Routing in Networks with Low Doubling Dimension

Ittai Abraham (Hebrew Univ. of Jerusalem)
Cyril Gavoille (LaBRI, University of Bordeaux)
Andrew V. Goldberg (Microsoft Research)
Dahlia Malkhi (Hebrew Univ. of Jerusalem, Microsoft Research)
The Compact Routing Problem

Input: a weighted network $G$
Output: a routing scheme for $G$

A routing scheme allows any source node to route messages to any destination node, given the destination’s network identifier.
Ex: Grid with X,Y-coordinates

Routes are constructed in a distributed manner

... according to some local routing tables (or routing algorithms)
Quality & Complexity Measures

Time vs. Space

- Near-shortest paths:
  \[ |\text{route}(x,y)| \leq \text{stretch} \cdot d_G(x,y) \]

- Size of headers and local routing tables

- **Goal:** constant stretch & polylog size tables
Labeled vs. Name-independent Routing Schemes

- **Name-independent**: Node identifiers are chosen by an adversary (the input is a graph with the IDs)

- **Labeled**: Node IDs can be chosen by the designer of the scheme (as a routing label whose length is a parameter)
... in a Path

- Name-independent: Fixed IDs in \{1, \ldots, n\}

Routing from 5 to some target \( t \)

- Stretch 9 with \( O(1) \) space \([\text{BYCR03}]\)
- Stretch \( 1 + \varepsilon \) with \( \text{polylog}(n) \) space \([\text{AM05}]\)
- Stretch 1 implies \( \Omega(n) \) bit space

Labeled routing is trivial! stretch 1 with \( O(1) \) space
Overview of Compact Routing

For arbitrary weighted networks

\[ \text{stretch: } \sim k \quad \text{space/node: } \sim n^{1/k} \]

\( n = \# \text{of nodes, integer parameter } k > 0 \)

In more details: LABELED case

[Thorup-Zwick STOC+SPAA ’01]

\( \tilde{\Omega}(n^{1/k}) \) space for stretch \( 4k-5, \ k \geq 2 \)

(essentially optimal trade-off / stretch \( 2k-1 \)?)
Overview of Compact Routing

In more details: NAME-INDEPENDENT case
(the most difficult variant)

[Awerbuch-Peleg FOCS ’90]
\( \tilde{O}(n^{1/k}\log \Delta) \) space for stretch \( O(k^2) \) where \( \Delta \)
is the aspect ratio of the network

\[ \Delta = \max d(u,v) / \min d(u,v) \quad (u \neq v) \]
Overview of Compact Routing

Aspect ratio might be very large

$\Delta = 2^n$ is possible!

**Scale-Free** routing scheme = size of tables and headers are independent of $\Delta$
Doubling Dimension

[Definition] A network (or a metric) has doubling dimension $\alpha$ if every radius-$2r$ ball can be covered by $\leq 2^\alpha$ radius-$r$ balls

- Euclidean networks have $\alpha=O(1)$
- Include bounded growing networks
- Robust notion
Doubling Dimension

► LABELED

Talwar (STOC ’04)
stretch: $1 + \varepsilon$
space: $O(1/\alpha\varepsilon)^{\alpha} \log^{2+\alpha}\Delta$

Chan et al. (SODA ’05)
stretch: $1 + \varepsilon$
space: $(\alpha/\varepsilon)^{o(\alpha)} \log n \log \Delta$

Slivkins (PODC ’05)
stretch: $1 + \varepsilon$
space: $(1/\varepsilon)^{o(\alpha)} \log n \log n \log \log \Delta$

This paper
stretch: $1 + \varepsilon$
space: $(1/\varepsilon)^{o(\alpha)} \log^3 n$
Doubling Dimension

► NAME-INDEPENDENT

Awerbuch-Peleg (FOCS ’90)
stretch: \(O(\log^2 n)\)  space: \(O(\log \Delta \log^2 n)\)

Konjevod et al. (PODC ’06)
stretch: \(9 + \varepsilon\)  space: \((2 + 1/\varepsilon)^{o(\alpha)} \log^2 \Delta \log n\)

This paper
stretch: 64  space: \(2^{O(\alpha)} \log^4 n\)
Doubling Dimension

► Also in this paper: Lower bounds

(Name-independent): If the stretch $< 3$ then the space must be $\Omega(\alpha n)$

(Labeled): If the stretch $= 1$ then the space must be $\Omega(\sqrt{n})$ even for bounded growth networks
Sparse Partition

[Lemma] For weighted network $G$ of doubling dimension $\alpha$ and every $\rho > 0$, one can construct (in polynomial time) a collection of “clusters” $\mathcal{H}$ (connected subgraphs) of $G$ such that:

1. [cover] Every radius-$\rho$ ball centered at $u$ is contained in some cluster $H \in \mathcal{H}$
2. [sparse] Every $u$ belongs to at most $4^\alpha$ clusters of $\mathcal{H}$
3. [small radius] $\forall H \in \mathcal{H}, \text{radius}(H) \leq 2\rho$
Intuition: Try & Fail

Design a (name-independent) routing scheme for distance at most $\rho$ nodes such that:

For any source $s$ and target $t \in G$

1. If $t$ is at distance $\leq \rho$ from $s$, then $t$ is discovered after a route of length $O(\rho)$
2. If $t$ is at distance $> \rho$ from $s$, a negative answer is reported back to $s$ after a walk of length $O(\rho)$

$\Rightarrow$ Trying with $\rho = 1, 2, 4, \ldots, 2^i \ldots$, any $t$ will be found with a constant stretch factor and with an increasing factor of $\log \Delta$ on the space.
Removing the “logΔ” factor

Construct a sparse partition as before, but store routing only for logn levels instead of log Δ

⇒ Dense-Sparse Partition

Radius-2^i clusters for a node s
Removing the “logΔ” factor

RANGE=levels that at least double the volume of the balls (at most log₂n ranges)

Dense range condition: if increasing the radius by a factor at most $2^5$ the volume at least double
Removing the “logΔ” factor

Searching a target \( t \) from \( s \) in \( s’ \) clusters:

FOR \( i=1 \) to \( \log n \) DO:
- IF range\((s,i)\) is dense, route with \textbf{dense}(i) routing.
- IF range\((s,i)\) is sparse, route with \textbf{sparse}(i) routing.

\textbf{Dense}(i) routing: search in \( O(\text{radius} \cdot \text{range}(s,i)) \) is OK.
\textbf{Sparse}(i) routing: negative answer reported faster (by sparsity stored information for that is small)
Conclusion

► Recently, in PODC’06 Konjevod et al. prove that for name-independent routing if stretch $<9-\varepsilon$ then space $\Omega(n^{\varepsilon/16})$ for some networks of doubling dimension 10.

► Can be do stretch $9+\varepsilon$ and scale-free polylog routing? (best stretch is 64).