parent, and $d_v \oplus \omega_u(v)$ if it chooses $v$ as a parent

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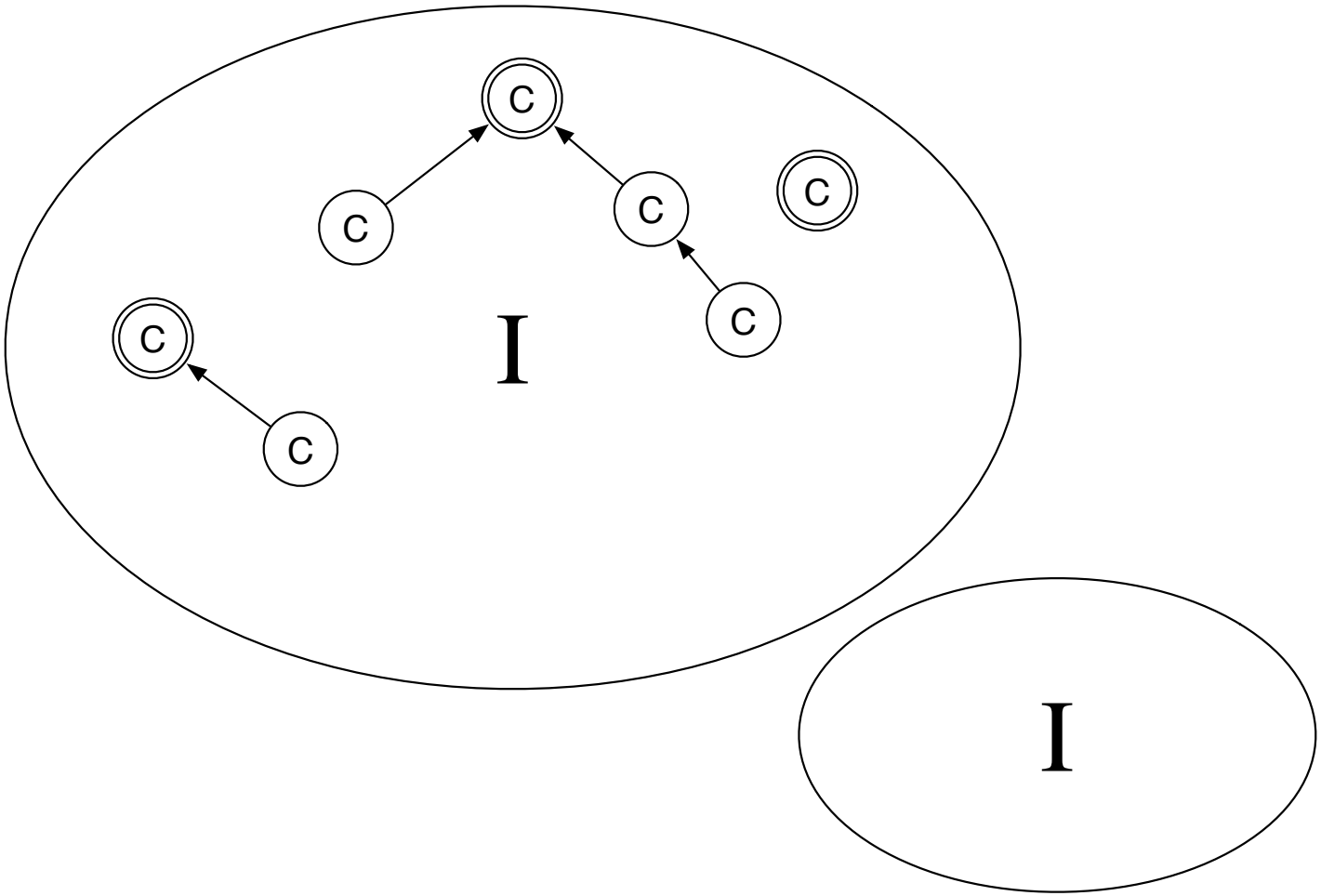
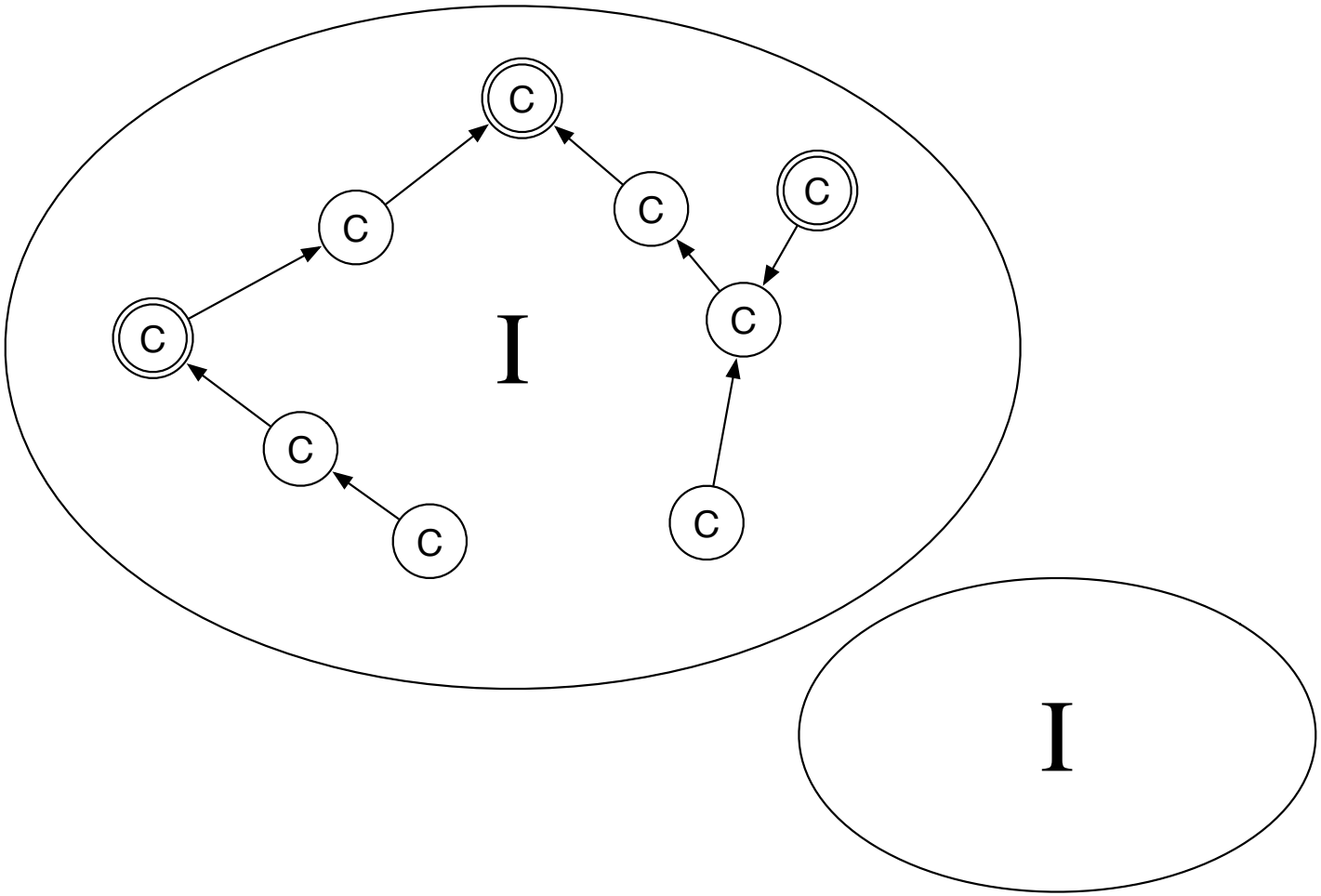



In parallel, rules $\mathbf{R_U}$ are executed to reduce the weight of the trees: when a process $u$ with status $C$ satisfies $P\_nodeImp(u)$, this means that $u$ can reduce $d_u$ by selecting another neighbor with status $C$ as parent.

# Typical Execution




**A candidate can lose its tree root condition** using $\mathbf{R_R}$, if it finds a sufficiently good parent in its neighborhood.

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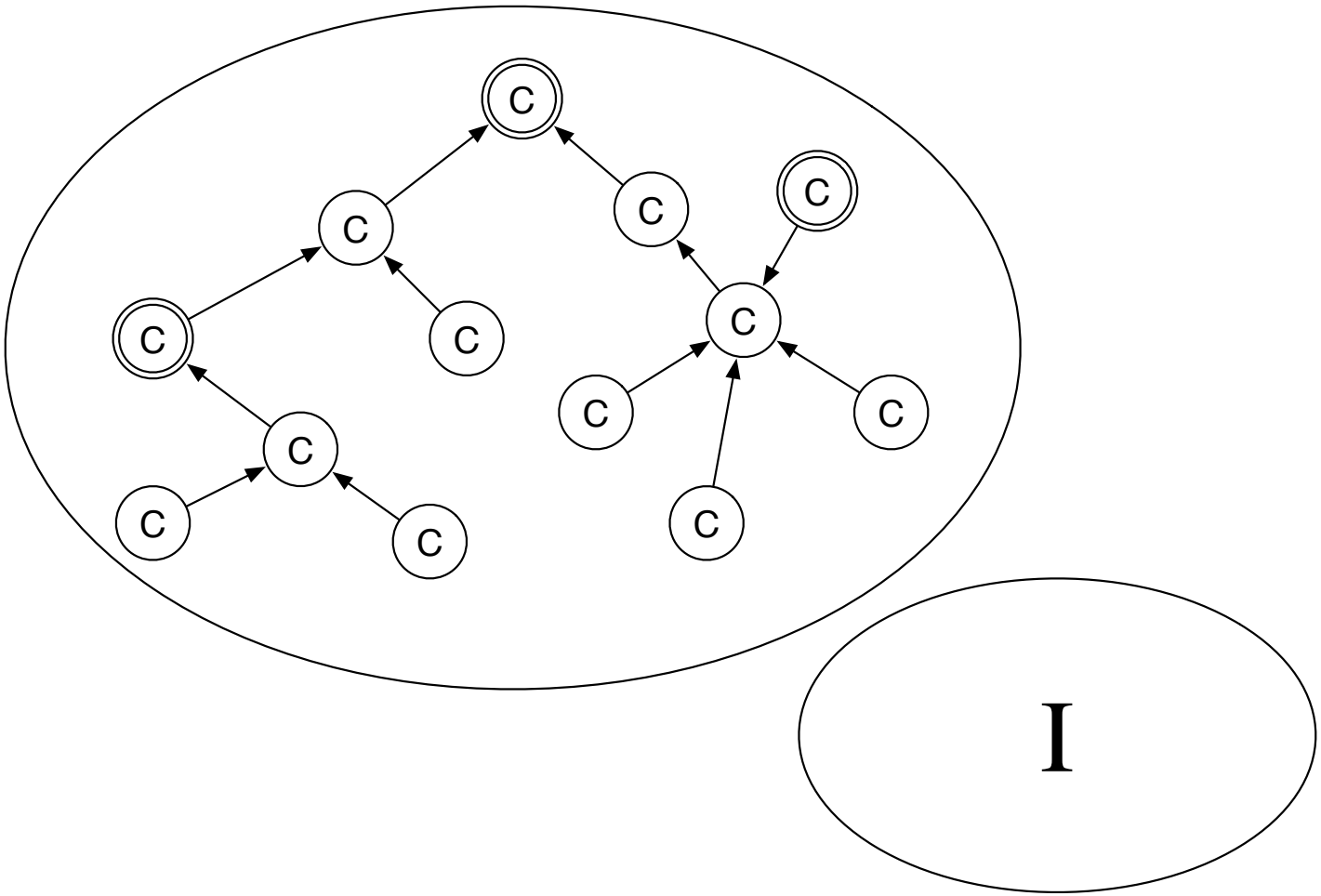


Overall, within at most $n_{\texttt{maxCC}}$ rounds, a terminal configuration is reached.

# Abnormal Roots

Inconsistencies are detected by some processes called Abnormal Roots

A process $u$ is an abnormal root if $u$ is not a normal root[§], $st_u \neq I$, and satisfies one of the following four conditions:

- its parent pointer does not designate a neighbor,
- its distance $d_u$ is inconsistent with the distance of its parent, or
- its status is inconsistent with the status of its parent.

[§]A normal root is any process $v$ such that $canBeRoot_v \wedge st_v = C \wedge parent_v = \perp \wedge d_v = distRoot(v)$.

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An abnormal root $u$ is alive if $st_u \neq EF$

An abnormal tree is a tree root at an abnormal root.

An abnormal tree is alive if it contains a node $v$ such that $st_v \neq EF$

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An abnormal tree is alive if it contains a node $v$ such that $st_v \neq EF$

**Main result:** No abnormal alive root (resp. tree) is created during the execution.

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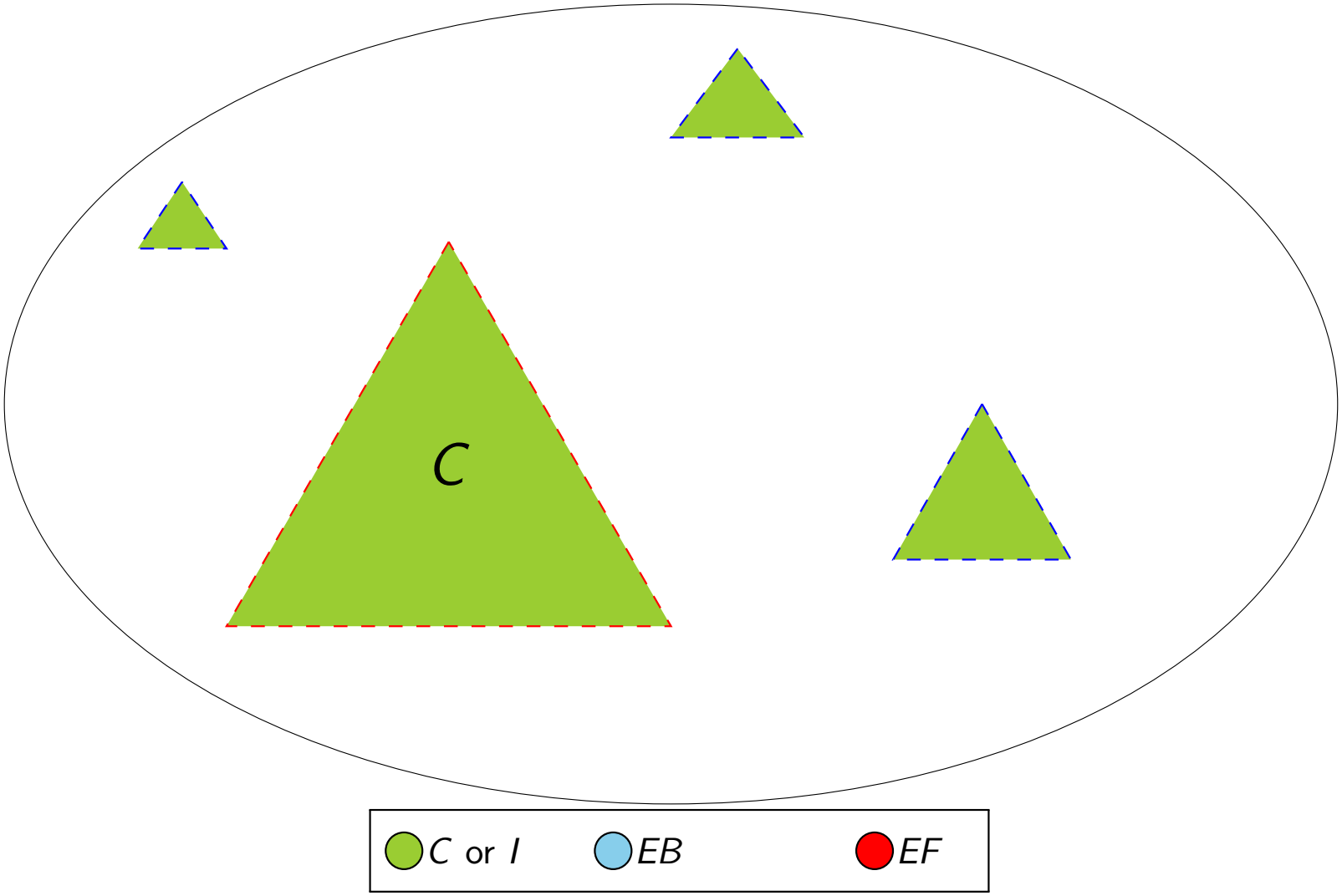
# Abnormal trees removal

Freeze before Remove

Variable $st_u \in \{I, C, EB, EF\}$

- $I$ means *Isolated*
  - a process of status $I$ can join a tree
- $C$ means *correct*
  - only processes of status $C$ in a tree can modify their parent pointers and
  - only by choosing a neighbor of status $C$ as parent
- $EB$: Error Broadcast
- $EF$: Error Feedback

# Freeze before remove



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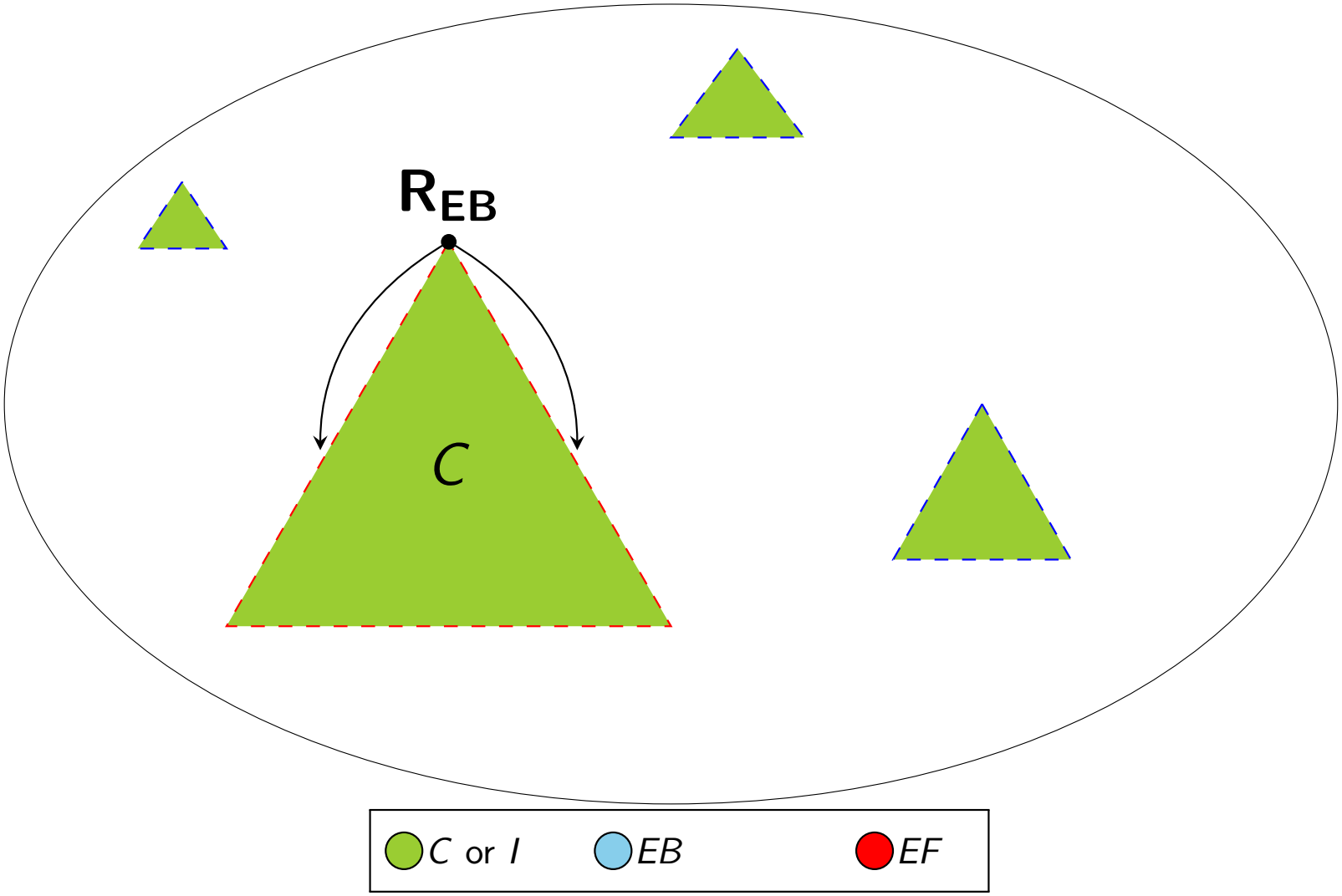

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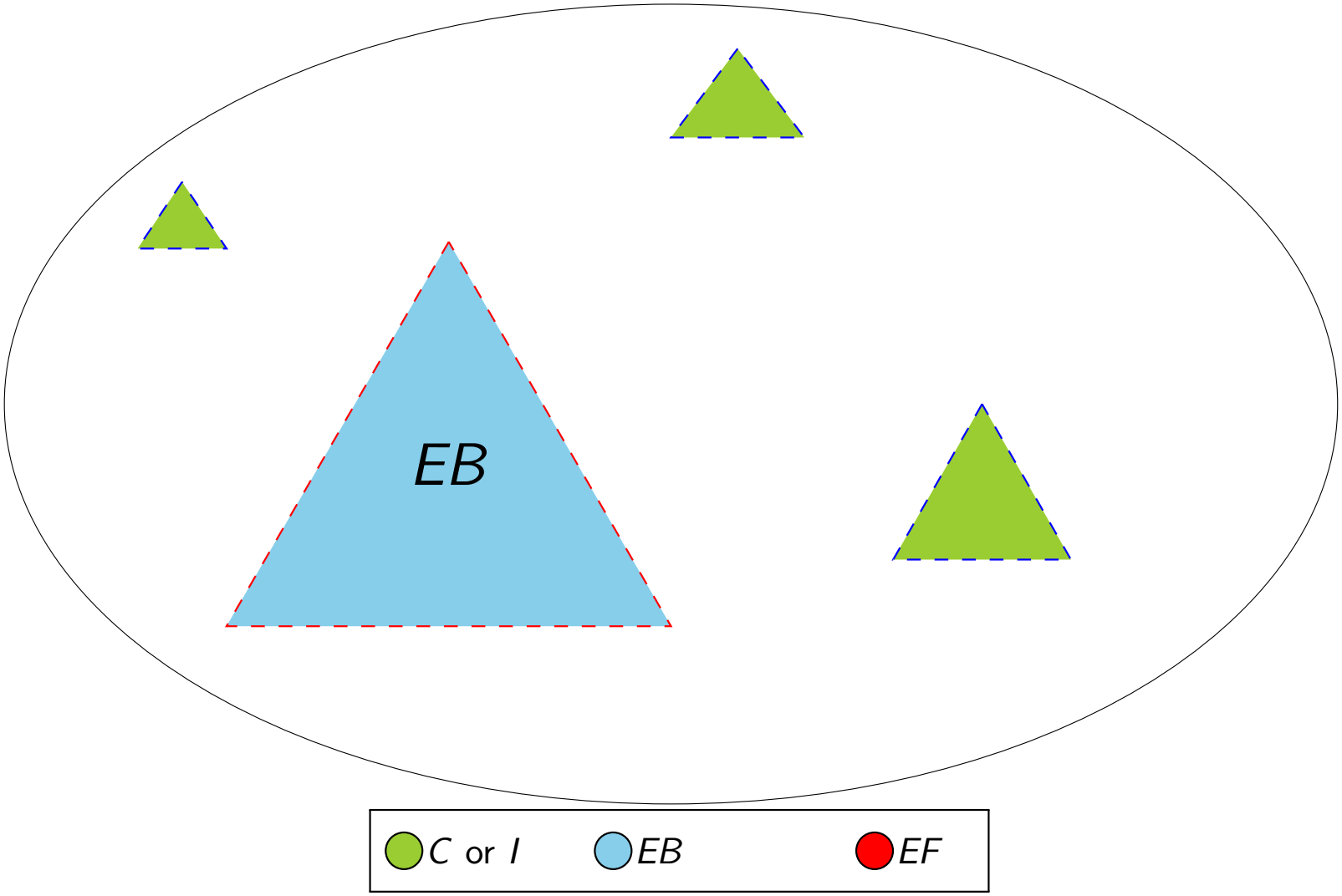

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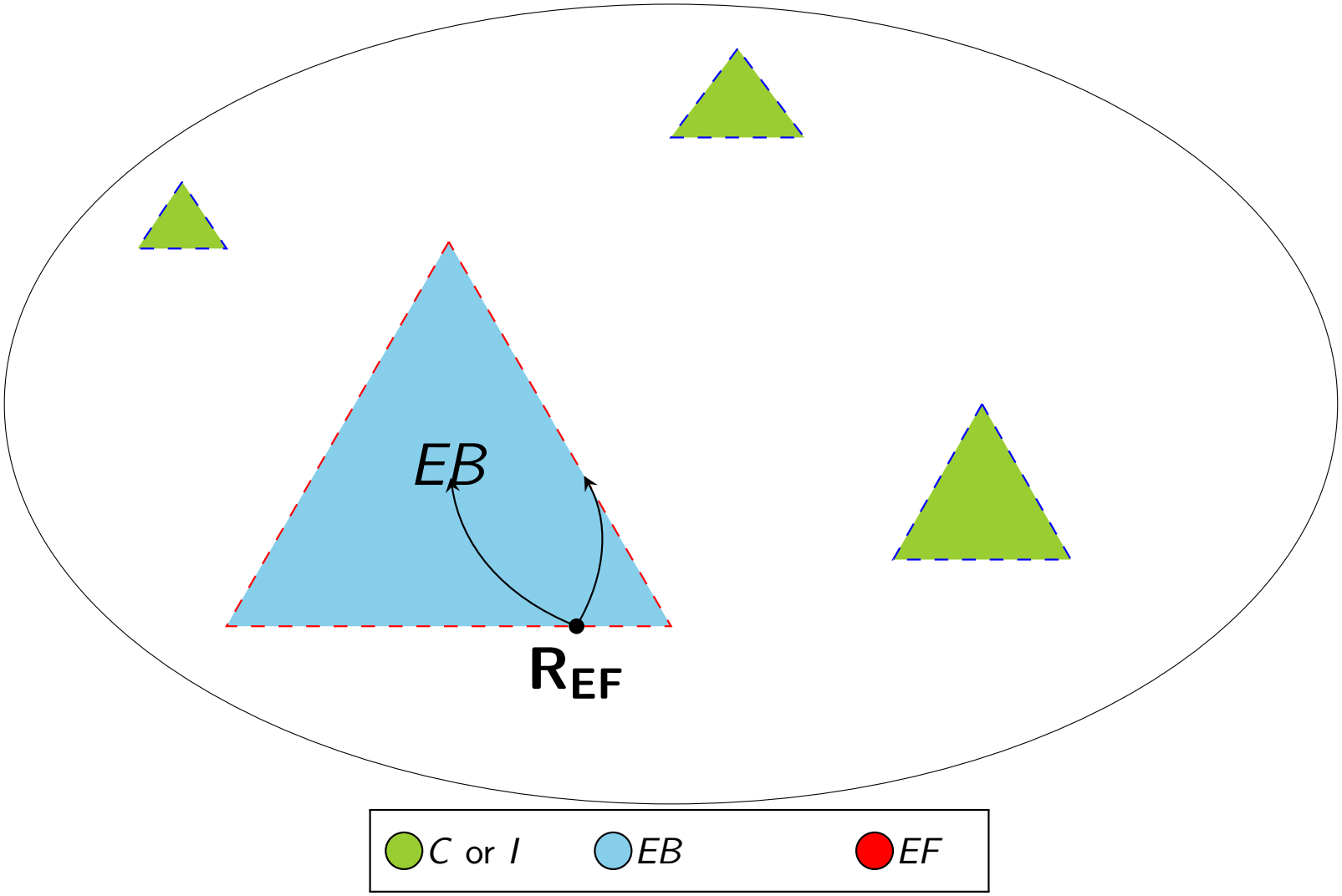

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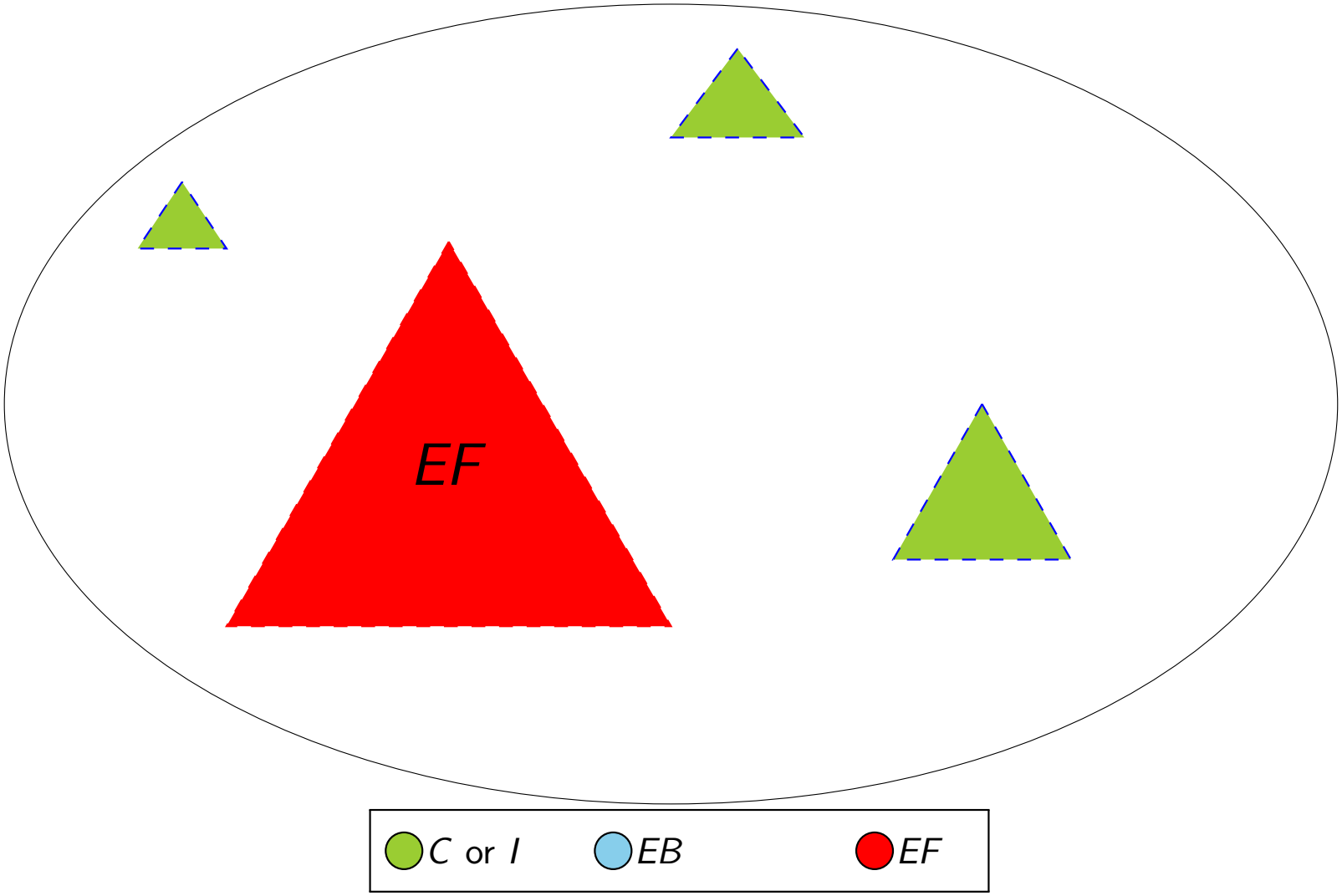

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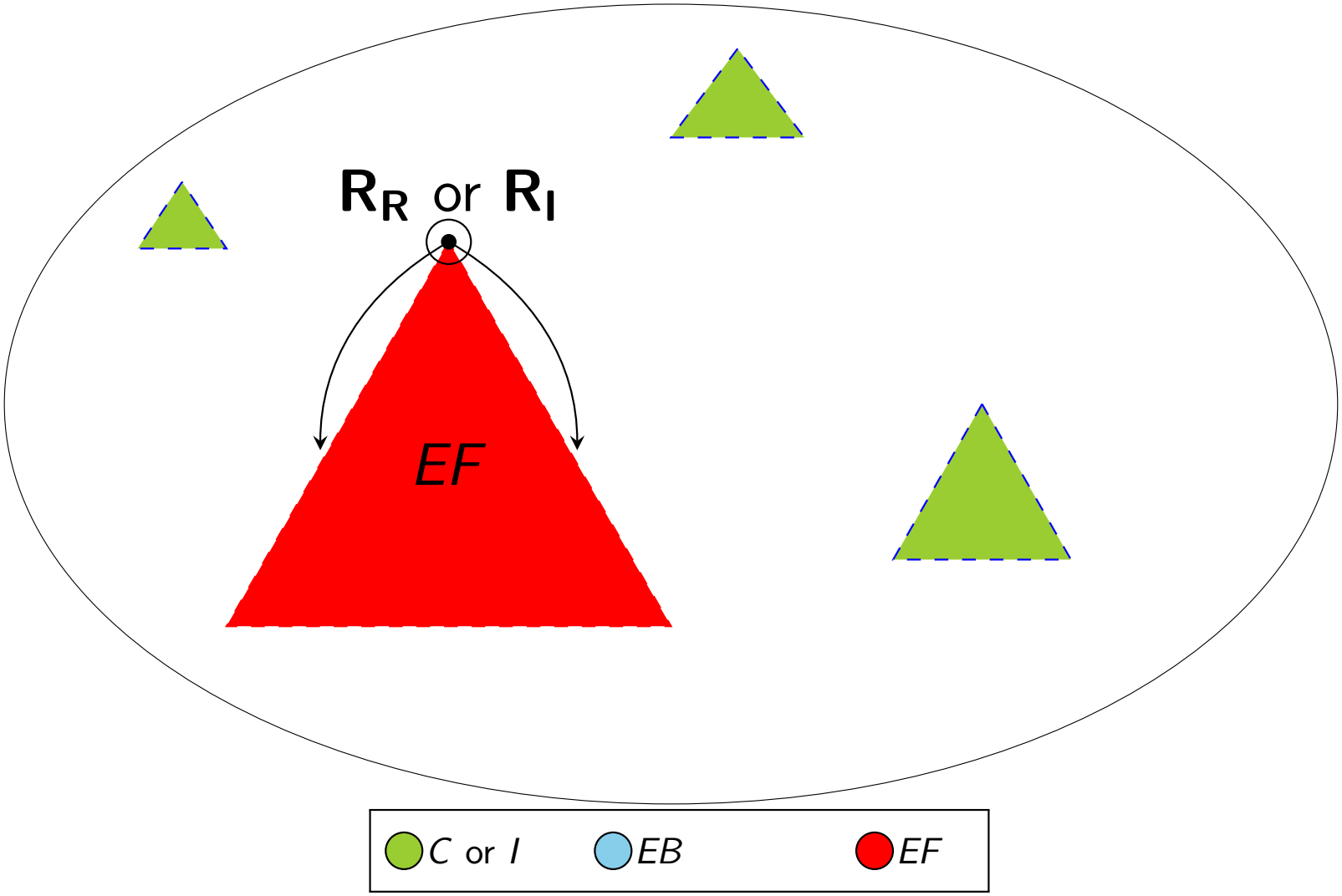

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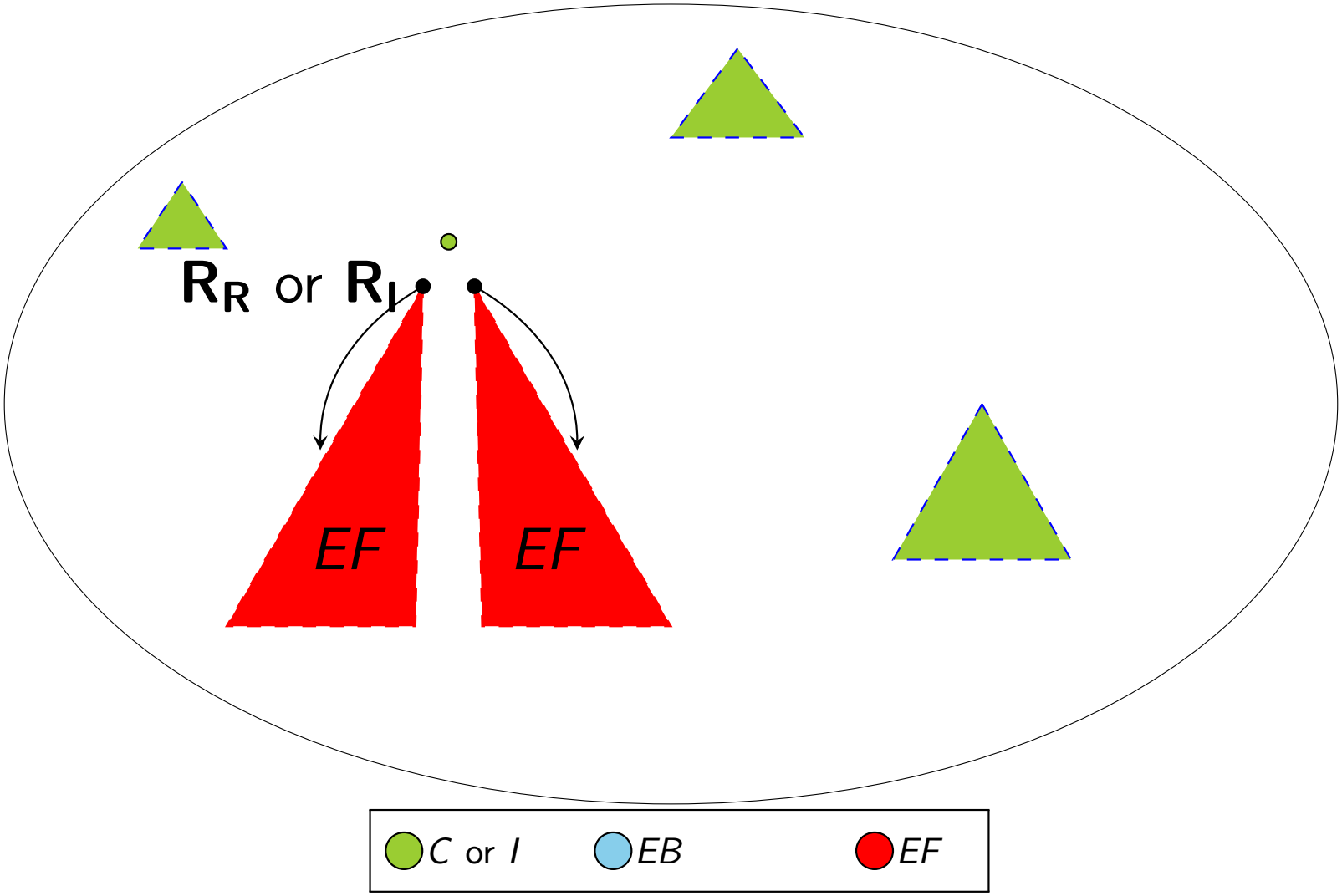

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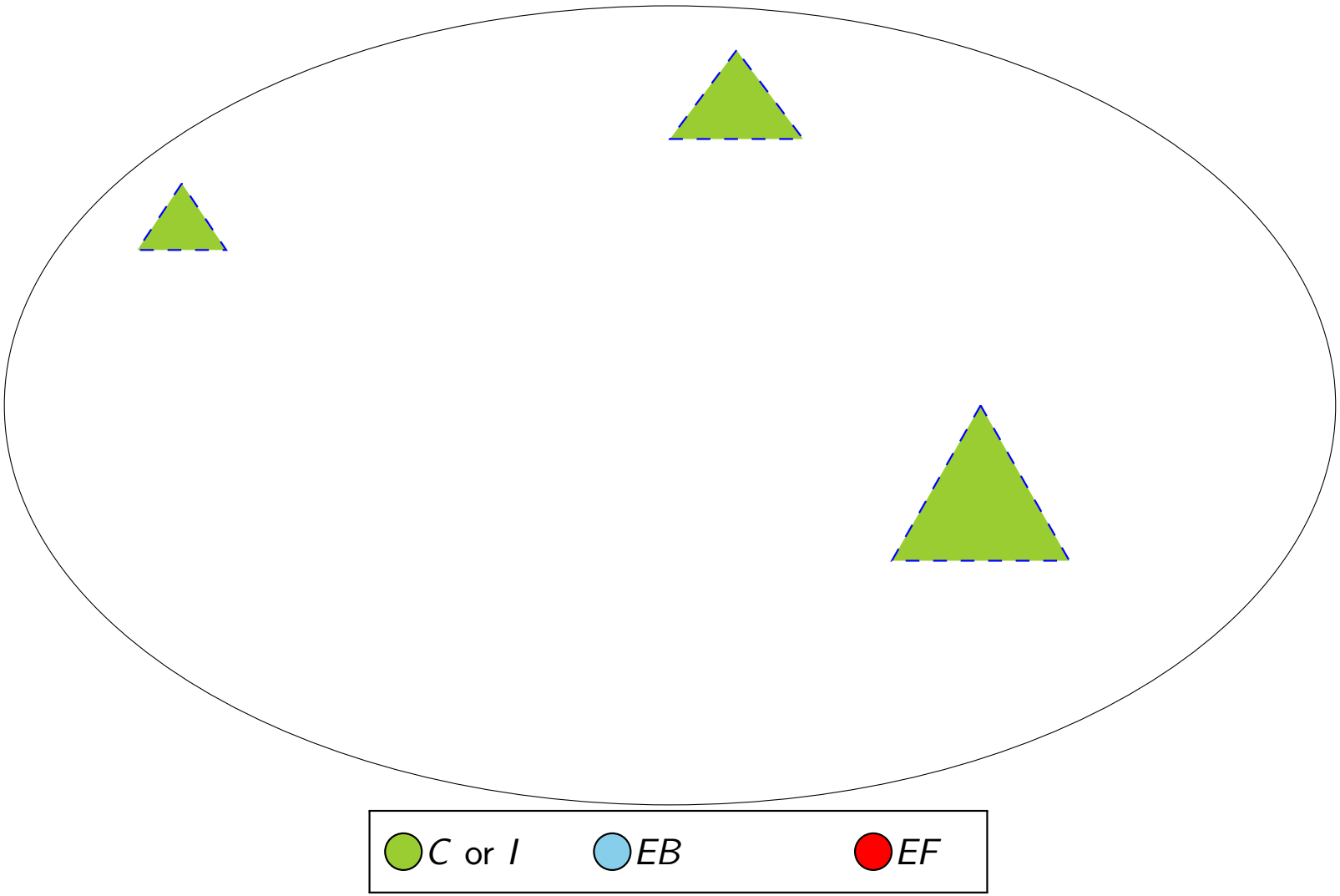


The definition of abnormal root should take to possible inconsistencies of variables *st* into account!

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$$O(4n_{\texttt{maxCC}}) \textbf{ rounds}$$

# Stabilization Time in Moves (1/2)

Let $GC$ be a connected component of $G$.

Let $SL(\gamma, GC)$ be the set of processes $u \in GC$ such that, in the configuration $\gamma$, $u$ is an alive abnormal root, or $canBeRoot_u \wedge distRoot(u) \prec d_u \wedge st_u = C$ holds.

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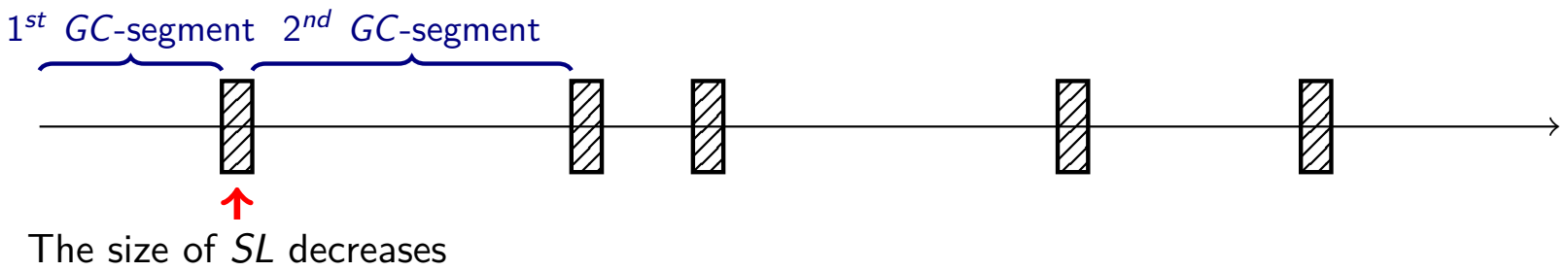


$\longrightarrow$ At most $n_{\texttt{maxCC}} + 1$ $GC$-segments in $GC$

# Stabilization Time in Moves (2/2)

Let $u$ be any process of $GC$. We proved that the sequence of rules executed by $u$ during a $GC$-segment belongs to the following language:

$$(\mathbf{R_I} + \varepsilon)(\mathbf{R_R} + \varepsilon)(\mathbf{R_U})^*(\mathbf{R_{EB}} + \varepsilon)(\mathbf{R_{EF}} + \varepsilon).$$

## Theorem 4

*If the number of $\mathbf{R_U}$ executions during a GC-segment by any process of GC is bounded by $nb\_UN$, then the total number of moves in any execution is bounded by $(nb\_UN + 4)(n_{maxCC} + 1)n$.*

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$nb\_UN$ is necessarily defined because $d_u$ decreases at each $\mathbf{R_U}(u)$ in a $GC$-segment.

## Theorem 5

*When all weights are strictly positive integers bounded by $W_{\max}$ $nb\_UN \leq W_{\max}(n_{maxCC} - 1)^2 + 1$*

# Thank you for your attention

- Stéphane Devismes, Colette Johnen, and David Ilcinkas. Silent Self-Stabilizing Scheme for Spanning-Tree-like Constructions. *Submitted to PODC'2018*.

  Technical report available online:

  `https://hal.archives-ouvertes.fr/hal-01667863`.