A Quick Overview on Name-Independent Compact Routing Schemes

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The Compact Routing Problem

Input: a network $G$ (an edge-weighted connected graph)
Output: a routing scheme for $G$

A routing scheme is a distributed algorithm that allows any source node to route messages to any destination node, given the destination’s network identifier

Space = size (in bits) of the largest local routing tables
Stretch = ratio between length of the route and distance
Goals

What we expect for a good routing scheme:

- Universal
- Low Space & Low Stretch
- Name-Independent

The destination enters the network with its name, which is either determined by the designer of the routing scheme (labeled), or set arbitrarily (name-independent).
For each integer \( k \geq 1 \), and every weighted graph, there is a name-independent routing scheme with stretch linear in \( k \) and space \( n^{1/k} \cdot \text{polylog}(n) \) space.

\( k = 1 \Rightarrow \text{BGP}: \) stretch 1, space \( n \cdot \text{polylog}(n) \)
\( k = 2 \Rightarrow [\text{AGMNT}]: \) stretch 3, space \( \sqrt{n} \cdot \text{polylog}(n) \)
\( k \geq 3: \) stretch \( \sim 64k \), optimal stretch for \( k = 3 \) is open
For each integer \( k \geq 1 \), there are weighted trees for which every name-independent routing scheme with space \( < n^{1/k} \) requires stretch \( \geq 2k + 1 \) and average stretch \( \geq k/4 \).
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Main differences with labeled routing schemes:
- stretch 1 with space $\text{polylog}(n)$ for weighted trees
- average stretch $O(1)$ with $\text{polylog}(n)$ for weighted graphs
- no difference between weighted/unweighted case
- general lower bound depends on the Girth Conjecture
### Parameterized Bounds
(graph parameter dependent)

<table>
<thead>
<tr>
<th>parameter</th>
<th>stretch</th>
<th>space</th>
</tr>
</thead>
<tbody>
<tr>
<td>growth $\alpha$</td>
<td>$1 + \varepsilon$</td>
<td>$\varepsilon^{-O(\log \alpha)} \cdot \text{polylog}(n)$</td>
</tr>
<tr>
<td>doubling dimension $d$</td>
<td>$9 + \varepsilon$</td>
<td>$\varepsilon^{-O(d)} \cdot \text{polylog}(n)$</td>
</tr>
<tr>
<td>minor-free size $r$</td>
<td>$4r^2$</td>
<td>$r!2^{O(r)} \cdot \text{polylog}(n)$</td>
</tr>
</tbody>
</table>

**Note:** these schemes do not depend on any specific pre-decomposition or structure driven by the parameter. It always works! If the parameter is small, stretch & space are low.

$\implies$ run the algorithm and see.
A Relevant Structure: Sparse Cover

An $(s, \delta)$-sparse cover for a graph $G$ is a set of clusters $\mathcal{C} \subset 2^V(G)$ such that for every $r \geq 0$: 
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If \(G\) has a \((s, \delta)\)-sparse cover, then it has name-independent routing scheme with space \(\delta \log D \cdot \text{polylog}(n)\) and stretch \(O(s)\).
Known Sparse Covers

<table>
<thead>
<tr>
<th>networks</th>
<th>stretch</th>
<th>density</th>
</tr>
</thead>
<tbody>
<tr>
<td>any graph, $k \geq 1$</td>
<td>$2k - 1$</td>
<td>$2kn^{1/k}$</td>
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<tr>
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<td>$1 + \varepsilon$</td>
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<td>$k$-path separable</td>
<td>$4$</td>
<td>$O(k \log n)$</td>
</tr>
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</table>

**Note:** the space bound depends on $\log D$. So, for weighted graphs the sparse cover based routing scheme may produce non-polynomial space.
Thank you!