

# A NEW SLOT ALLOCATION FOR ATFM

Vu Duong<sup>†</sup>

Frédéric Ferchaud<sup>†‡</sup>

Cyril Gavaille<sup>‡</sup>

Mohamed Mosbah<sup>‡</sup>

## INTRODUCTION

Once the aircraft is in the air, it is supervised by ATCo<sup>1</sup> to guarantee safe separation. The capacity is defined according to ATCo workload, responsible of air traffic in a sector of airspace, simplified in a number of aircraft per hour. In Europe, the Central Flow Management Unit (CFMU) is responsible for the flow management in order to guarantee the capacity of the sectors.

Air Traffic Flow Management (ATFM) is a service established with the objective of contributing to safe, orderly, and expeditious flow of air traffic. The ATFM is divided into three phases[1]:

- The strategic phase takes place between 18 month and seven days prior to the day of operation and includes research, planning and coordination activities. The output of this phase are the capacity plan for the following year.
- The pre-tactical phase is applied during 6 days prior to the days of operation and consists of planning and coordination activities. The output is the ATFM Daily Plan.
- The tactical phase is applied on the day of the operation. This phase updates the daily plan according to the actual traffic and capacity. The management of the traffic is achieved through slot allocation.

In Europe, to realize a flight between two airports, a company must submit a flight plan to the CFMU. The flight plan contains the following informations :

- aircraft identifier or call-sign,
- departure airport,
- arrival airport,
- desired take-off time,
- waypoint list (air road taken),
- flight level associated with each waypoint.

From flight plan information, the decision to activate traffic balancing demand is to be made for the sectors where necessary (number of aircraft greater than the capacity). In accordance with the principle of the "First filed-First Served", the system in charge of the regulation, namely Computer Assisted Slot Allocation (CASA), extracts all the flights entering the specified airspace and sequences them into the order that they would have arrived at the airspace in absence of any restriction.

On this basis, the Take-Off Time (TOT) is calculated. The Calculated TOT (CTOT) is then transmitted to the

airlines concerned and to the control tower at the departure airport.

In EUROCONTROL ATFM, the CFMU is in charge of the slot allocation for all flights and elaborates the daily planning of slots. A slot corresponds to a time window (-5 min, +10 min) of the CTOT during which the aircraft must take-off.

Unfortunately, as a matter of fact, uncertain operational events (weather conditions, technical failure, waiting passenger...) occurs daily and disturb the CFMU planning, leading to safety problems and sub-optimally used capacity. We call these events operational aleas. When an aircraft can not take its allocated slot, we say it has got an alea. To accommodate the aleas, we introduce the notion of absorption areas (AA). An AA is a number of slots, left unfilled during the slot allocation process, allowing the absorption of such disturbances with least modification of the planning.

Finding the best configuration of the AAs corresponds to balancing two complementary objectives: maximizing their effect and minimizing "load loss". Load loss correspond to the lost slots. The lost slots correspond to unused AAs or slots not taken by aircraft with aleas.

We implemented AAs in a prototype ATFM simulator, called SIVOR<sup>2</sup>[2].

Our experimental results confirm that AAs could increase the throughput. Then we developed a theoretical approach to formally prove these results. We show that, whatever the rate of aleas is, the flow with AAs is always higher than without them.

Another interest of AAs is that if we find an algorithm giving the best distribution of unfilled slots, then its implementation will neither change sector topologies, nor controller's work nor the flight plans submission procedure.

## I. SIVOR

### A. Obtained results

With SIVOR, we showed experimentally the utility of absorption areas.

If an aircraft lost its slot, we must reallocate it. This aircraft could perturb the initial planning and can cause excess of sector capacity. So some other aircraft could be delayed. With SIVOR, we observed that whatever the rate of aleas is, absorption areas decrease the delays due to aleas, and sometimes also increase the throughput (see Figure 1). The results were obtained for a capacity of 15 aircraft per sector and per hour. On the Y-axis of the upper graph of Figure 1, the 0 value corresponds to a capacity of 15 for the pretactical planning, 1 to 14, 2 to 13, and 3 to 12. We

<sup>†</sup>EEC : EUROCONTROL Experimental Centre

<sup>‡</sup>LaBRI : Laboratoire Bordelais de Recherche en Informatique

<sup>1</sup>Air Traffic Controllers

<sup>2</sup>Simulateur de VOI en Route

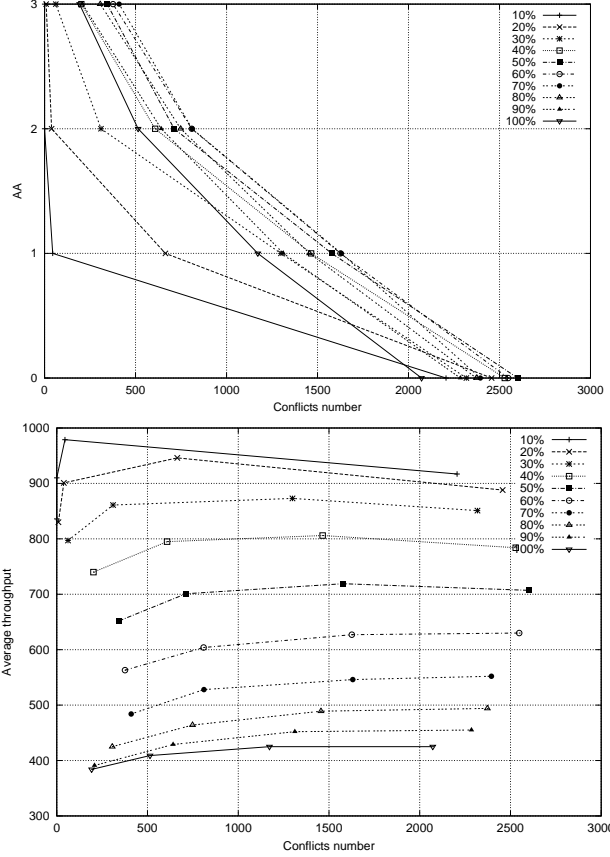


Fig. 1. Obtained Results with SIVOR showing that AAs are useful. On the X-axis we have the number of times we had to resolve an excess of capacity due to aleas. The Y-axis of the upper graph corresponds to the AA height used, and on the lower graph to the average throughput. For example, with 10% of uncertainty, the number of conflicts are reduced for one AA (AA1) and the throughput is increased. However, for AA2, the throughput is really decreased in relation to AA1 without a gain for the conflicts, in this case we choose the AA1 strategy.

only used SIVOR with continuous AAs, i.e., the capacity used for the pretactical planning is the same during all the simulation. The results show the benefits of AAs.

Then, we tried to set up AAs in a discontinuous way. For example: if we have a capacity of 15, we used a capacity of 13 during 15 min, and a capacity of 15 during 45 min. The results obtained were not conclusive, in particular because we had not taken into account dependencies between neighboring sectors. Indeed, if an aircraft that lost its slot, passing through several regulated sectors, the unfilled slots must be impacted, otherwise the unfilled slots are unusable.

We will use a model based on graph theory to adress this problem. If we manage to define a good representation of the “en-route” air traffic, then the known algorithms of the graph theory will enable us to take into account the neighborhood relationship of the sectors. We will probably be able to find the best distribution of the AAs in the pre-tactical planning, and thus to reduce the load loss.

## II. THEORETICAL APPROACH

An AA corresponds to one or more unfilled slots in the pre-tactical planning. Although AAs decrease the capacity, we will show theoretically, that this is not the case when some aircraft do not take their slots.

Let  $n$  be the total number of available slots. Then one will define various values, functions of  $n$  noted  $\text{throughput}_1$ ,  $\text{throughput}_2$ ...  $\text{throughput}_i$ , representing the average throughput observed in experiment  $i$ .

### A. Without dynamic reallocation of the lost slots by the uncertainty

Let  $\text{throughput}_1 = n$ . A throughput of  $n$  corresponds to the case without AAs and that all slots are taken as planned. However, this ideal case does not exist. Some aircraft do not take their slots, and we must reallocate them. Thus the throughput will be always lower than  $\text{throughput}_1$ .

Let  $p$  be the probability that an aircraft takes its slot.

In this part we assume that the uncertainty on a flight is known only at TOT, implying a lost slot. In this case, a slot is reallocated to the flight. If there is no one, the flight is canceled. Of course, this is a question of an analysis of strategy in an academic case. We will see later how to use some lost slots (dynamic reallocation).

Let  $\text{throughput}_2$  be the throughput obtained without AA and  $\text{throughput}_3$  with AA. Lastly, we note  $z$  the number of unfilled slots (AAs).

We have four types of slots:

- filled slot (slot allocated by the CFMU)
- unfilled slot (AAs)
- taken slot (slot used for an aircraft take-off)
- lost slot (unused slot)

We obtain an average throughput for each case :

$$\begin{cases} \text{throughput}_1 = n \\ \text{throughput}_2 = pn \\ \text{throughput}_3 = p(n - z) + \min\{z, (1 - p)(n - z)\} \end{cases}$$

For  $\text{throughput}_3$  we allocate  $(n - z)$  aircraft. We have  $p(n - z)$  aircraft taking-off in its slots, and we add those that find unfilled slots to take-off. We have two possibilities :

- we have more aircraft getting alea than unfilled slots
- we have less aircraft getting alea than unfilled slots

We seek to determine the best  $z$  according to  $p$  to have the best throughput for  $\text{throughput}_3$ . The maximum capacity is obtained when all the planes getting alea take unfilled slots. More precisely :

$$\begin{aligned} z &= (1 - p)(n - z) \\ n - z &= n - n \frac{1-p}{2-p} \\ z &= n \frac{1-p}{2-p} \\ \text{and } \text{throughput}_3 &= \frac{n}{1+(1-p)} \text{ with a such } z \end{aligned}$$

To show the AAs benefits, we must have  $\text{throughput}_3 - \text{throughput}_2 > 0$ .

We have :

$$\begin{aligned} \text{throughput}_3 - \text{throughput}_2 &= n - z - pn \\ \text{throughput}_3 - \text{throughput}_2 &= n\left(1 - \frac{1-p}{2-p} - p\right) \\ \text{throughput}_3 - \text{throughput}_2 &= n\frac{(p-1)^2}{2-p} \end{aligned}$$

The difference is always positive for all  $p \in [0, 1[$  thus  $\text{throughput}_3$  is always higher than  $\text{throughput}_2$  when there are aleas ( $p < 1$ ).

Note that we consider here that all AAs are used. Normally we have a ‘‘load loss’’. But in this part we consider it equal to zero, because the aim is precisely to make tighten this ‘‘load loss’’ towards zero.

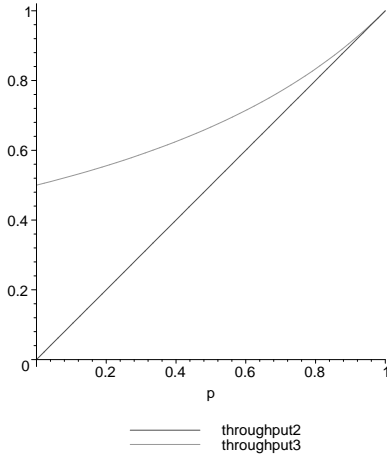


Fig. 2. Representation of  $\text{throughput}_2/N$  and  $\text{throughput}_3/N$  according to  $p$ .

Figure 2 shows that  $\text{throughput}_3$  is greater than  $\text{throughput}_2$ .

### B. With dynamic reallocation of the lost slots

It is considered now that we can anticipate some aleas and reallocate certain lost slots. We call these slots the recovered slots.

We note  $t_0$  the minimal time necessary to reallocate a slot left by an aircraft (see Figure 3).

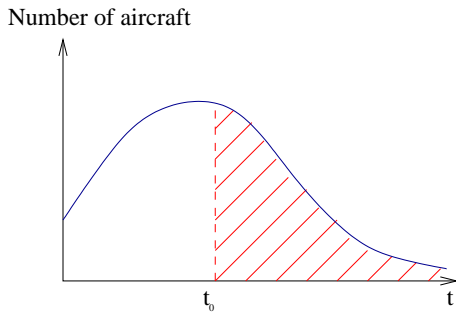


Fig. 3. Number of flights whose alea is known at a time  $t$  before take-off time. The area shaded in red represents the recoverable slots because the uncertainty is known at  $t \geq t_0$  before the slot time. We note  $q$  the proportion of the zone shaded in red compared to the integral of this curve.

Calculations are carried out for the optimal case, the case we wish to reach. It is considered that each time that one is informed of a risk at a time higher than  $t_0$ , then there is an aircraft (as long as there remain aircrafts undergoing uncertainty) which can take its place.

In this case, we have three types of planes, those not undergoing uncertainty ( $pn$ ), and those undergoing uncertainty ( $(1-p)n$ ), separate in two :

- Those where we know at a time higher than  $t_0$  that they will not take-off, slots which we can recover  $(1-p)qn$ .
- Those not taking-off and where one does not know it enough time in advance to recover their slots  $(1-p)(1-q)n$ .

	0	no	yes
$t_0$			
no		$(1-p)q$	X
yes		$(1-p)(1-q)$	p

Table 1-Table representing the proportion of aircrafts according to their answer ‘‘yes’’ or ‘‘no’’ at the time  $t_0$  and time 0 at the question: ‘‘will you take your slot?’’

We obtain new throughputs:

$$\begin{aligned} \text{throughput}_4 &= n(p + (1-p)q) \\ &\quad \text{planning} \quad \text{recovered slots} \\ \text{throughput}_5 &= \underbrace{p(n-z)}_{\text{absorption areas}} + \underbrace{(1-p)(n-z)q}_{\text{absorption areas}} \\ &\quad + \underbrace{\min\{z, (1-p)(1-q)(n-z)\}}_{\text{absorption areas}} \end{aligned}$$

With  $\text{throughput}_4$  the throughput obtained by the new strategy without AAs, and  $\text{throughput}_5$  with.

The maximum throughput is obtained when all the recovered and unfilled slots are used :

$$\left\{ \begin{array}{l} \text{we obtain } z = n\frac{(1-p)(1-q)}{1+(1-p)(1-q)} \\ \text{thus } \text{throughput}_5 = 1 - z = \frac{n}{1+(1-p)(1-q)} \\ \text{and } \text{throughput}_5 - \text{throughput}_4 = n\frac{(p-1)^2(q-1)^2}{1+(p-1)(q-1)} \end{array} \right.$$

$\text{throughput}_5$  is always higher than  $\text{throughput}_4$  for all  $(p, q) \in [0, 1[x[0, 1[$ . There is also a profit to pose AAs if we add a real time phase to the slot allocation. The real-time phase corresponds to recover slots ( $q > 0$ ). We showed that for the two strategies we increased the theoretical throughput in using AAs.

$$\begin{aligned} \text{throughput}_3 &= \frac{n}{1+(1-p)} \\ \text{throughput}_5 &= \frac{n}{1+(1-p)(1-q)} \end{aligned}$$

### III. THEORETICAL RESULTS WITH HIGH PROBABILITY

It is known how many AAs we have, but not if they will be all used and that all the planes undergoing an uncertainty will be able to find a slot. This section presents results with associated probability.

### A. Without AAs

#### THEOREM III.1

Let  $n$  be the number of slots, let  $a$  be the number of aircraft which we would like to let to take off. Let  $p$  be the probability that an aircraft takes its slot. Then we have, with a probability of at least  $1 - \frac{1}{n^c}$ ,  $\forall c > 0$ , at least  $pn - \sqrt{2c(1-p)n \ln(n)}$  aircraft which take-off under safety conditions without AAs.

*Proof:*

We consider the event "the aircraft gets alea" as  $+1$ , "the aircraft takes its slots" as  $0$ . We seek to determine the number of aircrafts which do not take their initial slot.

$$\text{Let } \left. \begin{array}{l} \Pr(X_i = 1) = 1-p \\ \Pr(X_i = 0) = p \end{array} \right\} \forall i \in \llbracket 1, n \rrbracket.$$

$$S_n = \sum_{i=1}^n X_i, \mathbb{E}(S_n) = (1-p)n$$

The random variable  $S_n$  represents the number of aircraft not taking-off in order to respect the safety conditions.

From Chernoff[5] we have:

$$\Pr(S_n < (1-\epsilon)\mathbb{E}(S_n)) < e^{-\mathbb{E}(S_n)\epsilon^2/2}$$

$$\text{Let } \epsilon = \sqrt{2c \frac{\ln(n)}{\mathbb{E}(S_n)}}:$$

$$\begin{aligned} \Pr(S_n < \mathbb{E}(S_n) - \sqrt{2c \frac{\ln(n)}{\mathbb{E}(S_n)} \mathbb{E}(S_n)}) &< e^{-c \ln(n)} \\ \Pr(S_n < \mathbb{E}(S_n) - \sqrt{2c \mathbb{E}(S_n) \ln(n)}) &< \frac{1}{n^c} \end{aligned}$$

and

$$\Pr(S_n > (1-p)n - \sqrt{2c(1-p)n \ln(n)}) > 1 - \frac{1}{n^c}$$

and symmetrically we have :

$$\Pr(S_n < (1-p)n + \sqrt{2c(1-p)n \ln(n)}) > 1 - \frac{1}{n^c}$$

■

### B. With AAs

The previous strategy is considered : when an aircraft undergoes uncertainty, then we look if there is an unfilled slot. If it is the case, it takes it, else we consider it as lost.

#### THEOREM III.2

Let  $n$  be the number of slots, and  $z$  be the number of AAs. Let  $p$  be the probability that an aircraft takes its slot.  $z = n \frac{1-p}{2-p}$  as previously demonstrated. There is, with a probability of at least  $1 - \frac{1}{n^c}$   $\forall c > 0$ , at least

$$\frac{n}{2-p} - \sqrt{2cn \frac{1-p}{2-p} \ln(n)}$$

aircraft which take-off in safety conditions.

*Proof:*

Let  $S_n \in \{-1, 0, 1\}^n$  be the continuation of these events :

- the slot was not affected in the planning :  $-1$ .

- the slot was taken by the aircraft initially affected :  $0$ .
- the aircraft undergoes uncertainty and could not take its slot :  $1$ .

let  $S_m \in \{-1, 1\}^m$ ,  $S_z \in \{0, 1\}^{n-z}$ .  $S_m$  and  $S_z$  come from  $S_n$ .  $S_m$  corresponds to the suit of lost and unfilled slots.  $S_z$  corresponds to the daily planning.

#### EXAMPLE III.1

- $n = 17$
- $z = 3$
- $S_n = 0, 1, 0, -1, 1, 0, 1, 0, 0, 1, 0, 1, -1, 0, 0, 0, -1$
- $S_m = 1, -1, 1, 1, 1, 1, -1, -1$
- $S_z = 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0$

$n$  and  $z$  are known, we want to determine  $m$  which corresponds to the number of elements of  $S_m$ .

$$\mathbb{E}(m) = \underbrace{z}_{\#(-1)} + \underbrace{(1-p)(n)}_{\#(1)}$$

$\mathbb{E}(m)$  breaks up in two parts, a fixed part noted  $m_f (= z)$  and an unfixed part noted  $m_{uf}$  ( $\mathbb{E}(m_{uf}) = (1-p)(n-z)$ ).

From Chernoff we have:

$$\Pr(m_{uf} < (1-\epsilon)\mathbb{E}(m_{uf})) < e^{-\mathbb{E}(m_{uf})\epsilon^2/2}$$

$$\text{With } \epsilon = \sqrt{2c \frac{\ln(n)}{\mathbb{E}(m_{uf})}}:$$

$$\Pr(m_{uf} < \mathbb{E}(m_{uf}) - \sqrt{2c \mathbb{E}(m_{uf}) \ln(n)}) < \frac{1}{n^c}$$

In the best of the cases, all the aircraft undergone uncertainty found a slot. The number of aircraft not having found a slot is equal to  $S_m = m - 2z$ .

We have with a probability of at least  $1 - \frac{1}{n^c}$  :

$$\begin{aligned} S_m &> \mathbb{E}(m) - \sqrt{2c \mathbb{E}(m_{uf}) \ln(n)} - 2z \\ S_m &> z + (1-p)(n-z) - \sqrt{2c(1-p)(n-z) \ln(n)} - 2z \\ S_m &> (1-p)(n-z) - z - \sqrt{2c(1-p)(n-z) \ln(n)} \end{aligned}$$

We showed that the best affectation of  $z$  according to  $p$  is given by (cf section II-A) :  $z = n \frac{1-p}{2-p}$ .

$$\begin{aligned} S_m &> n \frac{1-p}{2-p} - \sqrt{2cn \frac{1-p}{2-p} \ln(n)} - n \frac{1-p}{2-p} \\ S_m &> -\sqrt{2cn \frac{1-p}{2-p} \ln(n)} \end{aligned}$$

And symmetrically we obtain :

$$S_m < \sqrt{2cn \frac{1-p}{2-p} \ln(n)}$$

$$\sqrt{2cn \frac{1-p}{2-p} \ln(n)} < S_m < \sqrt{2cn \frac{1-p}{2-p} \ln(n)}$$

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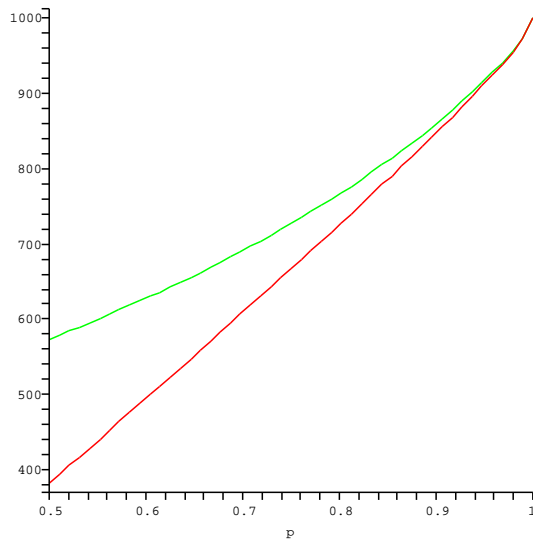


Fig. 4. Aircraft taking-off under safety condition with (upper curve) and without AAs for 1000 slots with a probability higher than  $1 - \frac{1}{1000^2} \cdot p$ .  $p$  is the probability that an aircraft takes its slot.

### C. Comparison

We see in Figure 4 the benefit of AAs. With the same number of slots, absorption areas can guarantee safety conditions for more aircraft than without.

Moreover, with AAs we can guarantee that all the aircraft allocated will take-off. It is not true without if we want guarantee the air traffic controllers workload.

## IV. CONCLUSION AND FUTURE WORK

The results obtained show the interest of AAs to improve air traffic throughput management. However, we did not seek the best unfilled slots distribution in the pretactical planning (find the best throughput according to delayed aircraft). The issue is to determine this distribution which guarantees a minimal load loss. For that, first we have to find the best slot allocation in one sector. Then we will determine the existing interactions between the sectors in order to minimize this load loss. Thus, all the aircrafts undergoing uncertainty must pass by the unfilled slots to not disturb other aircrafts and the initial planning.

The probability that an aircraft has got an alea does not have to be constant. It is a variable according to the hours of the day, and it can be estimated statistically. However the Chernoff's bounds remain useful under this assumption.

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