University of Bordeaux

The Compactness of Adaptive Routing Tables

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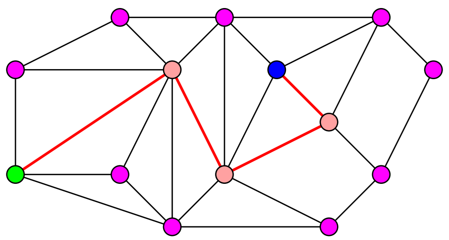
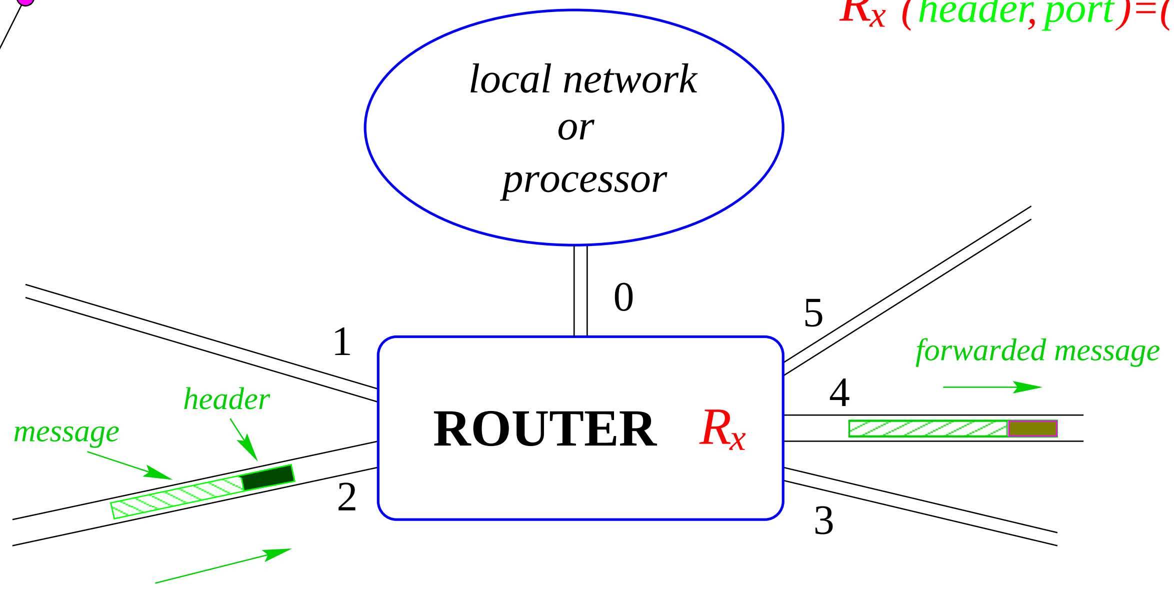
Routing Problem



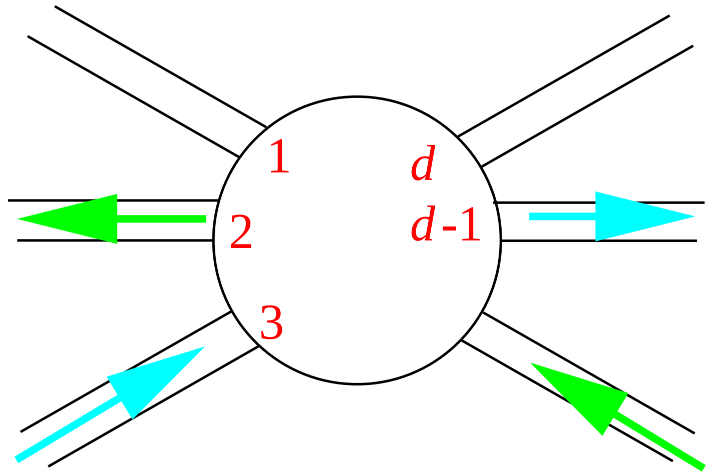
R_x (*header*, *port*) = (*header*, *port*)

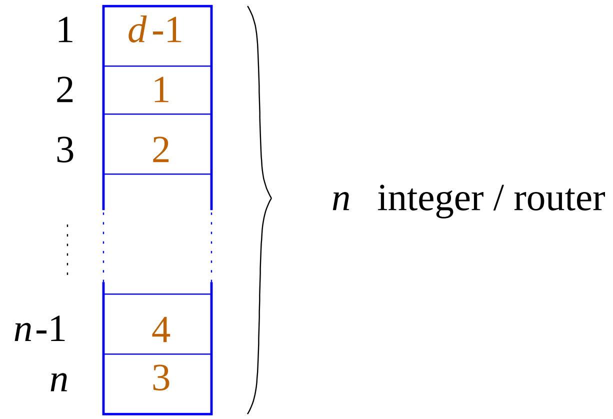


Constraints :

- bounded ressources (hardware / memory / buffers ...)
- high speed network (low latency / path lengths ...)

Routing Tables





message for the vertex 3

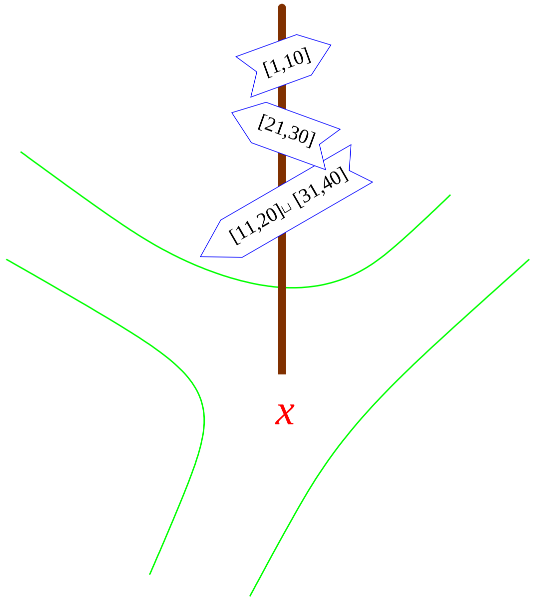
message for the vertex 1

- $n \log d$ bits/router,
- paths control (shortest paths),
- "direct" routing: time $O(\log n)$ (linear).

Compact Routing Tables

Intervals

[Santoro, van Leeuwen]



1	[1,10]
2	[21,30]
3	[11,20] [31,40]

d = degree(x)

k intervals

- $O(dk \log n)$ bits / router
- direct routing
- shortest path ? k ?

Adaptive Routing Tables

1	$2, 4,$ or 5
2	$\cdots$
$\vdots$	$\vdots$
n	$\cdots$

$\Rightarrow$ non deterministic

Adaptive routing $=$ non deterministic table $+$ Selection function S

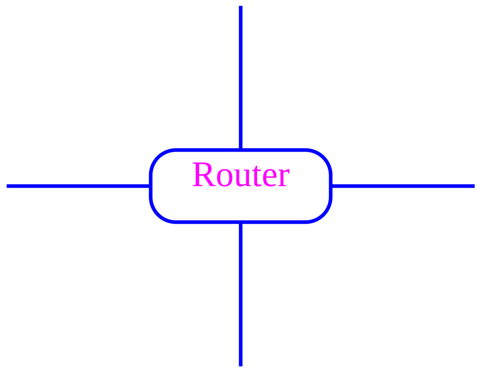
Selection function (e.g. at random, depending on the load,..., choose a shortest path)

$\Rightarrow$ need to store extra information for S.

Space Complexity Measure

Compactness = max. #intervals/link.

Routing table of compactness k = k-IRS .



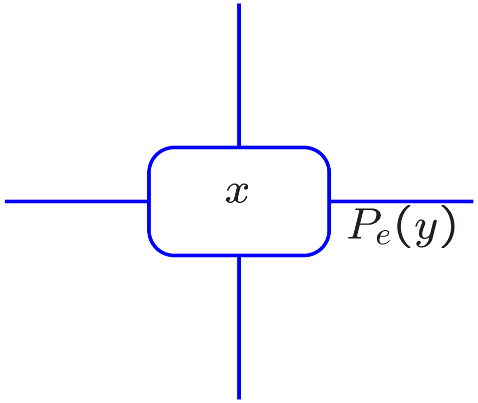
d=degree

Space complexity of the router $= O(dk \log n) + |S|$

depends on the selection function

Related Works

- Boolean Routing [FG'97]



1 link = 1 predicate

$P_e(y)$=TRUE iff $\exists$ shortest path
from x to y using e.

Complexity measure = length of P_e (in bits)

$P_e(y)$ iff $y \in [a,b]$; $y \in \{a_1, a_2, \cdots\}$
(simulation of interval routing and adaptive routing tables)

- Multipath Interval Routing [RŠ'97]
  E.g. : For bounded degree planar graphs, $O(\sqrt{n})$ intervals multipaths with optimal congestion.

- Characterization of all shortest path 1-IRS [FGNT'98]
  ($=$ Adaptive routing tables of compactness 1 encoding all the shortest paths)

- Broadcasting problem and IRS [EPTZ'97]
  $G \in$ all shortest path 1-IRS $\Rightarrow \exists$ Broadcasting protocol with $O(n)$ messages. In fact, $G \in$ shortest path 1-IRS $\Rightarrow O(n)$ messages [FGM'00].

Adaptivness : α

1	l_1^1	$\cdots$	l_1^α
2		$\cdots$	
$\vdots$	$\vdots$	$\cdots$	$\vdots$
n		$\cdots$	

$\longleftrightarrow$ EXACTLY α

Every destination appears on exactly α links

(α columns in the routing table)

If $\alpha = 1$, classical tables

If $\alpha \neq 1$, non deterministic tables

Notations:

$$\left.\begin{array}{ll}\text{Adaptiveness} & \alpha \\ \text{Compactness} & k\end{array}\right\} k\text{-IRS}_\alpha$$

$(\alpha = 1 \Rightarrow$ standard k-IRS$)$

Question #1

Does any graph supports a 1-IRS$_\alpha$, for every α ? (independently of the shortest paths).

Shortest path IRS$_\alpha$= In each table, there exists at least ONE shortest path for every destination.

Question #2

Find the lowest function $f_\alpha(n)$ s.t. every n-node graph supports a shortest path k-IRS$_\alpha$ and with $k \leqslant f_\alpha(n)$.

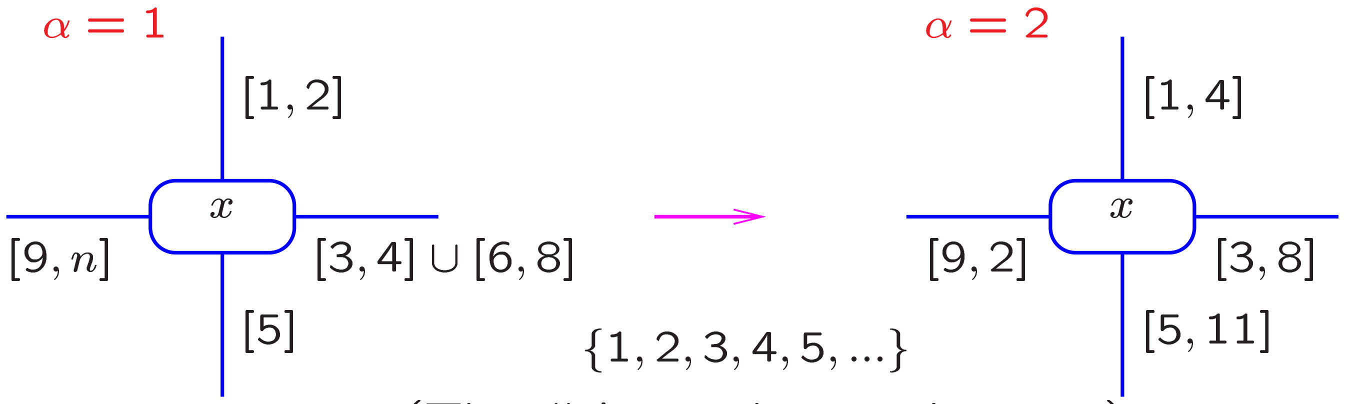
Answer for Question #1

Theorem 1. *For every* $1 \leqslant \alpha \leqslant \delta_{min}(G)$, $G \in$ 1-IRS$_\alpha$.

– If $\alpha > \delta_{min}(G)$, $\Rightarrow$ impossible.

– If $\alpha = 1$, the standard IRS $\Rightarrow$ use the proof of [SK'85], based on a spanning tree.

– By induction: transform any k-IRS$_\alpha$ into a k-IRS$_{\alpha+1}$.



(The # intervals may decrease)

Answer for Question #2

Remark. $\exists G_0$ *such that*

$G_0 \in$ *shortest path* $2 \log n - \mathrm{IRS}_2$ $(\alpha = 2)$, *and*

$G_0 \notin$ *shortest path* $n/8$-IRS_1 $(\alpha = 1)$.

$\Rightarrow$ growing of $f_1(n) \neq f_2(n)$.

BUT

Theorem 2. *(under* $1 \leqslant \alpha \leqslant \delta_{min}(G)$*)*

a) $\forall G \in$ *shortest path* k-IRS_α, $k \leqslant \frac{n}{\alpha}$.

b) $\exists G_0 \notin$ *shortest path* k-IRS_α, $k > \frac{n}{\alpha^c}$, c *constant* > 0.

Sketch of the proof of Theorem 2 b)

1. Assume $G_0 \in k$-IRS$_\alpha$. Knowing the labels, (intervals + node names), and with $O(n \log \alpha)$ extra bits, we can implement a standard shortest path routing tables on G_0.

$$T = \begin{array}{|c|}\hline 1 \\ \hline \vdots \\ \hline y \\ \hline \vdots \\ \hline n \\ \hline \end{array} \quad \underbrace{\overbrace{\begin{array}{|c|c|c|c|}\hline l_1^1 & l_1^2 & \cdots & l_1^\alpha \\ \hline & \vdots & \vdots & \vdots \\ \hline l_y^1 & l_y^2 & \cdots & l_y^\alpha \\ \hline & \vdots & \vdots & \vdots \\ \hline & & & \\ \hline \end{array}}}_{\alpha} \quad + \quad T' = \underbrace{\begin{array}{|c|c|}\hline 1 & \\ \hline \vdots & \vdots \\ \hline y & i \\ \hline \vdots & \vdots \\ \hline n & \\ \hline \end{array}}$$

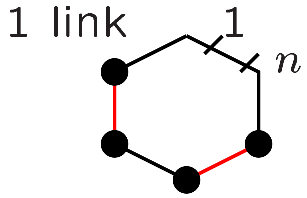
If l_y^i is a shortest path

k-IRS$_\alpha$

$d \log \binom{n}{2k}$ bits

$n \log \alpha$ bits $\quad (i \in \{1, 2, ...\alpha\})$

$T + T' \leqslant dk \log n/k + n \log \alpha$ bits



(there are at most $\binom{n}{2k}$ ways to choose k intervals of $\{1, ..., n\}$)

2. For every d, there exists a d-regular graph G_0 s.t. any (standard) shortest path routing tables needs $\Omega(n \log d)$ bits [GP'96] .

$$1. + 2. \quad \Rightarrow \quad dk \log(n/k) + n \log \alpha \ \geqslant \ n \log d$$

$$\text{choose } d = \alpha^c \quad \Rightarrow \quad \cdots$$

$$\Rightarrow \quad k \ \geqslant \ \frac{1}{4c\alpha^c} n \ \geqslant \ \frac{n}{\alpha^{c'}}, c' > 0$$

Conclusion

1. Extension

$$\left.\begin{array}{ll}\text{Adaptiveness} & \alpha \\ \text{shortest path, at least} & \beta\end{array}\right\} f_{\alpha,\beta}(n)\,?$$

2. Particular class of graphs

   Planar, bounded tree-width,...