

# A Quick Overview on Name-Independent Compact Routing Schemes

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# The Compact Routing Problem

**Input:** a network  $G$  (an edge-weighted connected graph)

**Output:** a **routing scheme** for  $G$

A *routing scheme* is a distributed algorithm that allows **any** source node to route messages to **any** destination node, given the destination's network identifier

**Space** = size (in bits) of the largest local routing tables

**Stretch** = ratio between length of the route and distance

# Goals

What we expect for a good routing scheme:

- Universal
- Low Space & Low Stretch
- Name-Independent

The destination enters the network with its **name**, which is either determined by the designer of the routing scheme (labeled), or set arbitrarily (name-independent).

# General Bounds

[Abraham, Gavoille et al.]

For each integer  $k \geq 1$ , and every weighted graph, there is name-independent routing scheme with stretch linear in  $k$  and space  $n^{1/k} \cdot \text{polylog}(n)$  space.

$k = 1 \Rightarrow$  BGP: stretch 1, space  $n \cdot \text{polylog}(n)$

$k = 2 \Rightarrow$  [AGMNT]: stretch 3, space  $\sqrt{n} \cdot \text{polylog}(n)$

$k \geq 3$ : stretch  $\sim 64k$ , optimal stretch for  $k = 3$  is open

# General Bounds: Lower Bounds

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Main differences with *labeled* routing schemes:

- stretch 1 with space  $\text{polylog}(n)$  for weighted trees
- average stretch  $O(1)$  with  $\text{polylog}(n)$  for weighted graphs
- no difference between weighted/unweighted case
- general lower bound depends on the Girth Conjecture

# Parameterized Bounds

(graph parameter dependent)

parameter	stretch	space
growth $\alpha$	$1 + \varepsilon$	$\varepsilon^{-O(\log \alpha)} \cdot \text{polylog}(n)$
doubling dimension $d$	$9 + \varepsilon$	$\varepsilon^{-O(d)} \cdot \text{polylog}(n)$
minor-free size $r$ (unweighted)	$4r^2$	$r!2^{O(r)} \cdot \text{polylog}(n)$

**Note:** these schemes do not depend on any specific pre-decomposition or structure driven by the parameter. It always works! If the parameter is small, stretch & space are low.

$\Rightarrow$  run the algorithm and see.

## A Relevant Structure: Sparse Cover

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If  $G$  has a  $(s, \delta)$ -sparse cover, then it has name-independent routing scheme with space  $\delta \log D \cdot \text{polylog}(n)$  and stretch  $O(s)$ .

# Known Sparse Covers

networks	stretch	density
any graph, $k \geq 1$	$2k - 1$	$2kn^{1/k}$
growth $\alpha$	$1 + \varepsilon$	$\varepsilon^{-O(\log \alpha)}$
doubling dimension $d$	$1 + \varepsilon$	$\varepsilon^{-O(d)}$
minor-free size $r$	$4r^2$	$r!2^{O(r)}$
$k$ -path separable	4	$O(k \log n)$

**Note:** the space bound depends on  $\log D$ . So, for weighted graphs the sparse cover based routing scheme may produce non-polynomial space.

Thank you!