

All your answers must be fully justified. The language for your answers can be either french or english, but not both! Documents (course and personal notes on paper) are allowed. The official lecture notes will be available for the test.

VERTEX COVER

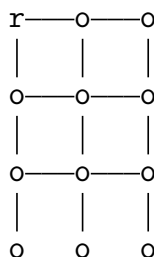
Question 1. Recall the definition of the minimum size VERTEX COVER problem.

A vertex u in a rooted tree T is called *internal* if u is not a leaf. Consider the following algorithm, which assumes that the graph G is connected.

Algorithme ApproxVC(G)

1. Compute a depth-first search spanning tree T for G .
 2. Return the set S of internal vertices of T .
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Question 2. Apply ApproxVC to the graph G_0 below so that the tree T has root r . Give the size of the set S returned. Specify T and S on a drawing of G_0 .



Question 3. Find a minimum size VERTEX COVER for the graph G_0 (a drawing is sufficient). Justify minimality.

Question 4. Prove that, for connected graphs, the algorithm ApproxVC is a 2-approximation for the minimum size VERTEX COVER. [Hint : Color the vertices of S .]

Question 5. Is the algorithm ApproxVC still correct if, instead of returning the internal vertices of T , it returns all vertices of T with degree > 1 ? Justify.

Question 6. Provide a critical instance for ApproxVC. [Warning! The critical instance must not be of constant size.]

Question 7. Briefly recall the principle of the Glouton algorithm for SET COVER, but applied to VERTEX COVER.

Question 8. Depending on a relevant parameter of G_0 , what theoretical approximation factor should the Glouton algorithm achieve on G_0 ?

Question 9. Apply the Glouton algorithm to G_0 . What is the size of the cover returned? Draw this solution.

DOMINATING SET

Question 10. *Recall the definition of the minimum size DOMINATING SET problem.*

Question 11. *Give the theoretical approximation factor of the Glouton algorithm applied to DOMINATING SET for a binary tree with n vertices.*

Question 12. *Provide a polynomial algorithm solving the minimum size DOMINATING SET problem for a forest, expressed as reduction rules. Justify.*

END.