Chp 4 - Communication Complexity

How much time is required to complete a dist. task in the b-CONGEST model?

$\leq b$ bit. — MSG are of size $\leq O(\log n)$ bit.

In general TIME $> t$ for 2 reasons:

1. The task requires that 2 proc. at distance $> t$ interact.

It's because the task is not "local"

2. the task requires that $\exists$ one edge where $> t \times b$ bits must traverse.

$\Rightarrow$ TIME $> t$

Exercice

Consider the following task: LEADER
In each connected component of the graph

$G =$

two components with diam ≥ 3.

select exactly one vertex per component.

1. Indicate what are the inputs and outputs of the LEADER problem?
2. Show that no LOCAL algorithm can solve LEADER in t rounds if G has at least 2 components of diameter $> t$.
3. What about graphs with 1 component? What about graphs whose components have diam $\leq t$?

Solutions

LOCAL assumptions. $n = |V(G)|$

1. Inputs: Each u, $ID(u) \in \{1 \dots n\}$

Outputs: Each $u \mapsto LEADER(u) = \{0,1\}$ s.t. $\exists!\, u$ per component s.t. $LEADER(u) = 1$.

(2)

3.

If $ID(u) = 1$, $LEADER(u) = 1$
otherwise ——— $= 0$.

2.

$LEADER(u) = 1$ iff u has smallest ID's among its neighbors. ?

By way of contradictions:
Assume there is a dist. algorithm $\mathbf{A}$ solving LEADER on graphs with c.c. of diam $> t$, in t rounds.

Intuition: Vertices need to "know" whether they 1 or >1 c.c.

A view argument.

Take G with 2 c.c. of diam. $> 2t$.

There is a contradiction, because $\mathbf{A}$ will act the same for u & v in G_0 and G_1.

1: we have to select only 1 vertex, not 2.

We can solve the problem in t rounds if c.c. have diam $\leq t$.

1. Collect everything at distance $\leq t$.
2. Choose the smallest ID.

$\{ID(u), ID(v), \dots\}$

Here we need $> t$ rounds because LEADER is not a "local" problem.

$\Rightarrow$ in the CONGEST this even worst. you need $> t$ to solve LEADER.

How to detect a C_3 or C_4?

G — inputs: $ID(u)\ \forall u$; outputs: $u \mapsto DETECT(u) \in \{T, F\}$

G has a C_3 or C_4 $\Leftrightarrow \exists u$, $DETECT(u) = T$

How much TIME to solve it in CONGEST model?

This is a "local" problem, it can be solved in 2 rounds in the $\boxed{\text{LOCAL}}$

Round 1

- SendAll($ID(u)$)
- $S = \bigcup_{v \in N(u)} Receive(v)$

Round 2

- SendAll(S) — this is $\deg(u) \cdot \log(n)$ bits in CONGEST.
- $S_2 = \bigcup_{v \in N(u)} Receive(v) = \{S_v : v \in N(u)\}$

$\Rightarrow$ There is a C_3 or C_4 is $\exists S_v, S_w$

$$(S_v \cup \{v\} \cap S_w \cup \{w\}) \setminus \{u\} \neq \emptyset \Leftrightarrow \exists C_3 \text{ or } C_4.$$

Next time will show that in the CONGEST model we can do with TIME $O(\sqrt{n})$.