Laboratoire Bordelais de Recherche en Informatique
Université Bordeaux I, 351, cours de la Libération,
33405 Talence CEDEX, France

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On Dilation of Interval Routing

by Cyril Gavoille

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Cyril Gavoille

LaBRI
Université Bordeaux I
33405 Talence Cedex, France
gavoille@labri.u-bordeaux.fr

Abstract

In this paper we deal with interval routing on n-node networks of diameter D. We show that for every D, there exists a network on which every interval routing scheme with routing path lengths bounded by $3D/2-3$ needs $\Omega(n/\log n)$ intervals per link. It improves the lower bound for the range of very large number of intervals. No result was known about the path lengths whenever more than $\Theta(\sqrt{n})$ intervals per link was used. Best-known lower bounds for a small number of intervals are $7D/4-O(1)$ for 1 interval [10], and $3D/2-O(1)$ up to $\Theta(\sqrt{n})$ intervals [3].

1 Introduction

The *dilation* of a routing scheme is the length of the longest routing path. Assuming that time cost of message delivery is function of the routing path length or of the number of routers crossed, dilation is a parameter of the worse-case time complexity. Concurrently, fast routers must be easily implemented with a small amount of hardware.

Interval Routing is a routing scheme implementing compact routing tables, and allowing fast routing decisions in each node [8, 12]. Interval routing consists in labeling nodes by an unique integer taken in $\{1,\ldots,n\}$, n the number of routers, and in assigning to each link at every router a set of intervals of destinations, such that any message can reach its destination from any source. Such a labeling scheme on a network G is so-called an *interval routing scheme* on G. Locally, a router finds the link to forward the message at a given destination by looking its link having an interval containing the destination. At each node, the intervals must be pairwise disjoint, and cover the set of all the labels, maybe excepted the label of the node it-self. The local routing decision time is bounded by $O(d\log k\cdot\log n)$ bit operations[1] whereas the space complexity of the router is at most $O(kd\cdot\log n)$ bits, for a router of d links, and if at most k intervals per link are used. In particular, such a routing scheme is "performing", *i.e.*, compact and fast, if the degree of the network and the number of intervals per link are both low[2] relatively to the number of routers of the network.

[1] For each link the router can perform a binary search among the k sorted intervals, every operations being on integers of size $O(\log n)$ bits.

[2] For instance k and d satisfying $kd/\log d = o(n/\log n)$ may provide a scheme more compact than the standard routing tables which need of $\Theta(n\log d)$ bits per router.

The interval routing scheme is used in the last generation of C104 routing chips used in the INMOS T9000 Transputer design [5]. Since the number of intervals is limited in each routing chip, we are interested in finding the minimum dilation for interval routing scheme using a fixed number of intervals. Dilation is expressed in term of the diameter of networks which is a common lower bound of dilation of all networks.

The following table summarizes the best lower bounds known, and our contribution about dilation of interval routing schemes using at most k intervals per link on n-node networks of diameter D. Note that for every network, there is an interval routing scheme with one interval and of dilation $2D$, where D is the diameter of the network. Indeed, it is sufficient to route along a minimum spanning tree of the network which supports an interval routing scheme [8].

Number of intervals	Dilation (lower bound)	Reference
$k = 1$	$7D/4 - 7$ (*)	[10]
$2 \leq k \leq \Theta(\sqrt{n})$	$3D/2 - 3$	[3]
$2 \leq k \leq \Omega(n/\log n)$	$3D/2 - 3$	Theorem 1

(*)In the original paper the lower bound was $7k - 1 = 7D/4 - 1$, for $D = 4k$, for every integer $k \geq 2$. We have corrected this bound into $7\lfloor D/4 \rfloor - 1 \geq 7D/4 - 7$ in order to give asymptotic results in D, and to compare all the bounds.

The next section presents the notations, and previous works about the dilation. In Section 3, we prove the main theorem. Also, we extend the result to diameters which depend on n. We prove a trade-off of $\Omega(n/(D\log(n/D)))$ intervals required for every interval routing scheme of dilation less than $3D/2 - 3$. In Section 4, we improve the multiplicative constant of the result of [2] about the maximum compactness of n-node networks. We give some open problems to conclude in Section 5.

2 Statement of the Problem

2.1 Notations

The model of networks is a undirected connected graph G, each node representing a router. The distance between any two nodes x and y is the minimum number of edges of paths connecting x and y, and is denoted $\mathrm{dist}(x, y)$. In all the rest of the paper, n will denote the order of the graph, and D its diameter.

An *interval* means a set of consecutive integers taken in $\{1, \ldots, n\}$, n and 1 being considered as consecutive. For every arc[3] e, all the intervals associated to e form a set of integers, *viz* a set of labels of destinations[4], and is denoted by I_e. An interval routing scheme R on a graph G is denoted by a pair $(\mathcal{L}, \mathcal{I})$, where $\mathcal{L}$ is the labeling of vertices of G, and $\mathcal{I}$ is the set of all the I_e's.

[3]For convenience, each edge of the graph is considered as a pair of two symmetric arcs.

[4]We will assume that intervals are *strict*, that is for every arc (x, y), the label of $x \notin I_{(x,y)}$. This restriction is without loss of generality in our framework, since we prove results for a large and asymptotic number of intervals. A non strict interval can always be split in a strict interval adding at most 1 interval per link.

2.2 A simple example

Let us consider the following example of interval routing on a graph G_0 of 7 vertices depicted on the left-hand side of Figure 1. Nodes are labeled by integers from 1 to 7, and intervals are assigned to each arc. If the node 5 sends a message to node 1, the message will successively be forwarded along the arc $(5,7)$, then along $(7,1)$, because $1 \in I_{(5,7)} = [7,2] = \{7,1,2\}$, and $1 \in I_{(7,1)} = [1] = \{1\}$. Each set of destinations I_e, e arc of G_0, is composed of at most two intervals of consecutive labels. One can check that every routing path on G_0 is of minimal length. Therefore, the dilation of this routing scheme is the diameter of G_0, here 2.

This interval routing scheme on G_0 is qualified of *shortest path* interval routing scheme because *all* the routing paths in the graph are of minimal length. A classical problem for interval routing is to compute the minimum number of intervals needed to guarantee a shortest path interval routing scheme on a given graph. A such number depends on the graph only, and is termed *compactness*. Compactness of a graph is at most $n/2$, because any set of destinations, and any labeling of these destinations, can not contain more than $n/2$ non consecutive integers. In [2], it has been built graphs of compactness at least $n/12$. Therefore $\Theta(n)$ is a tight lower bound of compactness of n-vertex graphs.

Actually, G_0 has no shortest path routing scheme with only one interval per link (see [1, page 793] for a proof). Hence the compactness of G_0 is 2. The right-hand side of Figure 1 shows another interval routing scheme on G_0, but with only one interval per arc. It has also a dilation of 2, the diameter. The *dilation* problem is, given a graph G, and an integer k, k being less than the compactness of G, to determine an interval routing scheme on G using at most k intervals per link which minimizes the longest routing path. This general question is important in practice whenever a low number of intervals is forced by the hardware of the router, and whenever message delivery time must be as short as possible.

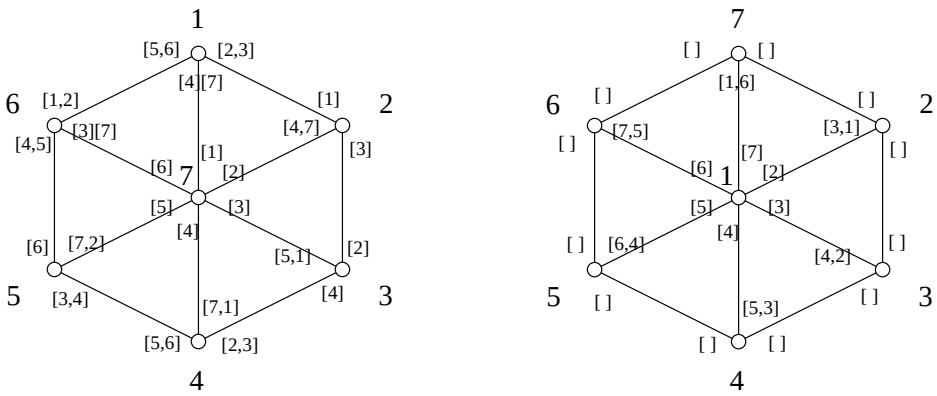

Figure 1: Two interval routing schemes of dilation 2 on the same graph G_0. (We denote by [ ] the *empty interval*, corresponding to an empty set of destination.)

Fundamentally, the compactness problem consists in measuring the compression of the "routing information" whenever paths are of minimum length. Its dual problem, the dilation problem, consists in measuring the efficiency of the routing scheme when the compression rate is limited, which defines the size of the routing information in each node. Both the problems contribute to design some trade-off between time and space used by a router in a communication network.

2.3 Related works

Lower bound for the dilation problem for interval routing was first addressed by Ružička in [6, 7]. He built a *globe-graph* on which every interval routing scheme with one interval has dilation at least[5] $3D/2$. This result was improved later by Tse and Lau in [10, 11], for one interval with a lower bound of $7D/4$ (based on an extension of the globe-graph), and generalized to more than one interval with a lower bound still of $3D/2$ up to $\Theta(\log n)$ intervals, and of $5D/4$ up to $\Theta(\sqrt{n})$ intervals. Other results are also mentioned in [9] for dilation in planar graphs. Recently, in [3], Kráľovič, Ružička, and Štefankovič improved the lower bound for the range of intervals from 2 up to $\Theta(\sqrt{n})$, with $3D/2$, using a *multi-globe graph*. No result was known for a larger number of intervals. Intuitively, more intervals are used, more dilation is low. Note that the best-known upper bound is $2D$, whatever the number of intervals used up to[6] $n/12$.

In this paper we extended the range of possible number of intervals up to $\Omega(n/\log n)$, and we prove a dilation of at least $3D/2 - 3$.

3 Construction of the Worst-Case

The main Theorem of this section is the following:

Theorem 1 *For every integer $D \geq 3$, there exists an n-vertex graph of diameter D on which every interval routing scheme of dilation less than $3D/2 - 3$ requires $\Omega(n/(D\log(n/D)))$ intervals.*

In Theorem 1, the number of intervals is expressed as a function of the diameter. It turns out more general results. For instance, Theorem 1 shows that for every constant k there exists an n-vertex graph of diameter $D = \Theta(n)$ on which every interval routing scheme using k intervals has a dilation $3D/2 - 3$. Note that all the previous results were proved for an arbitrary but fixed diameter. The result of [3] is improved just by applying Theorem 1 with D an arbitrary constant.

Corollary 1 *For every constant integer $D \geq 3$, there exists an n-vertex graph of diameter D on which every interval routing scheme of dilation less than $3D/2 - 3$ requires $\Omega(n/\log n)$ intervals.*

Sketch of the proof.
Basically we use a similar technique to establish lower bounds of the compactness, and to prove a lower bound of the dilation with a large number of intervals. Our construction is an adaptation of the graph defined in [2]. For the sketch we assume that D is a fixed constant. We build a graph which has the two following properties:

1) Some vertices require an interval routing scheme using $k = \Omega(n/\log n)$ intervals on an arc to route along the shortest paths between vertices at distance $t = \lfloor (D-1)/2 \rfloor$.
2) Any interval routing scheme which does not route along the shortest paths between these vertices has routing path lengths at least $3t$.

[5] To sharpen the discussion we drop the additive constant of D for the dilation.
[6] Actually, the upper bound is $2D$ up to $n/4 - O(\sqrt{n\log n})$ intervals according to Theorem 2.

Say in others words, any interval routing scheme of dilation $< 3t = 3D/2 - O(1)$ on this graph requires at least k intervals.

Our construction is function of a boolean matrix M, and of an integer D. It is denoted by $G_{M,D}$. More precisely, for every $p \times q$ boolean matrix M and for every integer $D \geq 3$ we define the graph $G_{M,D}$ as follows: we associate with each row i of M, $i \in \{1, \ldots, p\}$, a vertex v_i in $G_{M,D}$. At each column j of M, $j \in \{1, \ldots, q\}$, we associate a pair of vertices $\{a_j, b_j\}$ which are connected by an edge. For every $i \in \{1, \ldots, p\}$ and $j \in \{1, \ldots, q\}$, we connect v_i to a_j by a path of length $t = \lfloor (D-1)/2 \rfloor$ if and only if $m_{i,j} = 0$. Similarly, we connect v_i to b_j by a path of length t if and only if $m_{i,j} = 1$. See Figure 2 for an example. Note that the graph obtained by contraction of the edges $\{a_j, b_j\}$, $j \in \{1, \ldots, q\}$, of the graph $G_{M,D}$ is a complete bipartite graph $K_{p,q}$.

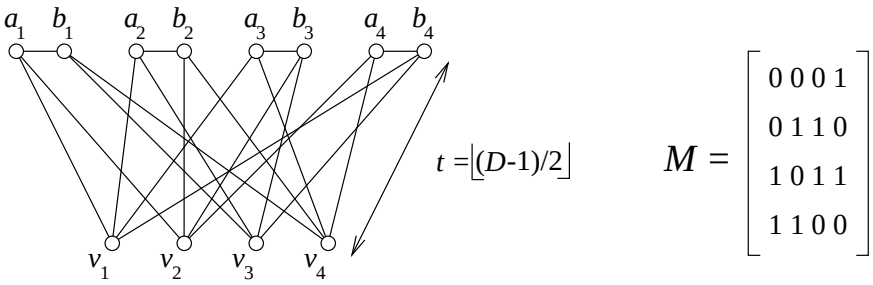

Figure 2: An example of the graph $G_{M,D}$, here of diameter $D \leq 2t + 1$.

We define the *compactness* of a boolean matrix M as the smallest integer k such that there exists a matrix obtained by row permutation of M having at most k blocks of consecutive ones per column. The first and the last entry of a column are considered as consecutive. For example, the matrix M described on Figure 2 has its first and its third column composed of one blocks of consecutive ones, whereas its second and its forth column are composed of two blocks (each one being composed of only one 1-entry). The reader can check that whatever one permute the rows of M, there exists at least one column with two blocks of consecutive ones. Therefore, the compactness of M is 2. Intuitively, on this example, any interval routing scheme using only one interval on all the arcs of the form (a_j, b_j) should have a dilation of $3t = 3D/2 - O(1)$.

In all the following, columns of boolean matrices are seen as binary strings. We need of the following lemma to prove Theorem 1:

Lemma 1 *For every sufficiently large integer p, there exists a $p \times (5\lceil \log_2 p \rceil)$ boolean matrix of compactness at least $p/5 - O(p/\log p)$.*

Proof. We use a counting argument which can be formalized by the Kolmogorov Complexity (see [4] for an introduction). Basically, the Kolmogorov Complexity of an individual object X is the length (in bits) of the smallest program, written in a fixed programming language, which prints X and halts. A simple counting argument allows to say that no program of size less than K can print certain X_0 taken from a set of more than 2^K elements.

Let $\mathcal{M}$ be the set of all the $p \times q$ boolean matrices. The total number of $p \times q$ boolean matrices is $|\mathcal{M}| = 2^{pq}$. Let us show that compactness of some matrices of $\mathcal{M}$ is linear in p for $q = 5\lceil \log_2 p \rceil$.

For every $M \in \mathcal{M}$, we define $\mathrm{cl}(M)$ the subset of the matrices of $\mathcal{M}$ obtained by row permutation of M. There exists a matrix $M_0 \in \mathcal{M}$ such that all the matrices of $\mathrm{cl}(M_0)$ have a Kolmogorov

Complexity at least $C = \log_2 |\mathcal{M}| - \log_2(p!) - 3\log_2 p$. Indeed, by contradiction let $M_0' \in \mathrm{cl}(M_0)$ be a matrix of Kolmogorov Complexity $C' < C$, for any $M_0 \in \mathcal{M}$. M_0 may be described by a pair (i_0, M_0'), where i_0 is the indexe of the row permutation of M_0' into M_0. Such an integer can be coded by $\log_2(p!)+2\log_2 p+O(1)$ bits ($2\lceil\log_2 p\rceil+1$ bits are sufficient to describe p in a self-delimiting way). Hence the Kolmogorov Complexity of M_0 would be at most $C'+\log_2(p!)+2\log_2 p+O(1) < \log_2 |\mathcal{M}|$, that is impossible for any matrix $M_0 \in \mathcal{M}$. Therefore, since the matrices of $\mathcal{M}$ have q columns, there exists a matrix M_0 such that every matrix obtained by row permutation of M_0 has a column of Kolmogorov Complexity at least

$$C_0 = \frac{1}{q}\left(\log_2 |\mathcal{M}| - \log_2(p!) - 3\log_2 p\right).$$

We have $\lceil\log_2(p!)\rceil = p\log_2 p - O(p)$, hence,

$$C_0 = p\left(1 - \frac{\log_2 p}{q}\right) - O\left(\frac{p}{q}\right).$$

It is well-known that every incompressible string of length K bits contains $K/4-O(\sqrt{K})$ occurrences of 01-sequences [4, Theorem 2.15, page 131]. Since each 01-sequence in a binary string starts necessarily a new block of consecutive ones, we can conclude that a column having a Kolmogorov Complexity K has at least $K/4 - O(\sqrt{K})$ blocks of consecutive ones. It follows that M_0 has a compactness at least $C_0/4 - O(\sqrt{C_0})$.

Therefore, we get the following lower bound of the compactness of M_0:

$$C_0/4 - O(\sqrt{C_0}) = \frac{p}{4}\left(1 - \frac{\log_2 p}{q}\right) - O\left(\frac{p}{q} + \sqrt{p}\right). \tag{1}$$

Choosing $q = 5\lceil\log_2 p\rceil$, we have:

$$C_0/4 - O(\sqrt{C_0}) = \frac{p}{5} - O\left(\frac{p}{\log p}\right),$$

that completes the proof. □

Remark. The proof of Lemma 1 is not constructive. As a result, we can prove only existence of such a worst-case graph $G_{M,D}$.

We are now ready to prove Theorem 1.

Proof of Theorem 1.
Let D be an integer ≥ 3, let p be an integer large enough, and let M be a matrix satisfying Lemma 1. We consider the graph $G_{M,D}$. Let $t = \lfloor(D-1)/2\rfloor$. Let us remark first that for every integer $D' > D$, and for every graph G of diameter D on which every interval routing scheme using k intervals has a dilation δ, there exists a graph G' of diameter D' on which every interval routing scheme using k intervals has a dilation at least δ. Indeed, to obtain G' it is sufficient to add a path of length $D' - D$ to any vertex of G which is of eccentricity D. Hence, to prove a lower bound for the dilation of graphs of diameter D it is sufficient to prove a lower bound for dilation in graphs of diameter at most D.

Fact 1. The diameter of $G_{M,D}$ is at most D.

Let us show that, for every pair of vertices x and x', $\mathrm{dist}(x,x') \leq 2t+1$. Let us consider two arcs (a_j,b_j), and $(a_{j'},b_{j'})$, and two vertices v_i, and $v_{i'}$. Let us assume that x is at distance α of v_i, $0 \leq \alpha \leq t$, on the path which connects v_i to a_j, or to b_j. Similarly, we assume that x' is at distance α' of $v_{i'}$, $0 \leq \alpha' \leq t$, on the path which connects $v_{i'}$ to $a_{j'}$, or to $b_{j'}$. Without loss of generality we assume that both paths are connected respectively to a_j, and to $a_{j'}$. Two paths connect x and x'. One goes trough a_j then $v_{i'}$, because necessarily there is a direct path between a_j and $v_{i'}$, or between b_j and $v_{i'}$, depending on the value of $m_{i',j}$. Its length is at most $t-\alpha+1+t+\alpha'$. By symmetry, there is another path which goes trough v_i then $a_{j'}$ of length at most $\alpha+t+1+t-\alpha'$. Thus, $\mathrm{dist}(x,x') \leq 2t+1+\min\{\alpha'-\alpha, \alpha-\alpha'\} \leq 2t+1 \leq D$, for every α, α'.

Fact 2. For any interval routing scheme $R=(\mathcal{L},\mathcal{I})$ on $G_{M,D}$, for every arc (a_j,b_j), and for every vertex v_i, if $m_{i,j}=1$, and $\mathcal{L}(v_i) \notin I_{(a_j,b_j)}$, then the dilation of R is at least $3t$. Similarly, if $m_{i,j}=0$, and $\mathcal{L}(v_i) \in I_{(a_j,b_j)}$, then the dilation of R is at least $3t$.

This results from the fact that there is no path shorter than $2t$ between the vertices v_i, between the vertices a_j, and between the vertices b_j.

Fact 3. Let k be the compactness of M, and $R=(\mathcal{L},\mathcal{I})$ be any interval routing scheme on $G_{M,D}$. If R uses less than k intervals per arc, then its dilation is at least $3t = 3\lfloor (D-1)/2 \rfloor \geq 3D/2-3$.

Assume R is fixed, and uses only $k-1$ intervals. Let j_0 be a column of M composed of at least k blocks of consecutive ones. Such a column exists because the compactness of M is k. Let us consider the sequence $u=(u_1,\ldots,u_p)$ defined by: for every $i \in \{1,\ldots,p\}$, $u_i=1$ if $\mathcal{L}(v_i) \in I_{(a_{j_0},b_{j_0})}$, and $u_i=0$ otherwise. Since $I_{(a_{j_0},b_{j_0})}$ is composed of at most $k-1$ intervals, u is composed of at most $k-1$ blocks of consecutive ones. Thus the column j_0 and the sequence u differ in at least one place. Set i_0 the indexe such that $m_{i_0,j_0} \neq u_{i_0}$. If $u_{i_0}=1$, then $\mathcal{L}(v_{i_0}) \in I_{(a_{j_0},b_{j_0})}$, and $m_{i_0,j_0}=0$. If $u_{i_0}=0$, then $\mathcal{L}(v_{i_0}) \notin I_{(a_{j_0},b_{j_0})}$, and $m_{i_0,j_0}=1$. We conclude by applying Fact 2.

The order of $G_{M,D}$ is $n=p+2q+pq(t-1)$, with $q=\Theta(\log p)$. For $D=3$, or $D=4$, that is $t=1$, $n=p+2q$. So $\log_2 n = \log_2 p + o(\log p)$, thus we have $p=n-O(\log n)$. For $D \geq 5$, that is $t \geq 2$, $n=\Theta(Dp\log p)$. In this case, $p=\Theta(n/(D\log p))$, thus $\log_2 p = \Theta(\log(n/(D\log p))) = O(\log(n/D))$, and finally $p=\Omega(n/(D\log(n/D)))$. Therefore for every $D \geq 3$, $p=\Omega(n/(D\log(n/D)))$. By Lemma 1, the compactness of M is linear in p. We apply Fact 3 to complete the proof. □

Theorem 1 allows to establish a trade-off between the order, the diameter of the graph, and the number of intervals required for a dilation $3D/2-3$. For instance, we saw that the matrix M of Figure 2 has a compactness 2. Fact 3 of Theorem 1 shows that the worst-case graph of Figure 2, which is of maximum degree 4 and of diameter $D \sim n/8$, has a dilation $3D/2-3 \sim 3n/16$ for 1 interval. However in general, the graph $G_{M,D}$ has a maximum degree $\Theta(p)$ which is at least $\Omega(n/(D\log(n/D)))$.

4 An $\frac{n}{4}$-Lower Bound for Compactness

In [2], it has been built a graph of compactness at least $n/12$ intervals. Their construction was *effective* in the sense that they gave, in a deterministic way, the connection between the vertices. We improve the multiplicative constant with a non-constructive worst-case. Let us recall that the

compactness of a graph G is the smallest integer k such that G supports a shortest path interval routing scheme using at most k intervals per link.

Theorem 2 *For every sufficiently large integer n, there exists an n-vertex graph of compactness at least $n/4 - O(\sqrt{n \log n})$.*

Proof. Let us consider the graph $G_{M,3}$, for a suitable $p \times q$ boolean matrix M. The graph $G_{M,3}$ is of order $n = p + 2q$. Let us choose $q = \lceil \sqrt{n \log_2 n} \rceil$. Like it is shown in Equation 1 in the proof of Lemma 1, there exists a $p \times q$ boolean matrix M of compactness k such that

$$k \geq \frac{p}{4}\left(1 - \frac{\log_2 p}{q}\right) - O\left(\frac{p}{q} + \sqrt{p}\right) = \frac{p}{4} - O\left(\frac{p \log p}{q} + \sqrt{p}\right).$$

Since $p = n - 2q$, and $p = \Theta(n)$, we have:

$$k \geq \frac{n}{4} - \frac{q}{2} - O\left(\frac{n \log n}{q} + \sqrt{n}\right) = \frac{n}{4} - O\left(q + \frac{n \log n}{q} + \sqrt{n}\right) = \frac{n}{4} - O\left(\sqrt{n \log n}\right).$$

The lower bound on the dilation proved in Theorem 1, is the length of routing paths between the vertices a_j and v_i which are at distance at most $t + 1 = 2$. By Fact 3 of the proof of Theorem 1, any interval routing scheme using $k - 1$ intervals is of dilation $3t = 3\lfloor (D-1)/2 \rfloor = 3$, and thus is not a shortest path routing scheme. □

To our best knowledge, $n/12$ remains the best worst-case construction, for every n, which does not used randomization. It will be interesting to know either all graphs have a compactness at most $n/4$, that would prove that $n/4$ is an optimal bound of the maximal compactness of n-vertex graphs.

5 Conclusion

We proved a dilation of $3D/2 - 3$ for every interval routing scheme using $O(n/\log n)$ intervals. Two questions remain open about the dilation problem:

1) is a $2D$ lower bound for dilation can be reached? More precisely, is there an n-node network of diameter D on which every interval routing scheme using 1 interval per link have a dilation $2D - O(1)$?
2) is a $(1+\varepsilon)D$ lower bound for dilation can be reach for interval routing schemes using at most $c \cdot n$ intervals per link, for some suitable non-null constants ε and c?

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