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Compact Routing Schemes with Low Stretch Factor

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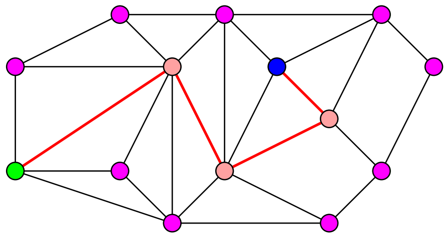
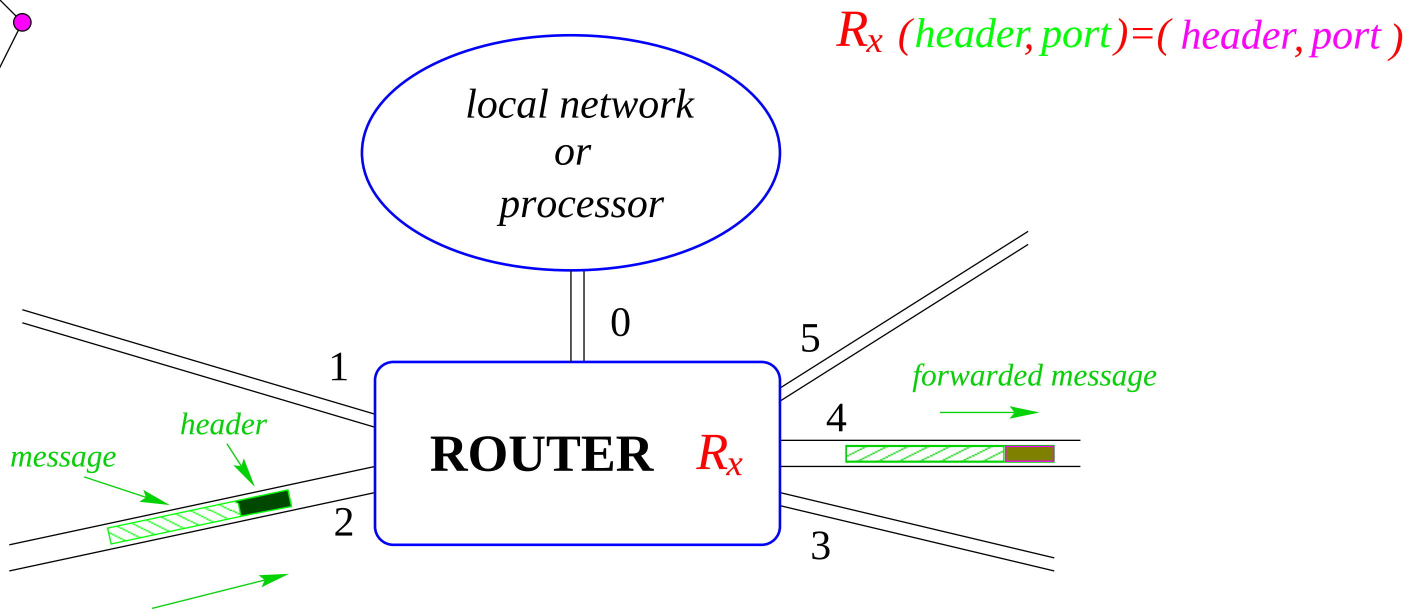
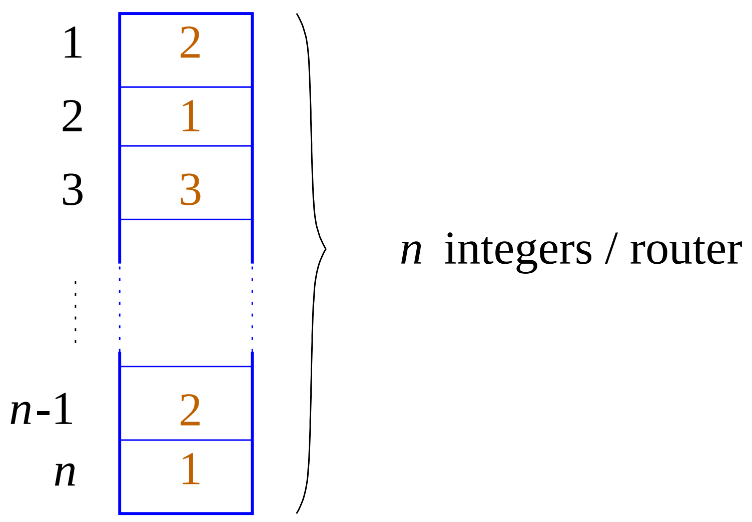




Constraints :

- bounded ressources (hardware / memory / buffers ...)
- high speed network (low latency / path lengths ...)

Solution : Routing Tables

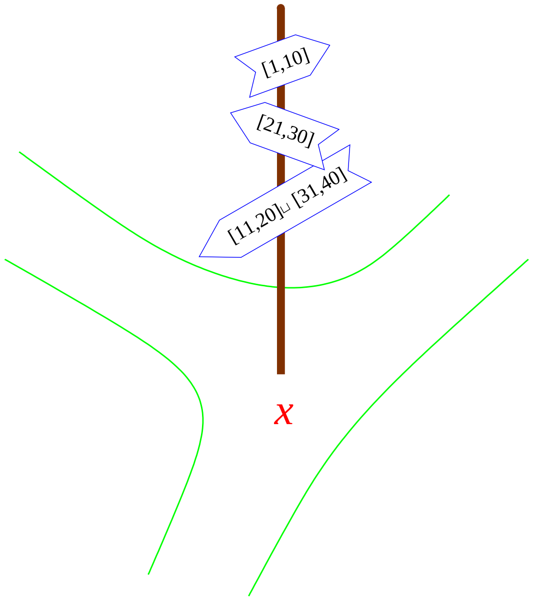


- $n^2 \log n$ bits in total
- control of the paths
- "direct" routing

Compact Tables

Intervals

[Santoro, van Leeuven]



1	[1,10]
2	[21,30]
3	[11,20] [31,40]

d = degree(x)

k intervals

- $O(dk \log n)$ bits / router
- direct routing
- shortest paths ? k ?

Negative Results

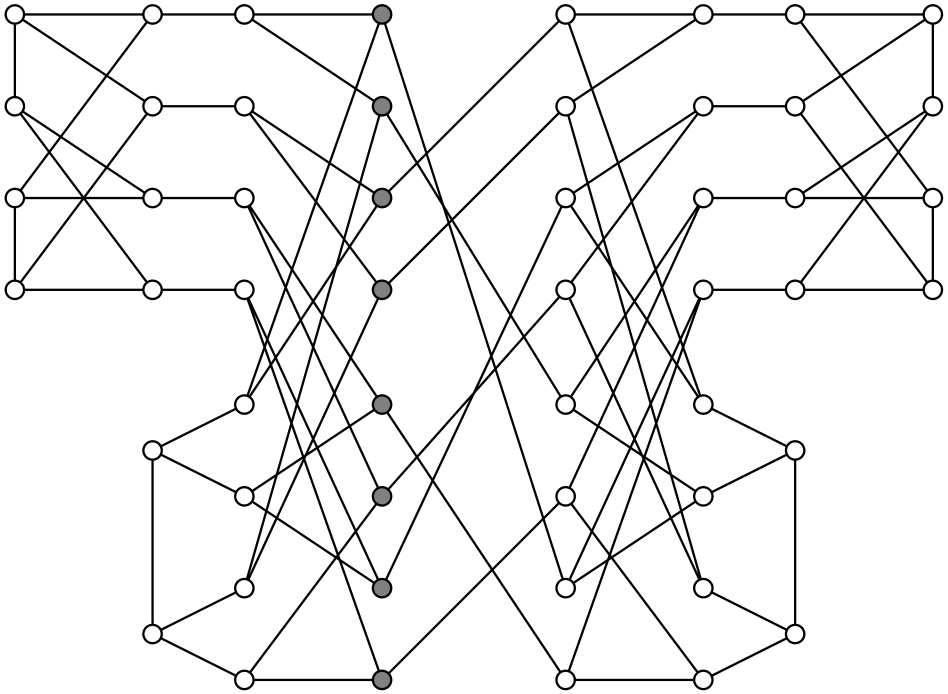
For every node labeling and for every choice of routing paths (shortest paths) there are some networks that require:

- $k \geq \frac{n}{4} - o(n)$ intervals (optimal) [MFCS '97]
- memory $\geq n \log n$ bits / router [PODC '96]

(for every coding strategy, Kolmogorov)

$\Rightarrow$ These are worst-cases ...

Cubic graph where $\Theta(n)$ intervals are required



G_{3,π_3}

Positive Results

For "almost" all the n-node graphs

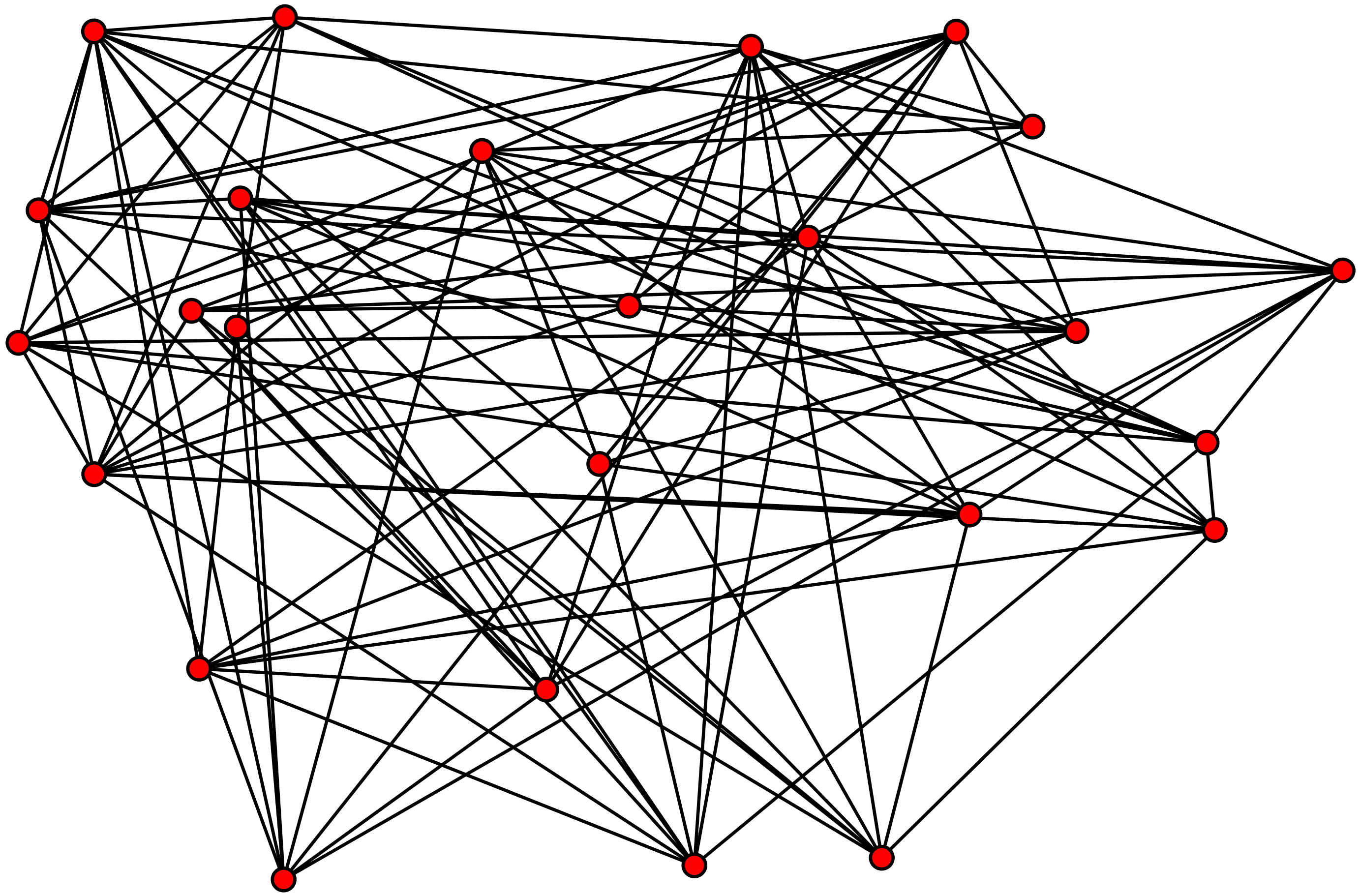
- $3n + o(n)$ bits / router suffice [Vitányi, *et al.* PODC '96]
(*Kolmogorov random graphs*)

- 1 interval suffices, but for $O(\log^3 n)$ edges where 2 are required.
$n + o(n)$ bits / router suffice [Gavoille, Peleg '98]

$\Rightarrow$ A theoritical "average" graph $\neq$ The internet network

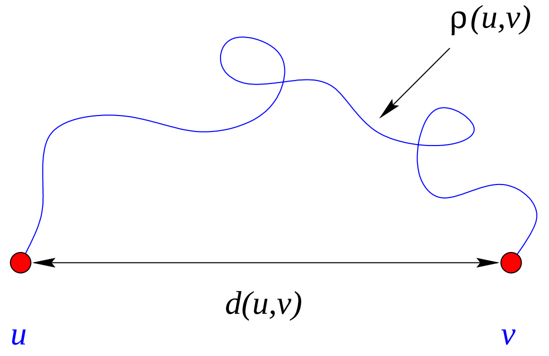
Diam $= 2$

$n = 25$



Hierarchical Routing

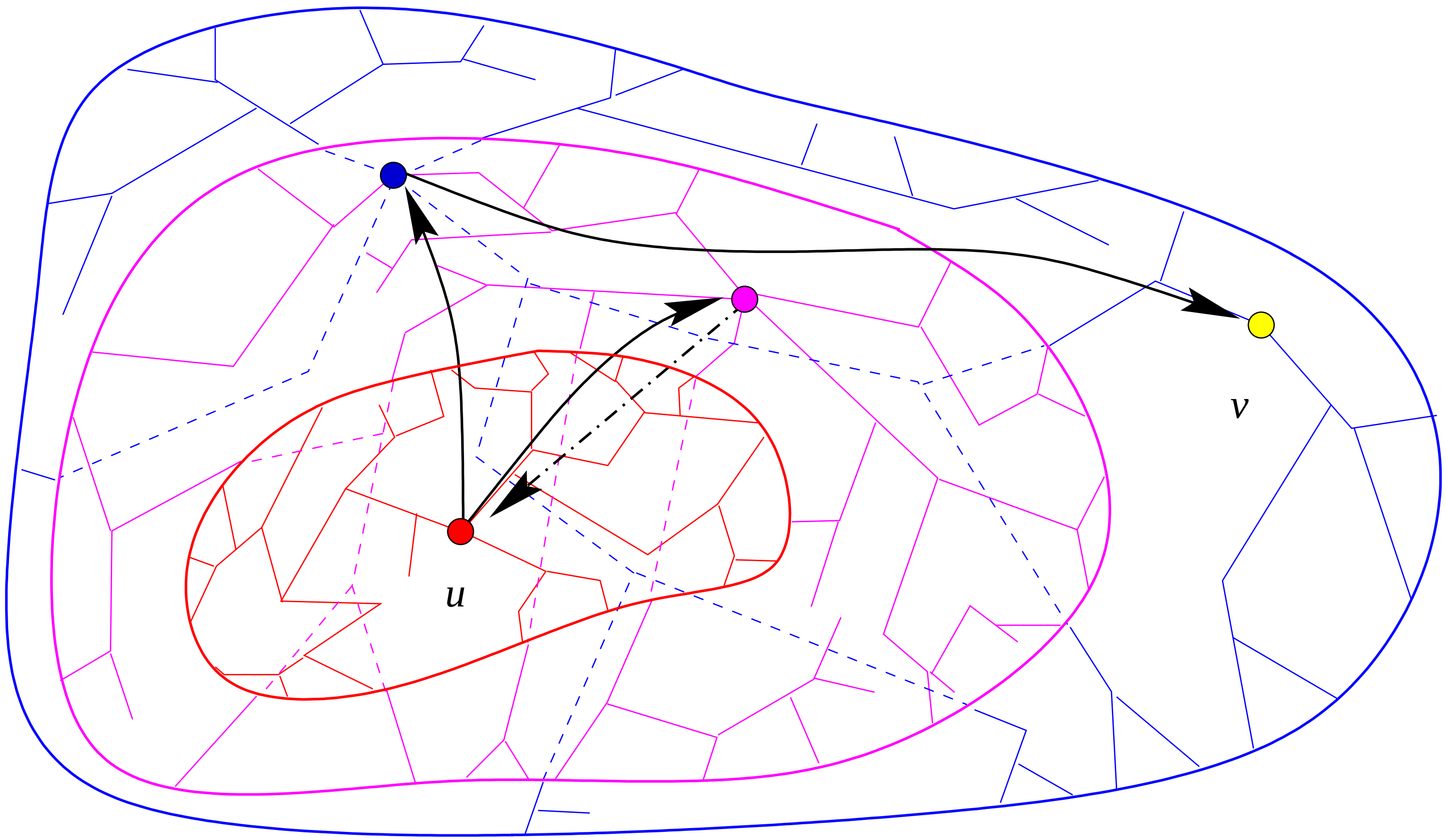
Trade-off between *memory* and *length* of the routes:



$$\text{stretch} = \frac{\rho(u,v)}{d(u,v)}$$

Awerbuch, Peleg [FOCS '90], k levels, $k \geq 1$

- memory $O(k\, n^{1/k} \log^2 n)$ bits / router
- stretch $O(k^2)$



- memory $O(k\,n^{1/k}\log^2 n)$ bits / router
- stretch $O(k^2)$
- non direct: header modifications

Compact and Direct Routing

[Eilam, Gavoille, Peleg PODC '98]

2 levels, $k = 2$

	[Awerbuch, *et al.*]
stretch ≤ 5	stretch ≤ 3
memory $\leq \sqrt{n}\log^{1.5} n$	memory $\leq \sqrt{n}\log^{2} n$
$\overline{\text{stretch}} \leq 3$	$\overline{\text{stretch}} \leq 3$
$\lvert\text{routes}\rvert \leq \lceil 1.5\,\text{Diam}\rceil$	
direct routing (intervals)	2 phases

Consider a collection $\mathcal{H}$ of subsets of size t of elements from a set $V = \{1, \ldots, n\}$. A set $P \subseteq V$ is said to *cover* the collection $\mathcal{H}$ if for every $B \in \mathcal{H}$, $B \cap P \neq \emptyset$.

The greedy algorithm starts with $P = \emptyset$ and iteratively adds to the set P an element in V occurring at the most uncovered sets. The (polynomial time) algorithm stops when P becomes a cover.

Lemma [Lovász '75]

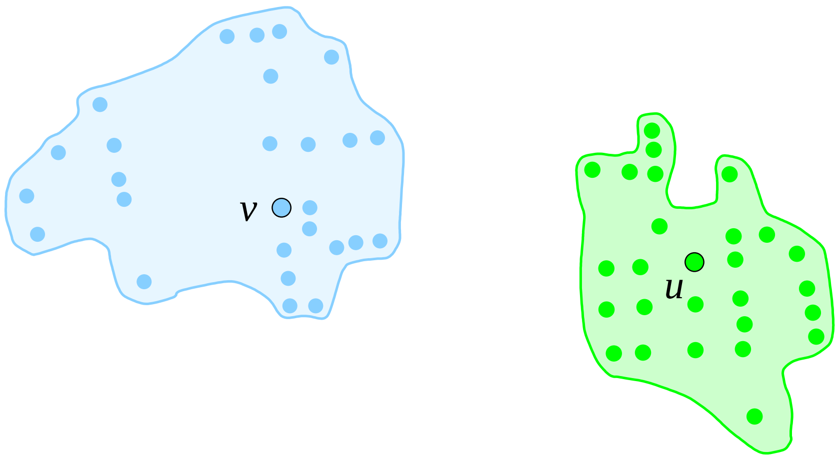
Let P be a greedy cover for $\mathcal{H}$. Then

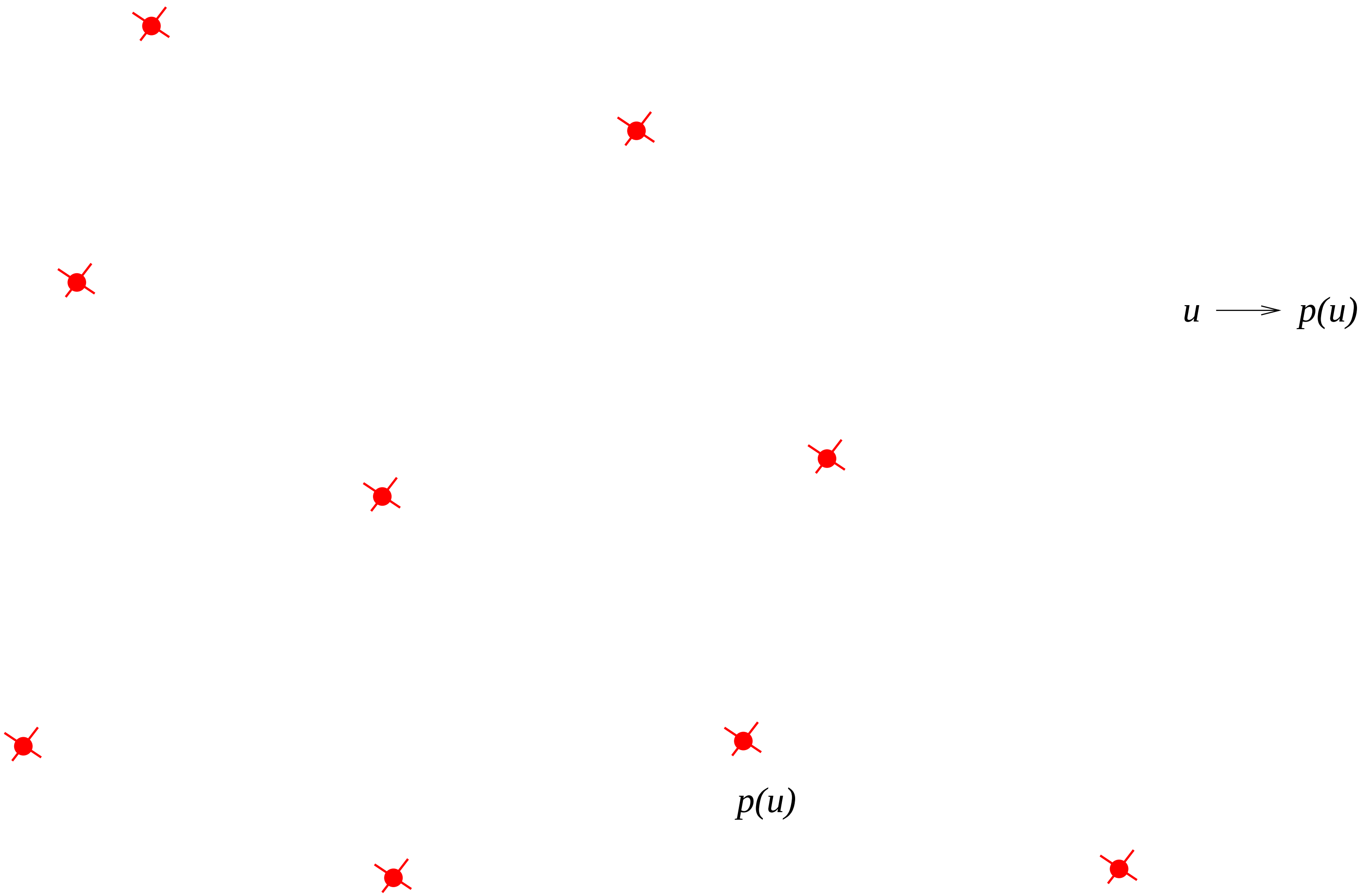
$$|P| < \frac{|V|(\ln|\mathcal{H}| + 1)}{t}$$

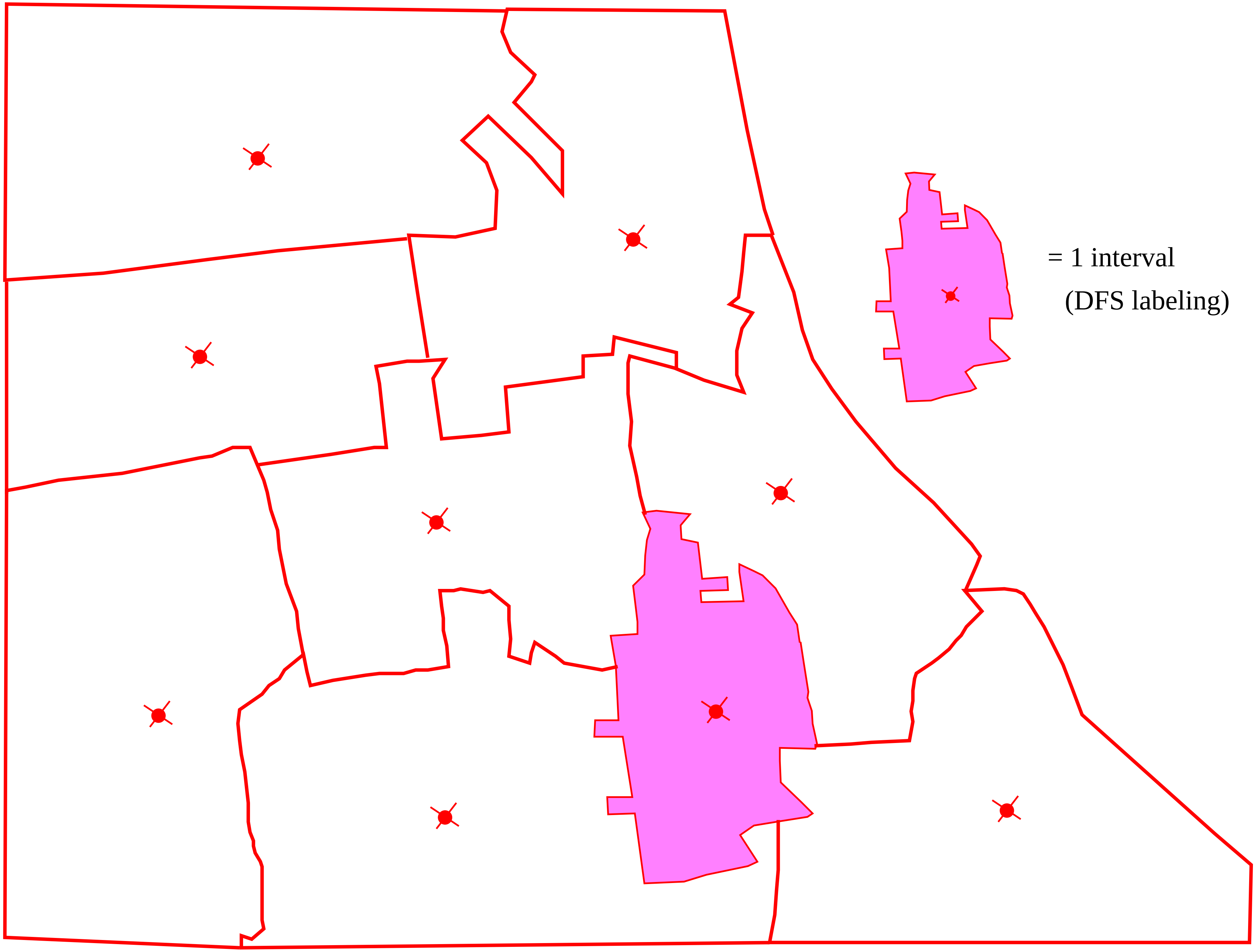
Since $|\mathcal{H}| = |V| = n$, chosing $t = \sqrt{n(1 + \ln n)}$

$|P| < \sqrt{n(1 + \ln n)}$

$u \longrightarrow B_u \qquad |B_u| = t$



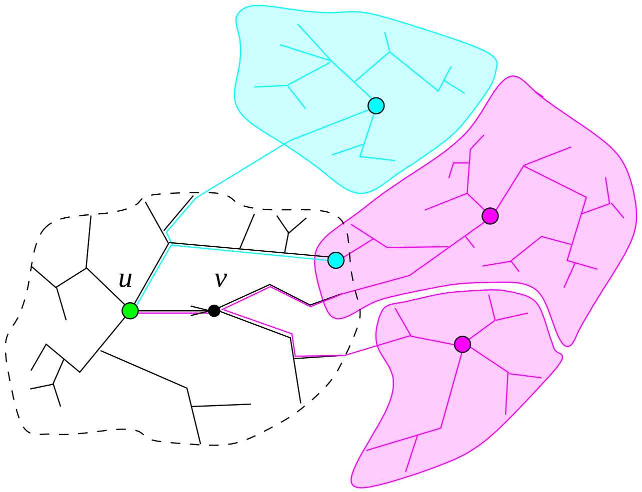

$u \longrightarrow p(u)$
$p(u)$

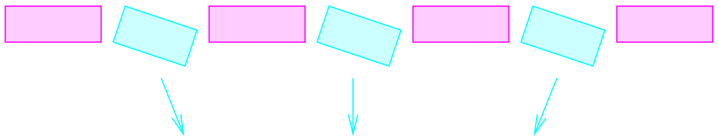

= 1 interval
(DFS labeling)

Analysis: # intervals

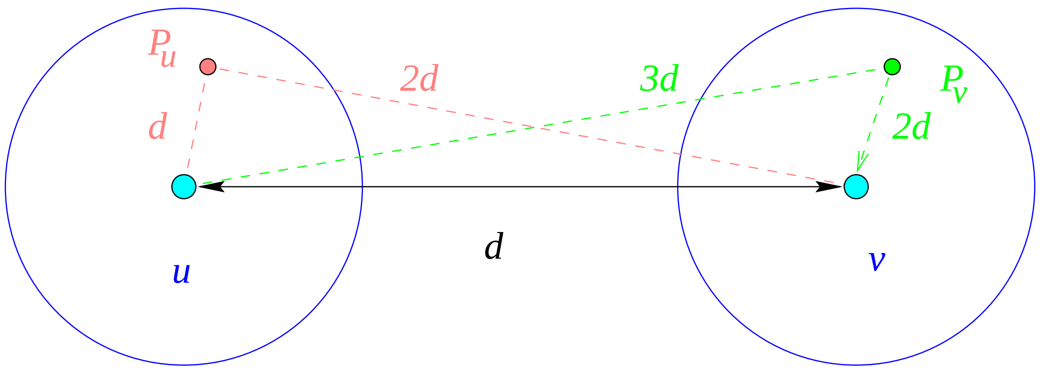
The arc (u, v) needs at most $1 + |P| + |\mathcal{B}_u| \times 2$

- $O(\sqrt{n \log n})$ intervals / arcs
- $O(\sqrt{n} \log^{1.5} n + \deg(u) \log n)$ bits / router
- $O(n^{1.5} \log^{1.5} n)$ bits in total





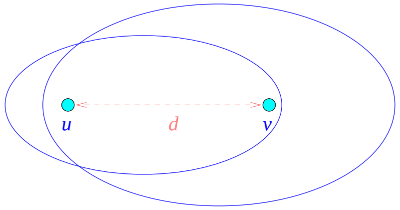
Analysis: stretch



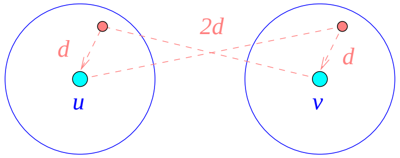
$$\rho(u,v) \le d(u,P_v) + d(P_v,v) \le 5d(u,v)$$

Average stretch: $\bar{s} \leq 3$

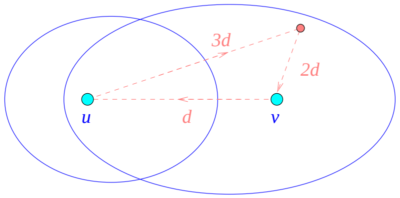
$$\rho(u,v) + \rho(v,u) \leq 6d(u,v)$$



$$\Rightarrow \quad 2d(u,v)$$



$$\Rightarrow \quad (3+3)d(u,v)$$



$$\Rightarrow \quad (5+1)d(u,v)$$

$$\Rightarrow \quad \bar{s} \quad \leq \quad \frac{1}{n(n-1)} \sum_{u<v} 6 \quad \leq \quad 3$$

Conclusion

Next step: a new direct hierarchical routing scheme

For every $k \geq 1$,

memory: $O(k\, n^{1/k} \log^{1-1/k} n)$ intervals

stretch: $s \leq \frac{4^k - 1}{3}$