Interval Routing in Reliability Networks

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Motivation

- *Interval routing - routing strategy wich allow compact routing tables.*

- *Problem of trade-offs between the memory requirements and the stretch of routing schemes.*

- *What is a realistic network ?*

- *Models of unreliable networks: worst case model versus probabilistic model.*

- *Random meshes: relation to the percolation.*

Reliability Networks

Extension of the standard Erdös-Rényi model of random graphs

$\mathbb{G}(B,p)$: *probability space of random graphs, where:*

B : *base graph*

$1-p$: *a failure probability*

Definition 1 *$G \in \mathbb{G}(B,p)$ is a uniform (labeled) random spanning subgraph of B as $V(G) = V(B)$ and $E(G) \subseteq E(B)$ such that:*

$$\Pr(G) \; = \; p^{|E(G)|}(1-p)^{|E(B)|-|E(G)|} \; .$$

$\mathbb{G}(M_s^r, p)$: *probability space of random r-dimensional meshes*

Routing Tables and Interval Routing

δ−`stretched path` - *the length of the path is at most the length of a shortest path between its extremities plus* δ

δ−`stretched routing scheme` - *scheme for which all the routes are* δ*-stretched paths*

(k, δ)*-IRS* = *k-IRS that is a* δ*-stretched routing scheme*

$\mathrm{IRS}_\delta(G)$ = *the smallest integer k such that G supports a* (k, δ)-IRS *(G is connected)*

$\mathrm{IRS}_\delta(G) = \max_i \mathrm{IRS}_\delta(G_i)$, *where the* G_i*'s are the connected components of G*

In particular:

$(k, 0)$*-IRS* = *shortest-path k-IRS*

$\mathrm{IRS}_0(G)$ = $\mathrm{compactness}(G)$

Previous Works on Random Graphs

Theorem *[Flammini, van Leeuwen and Marchetti-Spaccamela 1998]*

With high probability, a graph $G \in \mathbb{G}(n,p)$ *requires* $\Omega(n^{1-1/\Theta\sqrt{\log n}})$ *intervals per outgoing edge for* $p = 1/n^{1-1/\Theta\sqrt{\log n}}$.

Theorem *[Gavoille and Peleg 2001]*

Almost all graphs (i.e. graphs of $\mathbb{G}(n,p)$ *for* $p = 0.5$ *and with high probability) support a shortest-path* 2-*IRS.*

Open question: *Do almost all graphs support shortest-path* 1-*IRS ?*

Theorem *[Nehéz 2002]*

Lower bound for shortest-path k-IRS on random n-node tori is of the form $\Omega(\sqrt{\log n})$ *with high probability.*

Our Results

Theorem 1 *Let* $0 < p < 1$ *be a constant. With probability approaching to* 1 *a random mesh* $G \in \mathbb{G}(M_s^r, p)$ *verifies*

$$\mathrm{IRS}_\delta(G) = \Omega((\delta+1)^{1-r} r^{-3} (\log s)^{1-1/r}).$$

Experimental result: *Random square n-node meshes support shortest-path k-IRS with* $k = O(n^{0.827})$.

Proof of the Theorem 1

Main idea: *It is necessary to show that a random mesh contains a subgraph of δ-stretched path with a nonconstant compactness with high probability.*

Lemma 1 *If X is a subgraph of δ-stretched paths of G, then* $\mathrm{IRS}_\delta(G) \geq \mathrm{IRS}_\delta(X)$.

Lemma 2 *Let B be a connected graph with n nodes and maximum degree d, and let X be a m-subgraph of B with x nodes. Then, $G \in \mathbb{G}(B, p)$ contains X as subgraph of δ-stretched paths with probability approaching to 1 as $m \to \infty$. So, by Lemma 1,* $\mathrm{IRS}_\delta(G) \geq \mathrm{IRS}_\delta(X)$, *for every δ.*

Lemma 3 *Let G be a graph, and Π be a routing property. Let $P(u,v)$ be the set of nodes w such that there exists a path of $\Pi(G)$ from u to w that starts with the edge $\{u,v\}$. Let U and W be two disjoint node subsets of G such that for all distinct nodes $w, w' \in W$, there is $u \in U$ such that for each neighbor v of u it holds $w \notin P(u,v)$ or $w' \notin P(u,v)$. Then, every k-IRS with property Π on G must satisfy*

$$k \;\geq\; \frac{|W|}{\sum_{u \in U} \deg(u)}\;.$$

Proof of the Theorem 1

$\star$ - *graph G and its copy G' are connected by a path P_t*

$H_t^{r+1} = H_t^r \star P_t$, *with* $H_t^1 = P_t$, *it is:*

$$H_t^r \;=\; \underbrace{P_t \star P_t \star \cdots \star P_t}_{r\ times}$$

Lemma 4 $\mathrm{IRS}_\delta(H_t^r) = \Omega\left((\delta+1)^{1-r} r^{-2} t^{r-1}\right)$, *for all* $\delta \geq 0$ *and* $r \geq 1$.

Experimental Results

Random square n-node meshes support shortest-path k-IRS with $k = O(n^{0.827})$.

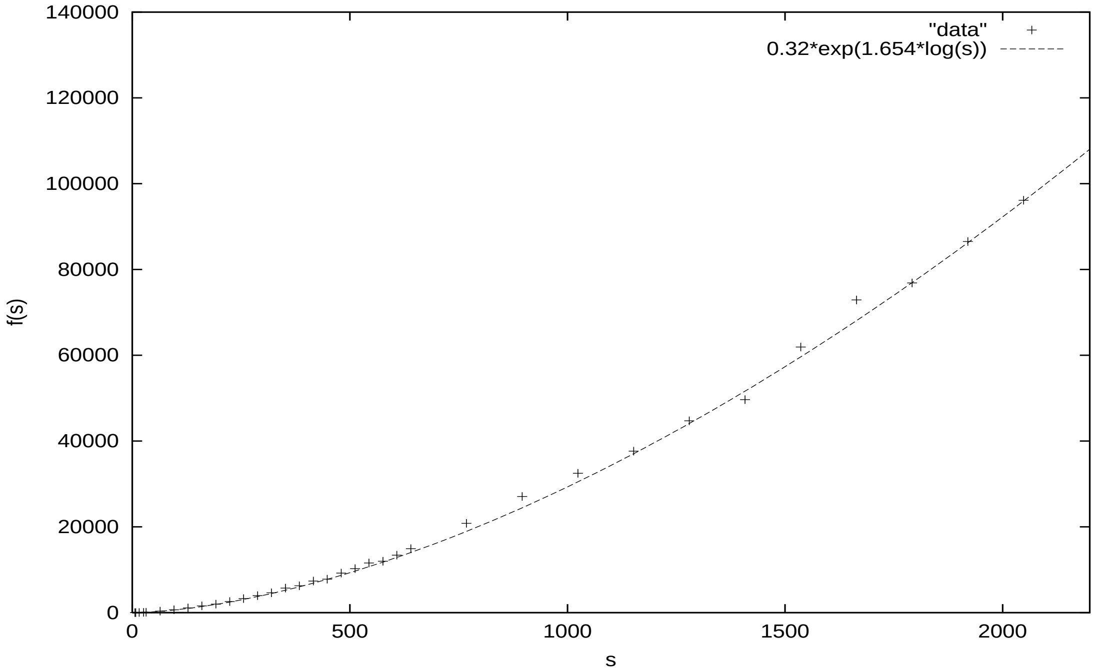

Figure 1: The size of the largest biconnected components of a random $s \times s$ mesh. The tests come from averaging 2000 experiments (for the small values of s) and up to 50 experiments (for the larger values of s).

Examples of the Computation of Our Algorithm

Directions for the Future Research and Open Problems

1. Extension to arbitrary routing strategies.

2. Extension to random B-graphs with different edge probabilities. Example: model $\mathbb{G}(B, p_r, p_c)$ where B is a mesh.

3. Can we have a base graph B of high compactness, say $\Omega(n^{\varepsilon})$, such that random B-graphs have low compactness (with high probability)?

4. What is the compactness of a general n-node random B-graph for $p = 0.5$ (still with high probability)? Is $\Omega(n)$ possible? Same question if B is a bounded degree graph? or a planar graph? or if B is the hypercube?

5. Is H_t^2 the worst-case sub-mesh for the compactness?

6. The most interesting question remains: is there an n-node planar graph with compactness larger than $\sqrt{n}$?

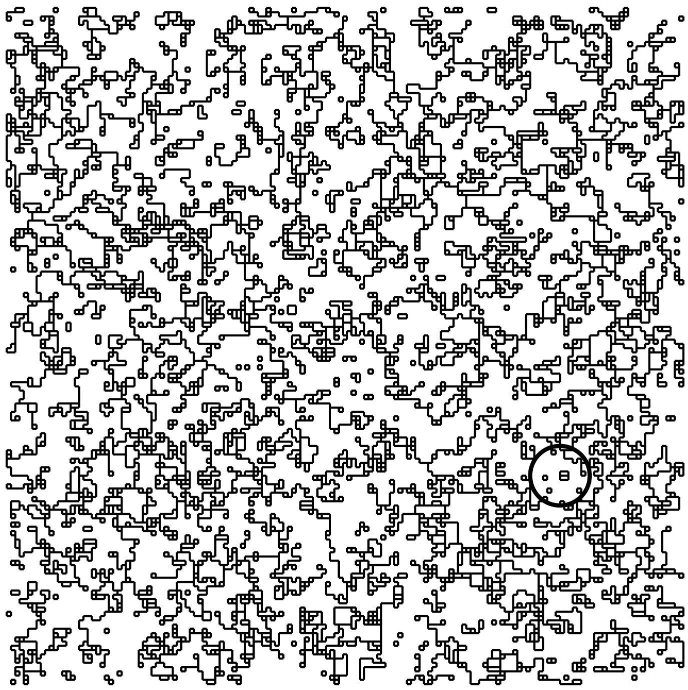

Figure 2: 160×160 random mesh with $p = 0.50$ without trees and with 672-node largest biconnected component.