

UE: APPLIED ALGORITHMS - 4TIN915U

(test – duration 1h30)

– *Documents (course and personal notes) allowed* –

Course Questions

Question 1. *Give at least one advantage and one drawback for algorithms based on dynamic programming.*

Question 2. *Same question for algorithms for graphs of small tree-width.*

Consider now the cycle graph C_n where its vertices are v_0, v_1, \dots, v_{n-1} and its edges are the pairs $\{v_i, v_{(i+1) \bmod n}\}$.

Question 3. *Give a tree-decomposition of width 2 for C_5 . (Give the tree, the bags and their contents, and check that all properties hold.)*

Question 4. *Generalize Question 3 by describing a tree-decomposition of width 2 for C_n .*

Covering Points by Ellipsis

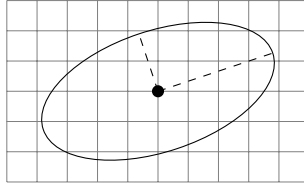
We consider the decision problem ELLIPSE COVER defined as follows:

Instance: A set of $P \subset \mathbb{R}^2$ of n points in the plane, and $k \in \mathbb{N}$.

Question: Does P has an ellipse cover of size k ? i.e., are there k ellipsis in the plane such that every point of P belongs to at least one of these ellipsis?

We will also consider the optimization variant, called MINIMUM ELLIPSE COVER, that asks to find the minimum k such that P has an ellipse cover of size k .

Recall that an *ellipse* is a close curve in the plane that looks like to a circle that has been flattened by two antipodal points. It can be defined by a center c (like a circle), two orthogonal radii r_x, r_y (for a circle $r_x = r_y$) and a slope α indicating by how much the curve is rotated ($\alpha = 0$ for a circle). See hereafter for such an example.



For the ellipse drawn above, the center c is the black middle point, $r_x = 4$, $r_y = 2$, and $\alpha = \arctan(1/3) \approx 18.4^\circ$. We can check that the set of all the points $(x, y) \in \mathbb{R}^2$ belonging to an ellipse satisfies a quadratic equation of the form $ax^2 + by^2 + cxy + dx + ey + f = 0$, where the real coefficients a, b, c, e, d, f depend on the parameters c, r_x, r_y, α defining the ellipse.

A set of points is said in *general position* if it has no three points aligned. It is well-known that any three points in general position determines a unique circle. This is different for an ellipse.

Question 5. *Construct four points in general position such that no ellipse can contain these four points. Explain why?*

Question 6. *Explain why four points in general position are not sufficient to uniquely determine an ellipse, whereas an ellipse can contain these same four points.*

In the remaining we will assume that five points are always sufficient to uniquely determine an ellipse. Moreover, you can assume to be given a routine $\text{Check}(Q)$ that, given a set Q of at most five points, returns the six coefficients of the quadratic equation of an ellipse passing by all the points of Q (if any), or that returns \perp if no such ellipse exists.

Question 7. *Describe an algorithm for solving ELLIPSE COVER for $k = 1$. What is its time complexity?*

Question 8. *Give a reduction rule showing that ELLIPSE COVER, whenever k is a parameter, admits a kernel. (A complete justification is required here! You have to show that your reduction is valid and is polynomial (give its complexity), and you have to express the size of the kernel.)*

Question 9. *Explain to which general problem seen in the course MINIMUM ELLIPSE COVER is a particular case. In particular, what are the sets B, \mathcal{F} , and their size $|B|, |\mathcal{F}|$?*

Question 10. *Explain how to approximate MINIMUM ELLIPSE COVER in polynomial time. What is the approximation factor?*

Question 11. *By combining answers of Question 8 and Question 10, describe an $O(\log k^*)$ -approximation for MINIMUM ELLIPSE COVER, where $k^* < n$ is the optimal size of the cover.*

FIN.