University of Bordeaux

The Compactness of Interval Routing For Almost All Graphs

Cyril Gavoille *and* David Peleg

NOTATIONS

3 models

ADVERSARY

Labels are fixed (by an adversary), and cannot be permuted.

$$IRS_A(G) = \max_{\mathcal{L}} \left\{ \min_{R} \left\{ \max_{e \in E(G)} \#\text{interval}(e) \right\} \right\}$$

R: shortest path routing table on G

RANDOM

Labels are randomly permuted.

$$IRS_R(G)$$

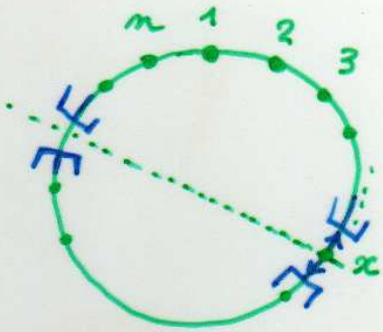
DESIGNER

Labels can be chosen in order to achieve the smallest number of intervals.

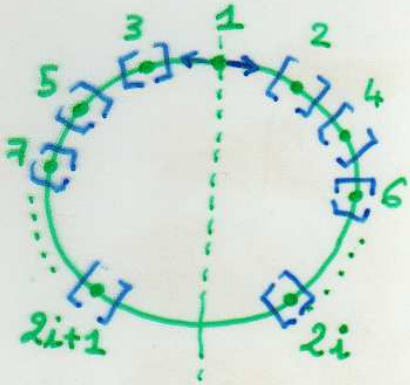
$$IRS(G)$$

Ex: n-node cycle, C_n

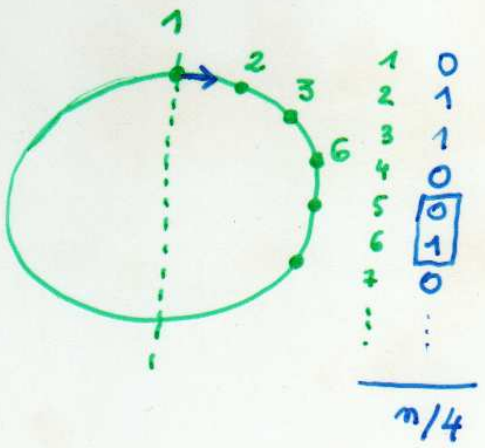
$IRS(C_n) = 1$



$IRS_A(C_n) = \frac{n}{2}$



$IRS_R(C_n) = \frac{n}{4}$ w.h.p.



(Actually: $IRS_R(G) \leq n/4 + o(n)$ $\forall G$.
[Gavoille, Peleg '97])

RESULTS

For at least a $1-\frac{1}{n}$ fraction of all the n-node graphs:

~~$2 \leq$~~ $IRS_A(G) \leq 3$

✓ $IRS_R(G) = 2$

✓ $1 \leq IRS(G) \leq 2$

(It is 1/edge excepted for $o(\log^3 n)$ edge/node where it is 2 intervals)

TECHNIQUE

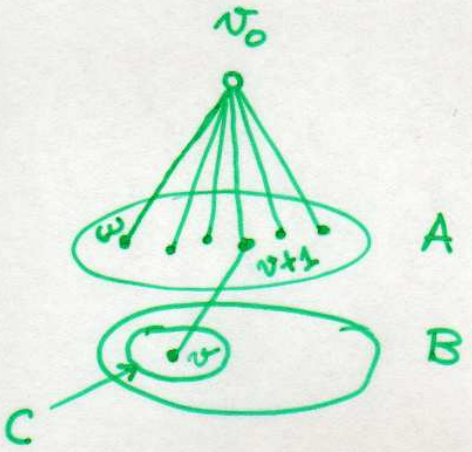
$\mathcal{G}_{n,1/2}$: set of random n-node graphs,

$\forall \{x,y\} \subset V(G)$, $Pr(\{x,y\} \in E(G)) = 1/2$

PROPERTY: If a property $\mathbf{P}$ holds for $G \in \mathcal{G}_{n,1/2}$ with probability $1 - o(1)$ (whp), then "almost all" the graphs have the property.

Ex: Almost all the n-node graph have diameter 2.

$IRS_R(G) \leq 2$



$|A| \sim \frac{n}{2}$

$|B| \sim \frac{n}{2}$

$C = \{ v \in B \mid v+1 \in A \text{ and } (v, v+1) \in E \}$

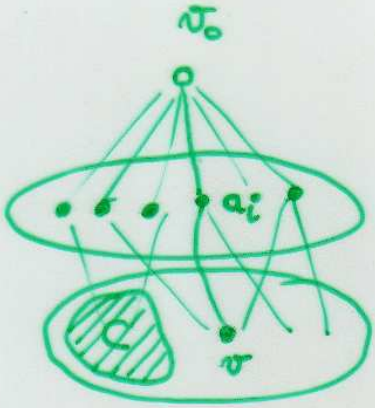
$X = B \setminus C$

whp $\quad |C| \sim \frac{n}{8}$

1. Label (v_0, ω) with $[\omega]$

or $[\omega, \omega-1]$ if $\omega - 1 \in C$

$IRS_R(G) \leq 2$ (cont'd)



2. For each $v \in X$ chose distinct $a_i \in A$.
$Q = Q \setminus \{a_i\}$, and repeat.

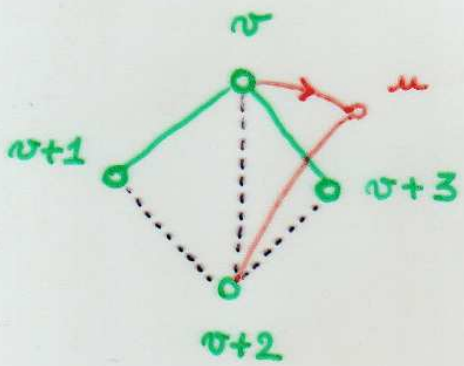
At any step $|Q| \geq |A| - |X| \geq \frac{1}{8} n$

$\Pr(\text{fail the } i\text{th step}) \leq \left(\frac{1}{2}\right)^{\frac{n}{8}}$

$\Downarrow$

$\Pr(\text{fail } \forall i \; \forall v_0) < c^{-n}$ ∎

$IRS_R(G) \geq 2$



$P = \left(\frac{1}{2}\right)^5$

o....o: no edge

o—o : edge

- Diameter 2 $\Rightarrow$ $\exists u$, $(u,v), (u, v+2) \in E$
- $u \notin \{v+1, v+3\}$

$\Rightarrow$ $I(u, v+2)$ has 2 intervals

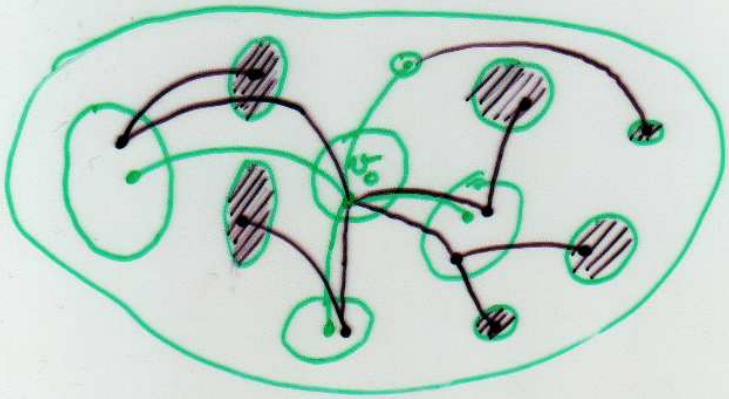
This pattern has probability $\left(1/2\right)^5$ to happen, and it independently occurs $\frac{n}{4}$ in G.

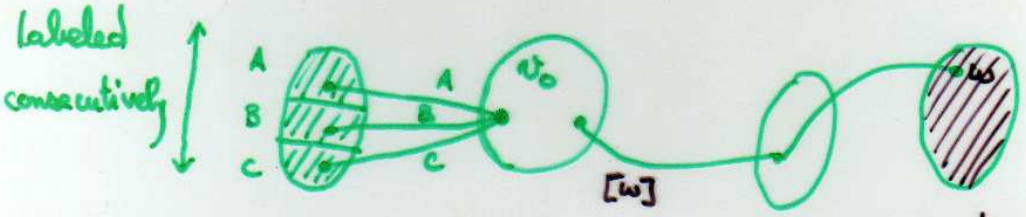
$\Rightarrow$ P_n (never occurs) $\to 0$

$\Rightarrow$ It occurs w.h.p. ∎

DESIGNER MODEL

clique decomposition of G





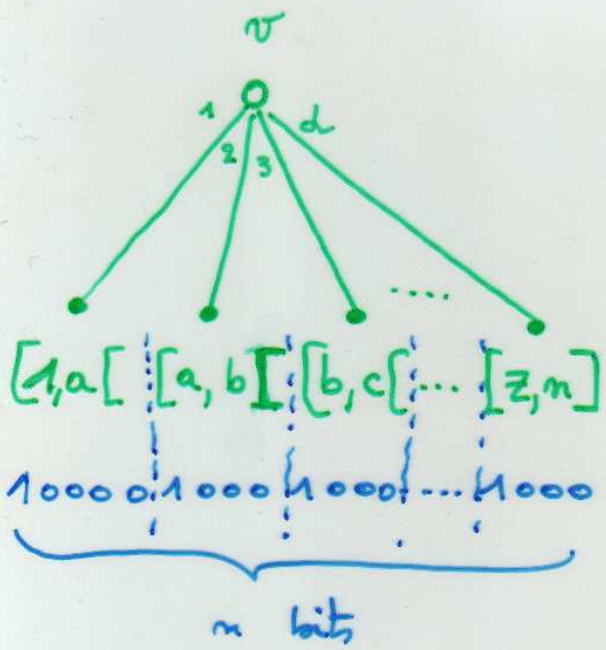
remote cliques (add 1 interval)

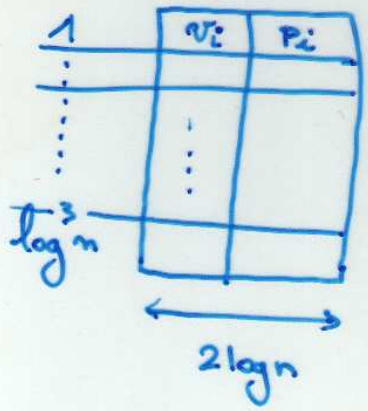
- $k \leq 2 \log n$ wh.p.
- # cliques of size k is at most $2^{k+2} \ln n$
- there is at most $O(\log^2 n)$ remote cliques

⇓

- There is at most $O(\log^3 n)$ nodes in the remote cliques

$n + o(\log^4 n)$ bits





$O(\log^4 n)$ bits in total

- Use first the exception node table ($\log^4 n$ bits)
- then use 1-IRS table (n bits)

CONCLUSION

$IRS(G) = 1$

for almost every G ?