

Energy-Efficient Intrusion Detection with a Barrier of Probabilistic Sensors: Global and Local

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Abstract—Intrusion detection is a significant application in wireless sensor networks (WSNs). S. Kumar *et al* have introduced the concept of barrier coverage, which deploys sensors in a narrow belt region to guarantee that any intrusion across the region is to be detected. However, the practical issues have not been investigated such as scheduling sensors energy-efficiently while guaranteeing the detection probability of any intrusion across the region based on probabilistic sensing model. Besides, the intruders may be humans, animals, fighter planes or other things, which obviously have diverse moving speeds. In this paper, we analyze the detection probability of arbitrary path across the barrier of sensors theoretically and take the maximum speed of possible intruders into consideration since the sensor networks are designed for different intruders in different scenarios. Based on the theoretical analysis of detection probability, we formulate *Minimum Weight ϵ -Barrier Problem* about how to schedule sensors energy-efficiently and prove it is NP-hard. We propose both global and local solutions to the problem. The global solution called *Minimum Weight Barrier Algorithm* is a bounded approximation algorithm, based on which a localized protocol for energy-efficient scheduling is designed. To evaluate our design, we analyze the performance of our approaches theoretically and also perform extensive simulations to demonstrate the effectiveness of our proposed algorithm.

Index Terms—Wireless sensor networks, localized protocol, energy-efficient, barrier coverage, detection probability.

I. INTRODUCTION

WIRELESS sensor network (WSN) is considered to be a large scale network which is composed of a large amount of sensors to perform pervasive sensing. Recently, the research on how to improve the energy efficiency and quality of service in WSNs attracts lots of attention [1]–[4]. Due to recent advances of wireless communication and hardware device technologies, WSNs are expected to be widely applied in security areas such as intrusion detection and border surveillance. The main goal of these applications is to detect intruders when they cross a barrier such as a border, which is usually a long belt region. A barrier consists of a large number of sensors such that any intrusion crossing it will be detected. This is referred to as *barrier coverage* in the literature [5].

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Due to sensors' limited resources, it is critical to use sensors in an energy-efficient way. No exception to this rule for barrier coverage. However, all previous works known to us on barrier coverage are based on the 0/1 disc sensing model [6]–[9], which is only a coarse and even unrealistic approximation to the real sensor's sensing behavior. Probabilistic sensing model has been proposed as a more accurate sensing model [10]–[12] and has been widely adopted to analyze the quality of coverage in WSNs [13]–[17]. The probabilistic sensing model assumes that the sensing probability of a sensor is a decreasing function of the sensing distance. In this paper, we adopt the probabilistic sensing model.

Requirements of barrier coverage are different in a variety of scenarios since a prospective intruder's maximum speed is a factor that may significantly affect the detection probability [18]. Sensors perform sensing task discretely, for various reasons: i) sensors with an on-board micro processor unit such as Mica2 and Imote2 sample at a certain rate; ii) in a directional sensor network, directional sensors (e.g., cameras or radars) need to rotate and change sensing directions [19]; iii) sensors may be set to sleep periodically to save energy. In summary, physical points in a monitored region are detected discretely and periodically. Thus, fast-moving intruders along a path are less likely to be detected than slower-moving ones along the same path. We argue that the maximum speed of intruder is a necessary factor to be considered in the design of sensor barrier. For example, sensor networks designed for monitoring the intrusion of humans in a house security system could be different from that for monitoring the intrusion of fighters in a battle field since civilians and soldiers typically have different maximum moving speeds. So we analyze the detection probability of intruders with diverse maximum moving speeds when crossing the barrier, which to the best of our knowledge has not been investigated before. Based on the analysis of detection probability, we define ϵ -barriers. A set of sensors deployed in a belt region A is said to provide an ϵ -barrier if every crossing path in A has a detection probability at least ϵ , given a known maximum moving speed of possible intruders.

If sensors are randomly deployed, the required number of sensors to form an ϵ -barrier is typically much greater than the optimal value. Scheduling sensors to simultaneously guarantee ϵ -barrier coverage and energy efficiency is an important, nontrivial problem. We are the first to study the problem using the probabilistic sensing model. We prove the problem NP-hard and propose an efficient approximation algorithm with a provable bound called *Minimum Weight Barrier Algorithm* (MWBA). With the 0/1 disc model, it is impossible

to guarantee global barrier coverage in a localized way [5]. To surmount this impossibility, the notion of local barrier coverage is proposed in [8]. In this paper, we show that, with the probabilistic sensing model, there is no need to resort to *local* barrier coverage. Indeed, we will describe a *localized* algorithm, called *Localized Minimum Weight Barrier Protocol* (LMWBP), that provides a *global ϵ -barrier*. Besides, our localized protocol can also be applied in a difficult problem discussed in [5], [8]: How to guarantee k -barrier coverage in a localized way with 0/1 sensing model assumption? It has been shown that it is infeasible to achieve global optimal solution in a localized way in [5]. To solve the problem, our localized protocol LMWBP can provide global k -barrier coverage in a sub-optimal way.

Our contributions are summarized in the following:

- 1) We for the first time analyze the detection probability of arbitrary paths across a barrier. The sensing model is constructed based on real sensing patterns of sensors. Equipped with such analytic results of detection probability, we show how to provide energy-efficient ϵ -barriers with randomly deployed sensors.
- 2) A *Minimum Weight Barrier Algorithm* has been designed to solve the scheduling problem in polynomial time. We theoretically prove that the approximation bound of the algorithm is $O(\rho)$, where ρ is the line density of sensors, i.e., the number of sensors N over the length of the belt region.
- 3) We further design a localized protocol that provides global ϵ -barrier coverage energy-efficiently. This is a solution to a difficult problem discussed in [8]: How to guarantee barrier coverage energy-efficiently in a localized way? Instead of confined to "local" barrier coverage, we maintain global coverage, while guaranteeing a pre-specified level of detection probability $\epsilon < 1$.

The rest of the paper is organized as follows. We discuss related work in Section II, followed by the formulation of *Minimum Weight Barrier Problem* in Section III. Section IV analyzes the problem theoretically. Section V spells out our algorithm design and analysis. Section VI proposes a localized protocol for the problem. Simulation results are presented in Section VII, and concluding remarks in Section VIII.

II. RELATED WORK

The concept of barrier coverage was first proposed in [5]. It introduced the concept of *weak barrier* and *strong barrier*. The goal of barrier coverage is to detect intrusion with a barrier of sensors. The reliable density of barrier coverage in thin strips with finite length is estimated in [20]. In random sensor deployments, scheduling the activation of sensors alternatively to form barriers can help prolong the network lifetime. Several sensor scheduling algorithms for barrier coverage have been proposed in the literature [6]–[9], [21]. In [7], a distributed algorithm is proposed to construct strong barriers. The problem of maximizing network lifetime in barrier coverage is studied in [6]. The authors designed an algorithm called *Stint* to achieve the optimal lifetime of homogenous network and an algorithm *Prahari* for heterogenous network while providing k -barrier coverage to the belt region. In [8], a localized

algorithm is proposed to increase the lifetime of a network deployed for barrier coverage, but the localized version can not guarantee the barrier coverage.

To model the sensing behavior of a sensor, the aforementioned papers adopt a simple 0/1 sensing disc, which is only a coarse approximation to the real sensing phenomenon. The problem of barrier coverage with a probabilistic sensing model is studied in [22]; but the authors consider only the condition of *weak barrier coverage* as defined in [5], which assumes that the intruder crosses the barrier along a straight line. In practice, that can hardly be the case — intruders could take any path. In addition, they do not consider the intruder's speed, which we believe is an important factor to consider when designing a sensor network for barrier coverage.

The paper [23] discusses the *Worst Case Coverage-Maximal Breach Path*. The problem is to maximize the minimum distance among the distances from any point in the path to its closest sensor. It is a different problem from what to be discussed in this paper.

Exposure has been proposed earlier to characterize the quality of coverage towards intrusion [24], [25]. Exposure is defined as the integral of a sensing function along a path. The authors discuss how to find a path with minimum exposure in the area based on grid-approximation. However, it can not guarantee the minimum exposure unless the number of grids approaches infinity, which can not be applied in our problem.

III. PRELIMINARY AND PROBLEM FORMULATION

A. Sensing Model

Several types of sensors (e.g., acoustic sensors) detect intruders by measuring received energy, which attenuates with increasing distance between the sensor and the target. Considering the uncertainty of signal detection process, the probabilistic sensing model, which assumes that the detection probability is a continuously decreasing function of the sensing distance, has been widely adopted in the literature of analysis of sensing coverage [11], [13]–[17]. Denote by $p_i(j)$ the probability that sensor i is able to detect a target at position j . The probabilistic sensing model assumes that $p_i(j) = \lambda(d)$ for some function $\lambda(\cdot)$, where d denotes the distance between sensor i and physical location j . Several empirical formulas have been proposed for $\lambda(\cdot)$, e.g., the exponential attenuation probabilistic model [10], [26] and the cubic attenuation model [14]. In practice, $\lambda(\cdot)$ can be estimated from training data.

Almost all existing probabilistic sensing models (including the aforementioned exponential attenuation and cubic attenuation models) use a λ function that is concave with a positive second-order derivative. Thus, the following assumption, which we will need in this paper, is not unrealistic or uncommon.

Assumption 1: The sensing function $\lambda(d)$ is concave in the interval where $\lambda(d)$ attenuates as d increases, and its second order derivative is positive (i.e., $\lambda''(d) > 0$).

Sensors usually *sense* at a fixed sampling rate. They collect information within their sensing range intermittently and periodically. We use the terms *sampling frequency* and *sampling period* to indicate the following.

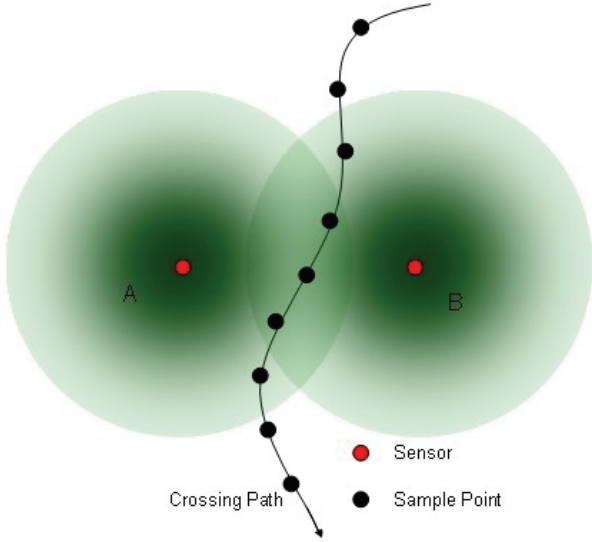


Fig. 1. The situation when an intruder crosses through two sensors.

Definition 1: Sampling Frequency: The number of samplings in a unit of time.

Definition 2: Sampling Period: The length of time between two consecutive samplings.

If f and T are the sampling frequency and sampling period, respectively, then $f = 1/T$.

B. Network Model

We consider a WSN deployed in a belt region A which is long and narrow. The WSN consists of N sensors, randomly deployed. Each sensor can only communicate with other sensors within a range of R , which is referred as its *communication range*. Sensors are assumed to be homogeneous, having the same sampling frequency f , but not necessarily synchronized (i.e., sensors do not necessarily sense at the same time). The barrier network is designed for possible intruders with a known maximum speed V_{max} .

We adopt the definition of *crossing path* in [5].

Definition 3: (Crossing Path) A curve in region A is said to be a *crossing path* if it crosses the complete width of A .

The detection probability of a crossing path l , denoted by p^l , is the probability that an intruder moving at speed no more than V_{max} , along l , will be detected by at least one sensor in the WSN. With discrete sensing, a path can be viewed as a set of points (as illustrated in Fig. 1) at which the intruder is sampled by sensors.

The WSN is deployed to detect intruders that may cross the belt region A with a maximum speed V_{max} .

Definition 4: (ϵ -barrier coverage) A set of sensors C provides ϵ -barrier coverage to belt region A if the detection probability of every crossing path of A is at least ϵ . (That is, $p^l \geq \epsilon$ for all crossing paths l .) C is said to be an ϵ -barrier along A .

C. Problem Statement

Suppose that a WSN of N sensors has been deployed over a belt region. We are interested in scheduling the sensors

such that they provide ϵ -barrier coverage in an energy-efficient fashion to the region. The goal is to prolong the WSN's lifetime.

To that end, the operation time is divided into (time) slots. At the beginning of each slot, each sensor i is assigned a cost/weight w_i . The cost/weight of a set of sensors C is the sum of weights of all sensors in C , i.e., $w(C) = \sum_{i \in C} w_i$. Relatively to a weight assignment, minimum weight ϵ -barrier is defined as follows..

Definition 5: Minimum weight ϵ -barrier: Given a set of sensors $\{1, 2, \dots, N\}$ with corresponding weights w_1, \dots, w_N deployed in a belt region A . A minimum weight ϵ -barrier C^* is a set of sensors which forms an ϵ -barrier and has a minimum total weight among all ϵ -barriers.

In each slot, we activate a minimum weight ϵ -barrier C^* . This problem, called the Minimum Weight Barrier Problem, is formally stated below.

Definition 6: Minimum Weight Barrier Problem: Given are a belt region A and a set of homogeneous sensors $\{1, 2, \dots, N\}$ each with a common sampling frequency f and communication radius R . Sensor i is also associated with a location, as well as an energy supply B_i . The *minimum weight barrier problem* is to choose a minimum weight ϵ -barrier against possible intruders with maximum speed V_{max} , where ϵ and V_{max} are pre-specified values (determined by applications).

There are the subproblems, each interesting and nontrivial. First, is there any ϵ -barrier (against speed V_{max} intruders) at all that can be constructed from the given deployed sensors? If the answer is positive, then we have the second question: how to find a minimum-weight ϵ -barrier? We will address these two questions in Sections IV, V and VI. But first let's discuss how to assign weights/costs to sensors.

D. Weight Assignment

Assume that each sensor initially has B units of energy, and consumes 1 unit of energy per slot if active. A sensor in *sleep* mode consumes only negligible energy; thus for simplicity, we assume it consumes zero energy when in sleep. Sensors with residual energy less than 1 unit can not be activated any more.

At the beginning of each time slot, let B_i denote sensor i 's residual energy, and let $\gamma_i = 1 - B_i/B$, which satisfies $0 \leq \gamma_i \leq 1$ and indicates i 's energy consumption ratio. We adopt the weight assignment scheme as proposed in [27]:

$$w_i = \begin{cases} \theta^{\gamma_i}, & \text{if } \gamma_i < 1 \\ \infty, & \text{if } \gamma_i = 1 \end{cases} \quad (1)$$

where θ is a constant greater than one. It captures the intuitive idea that sensors with less residual energy are more precious (expensive) and thus have a high weight (cost). Sensor i is specially marked by assigning infinite weight if it has no residual energy, when $\gamma_i = 1$, since it can not be activated any more.

IV. THEORETICAL ANALYSIS

This section discusses how to efficiently determine whether a set of sensors provides ϵ -barrier coverage against intruders with a given maximum speed V_{max} .

It is a challenging issue and there is no existing efficient approach. The approach in [24] divides the belt region into grids and calculate the exposure (similar to the detection probability) between each two grid points approximately by assuming that the target should move along the segment between these two grid points. Then a minimum exposure path across the region can be found by finding the shortest path from one side to another among all points based on the approximate exposure between each two points. However, it still can not find the path with minimum detection probability since it only considers the paths along the grid points and the exposure between grid points is only a course approximation. It can be accurate if the amount of grids approaches infinity, which is infeasible. Due to the limit of existing methods, we proceed to analyze the lower bound of detection probability theoretically in this section.

A. Detection Probability and Detection Gain

By definition, a WSN provides ϵ -barrier coverage if every crossing path's detection probability is no less than ϵ . Unfortunately, it seems difficult to determine a crossing path's exact detection probability, let alone the uncountable number of such paths. Our approach is to identify those crossing paths with minimal detection probabilities and check if these probabilities are at least ϵ .

Let C be the set of all activated sensors in the WSN. Let p_i^l be the detection probability of crossing path l by sensor $i \in C$. An intruder moving along path l is not detected by C iff it is not detected by any single $i \in C$. Thus, the detection probability of crossing path l , denoted by p^l , satisfies

$$1 - p^l = \prod_{i \in C} (1 - p_i^l) \quad (2)$$

Now consider an individual p_i^l , which not only depends on i and l but also on the target's moving speed — the higher speed, the smaller p_i^l . Since our objective is to provide ϵ -barrier coverage, we have to consider the worst case when targets move at the maximum speed V_{max} . Considering the sensor's sampling frequency, the path of a moving target is viewed as a set of sampled points spaced by V_{max}/f , as illustrated in Fig. 1. Let Ω_i^l be the set of sampled points for path l and sensor i . Then p_i^l satisfied

$$1 - p_i^l = \prod_{j \in \Omega_i^l} [1 - p_i(j)] \quad (3)$$

where $p_i(j)$ is the detection probability of point j by sensor i . Substituting Eq. 3 into Eq. 2 yields

$$1 - p^l = \prod_{i \in C} \prod_{j \in \Omega_i^l} [1 - p_i(j)] \quad (4)$$

Linearizing the formula by taking logarithm on both sides, we have

$$\ln(1 - p^l) = \sum_{i \in C} \sum_{j \in \Omega_i^l} \ln[1 - p_i(j)] \quad (5)$$

For ϵ -barrier coverage, it is required $p^l \geq \epsilon$, or equivalently, $\ln(1 - p^l) \leq \ln(1 - \epsilon)$, which by Eq. 5 translates to

$$\sum_{i \in C} \sum_{j \in \Omega_i^l} \ln \frac{1}{1 - p_i(j)} \geq -\ln(1 - \epsilon) \quad (6)$$

The term $\phi_i(j) = \ln \frac{1}{1 - p_i(j)}$ is called the **detection gain** of the physical point j by sensor i , and the term $\varphi(\epsilon) = -\ln(1 - \epsilon)$ the **aggregate gain threshold**. Also, call $\phi_i^l = \sum_{j \in \Omega_i^l} \phi_i(j)$ the detection gain of path l by sensor i ; and $\phi_C^l = \sum_{i \in C} \phi_i^l$ the total detection gain of path l (by C). Using these terms, it follows from Eq. 6 that C forms an ϵ -barrier if for every crossing path l ,

$$\phi_C^l \geq \varphi(\epsilon). \quad (7)$$

Remarks: Detection gain $\phi_i(j)$ is a monotonically increasing function of detection probability $p_i(j)$. Either can be used in the study of ϵ -barrier coverage. We choose to use detection gains simply because of its linearity in aggregating individual gains. Note that while detection probability ranges from 0 to 1, detection gain goes from 0 to infinity. If $p_i(j) = 1$, then $\phi_i(j) = \infty$. Both indicates that sensor i can always cover physical point j .

B. Scenario Analysis

We now analyze the scenario of multiple sensors. Let's start with the simplest case where there are only two sensors located at point m and k respectively. We will show that the crossing path with the least detection gain is along the perpendicular bisector of the segment between m and k . This claim may seem obvious. Actually, it is not obvious, and even *not* always true. It is true if Assumption 1 holds, which as mentioned earlier is the case for most known sensing functions. So, in the rest of the paper, we adopt Assumption 1 for the sensing function $\lambda(d)$. Before analyzing the detection gain of a path, we have such a lemma of concave function below. The detection gain is defined as $g(d) = -\ln[1 - \lambda(d)]$ where $\lambda(d)$ is the sensing function.

Lemma 1: Assume variable x and $0 \leq x \leq r$. For any fixed y , the function $\gamma(x) = g(\sqrt{x^2 + y^2}) + g(\sqrt{(r - x)^2 + y^2})$ has a minimum when $x = r/2$.

Proof:

We assume $d(x) = \sqrt{x^2 + y^2}$. To find an extremum of $\gamma(x)$, take the derivative of the function and make it zero:

$$\gamma'(x) = g'[d(x)]d'(x) - g'[d(r - x)]d'(r - x) = 0 \quad (8)$$

Denote $g'[d(x)]$ by $g'(d)$, we have $g'(d) = \frac{\lambda'(d)}{1 - \lambda(d)}$ and its derivative $g''(d)$

$$g''(d) = \frac{\lambda''(d)[1 - \lambda(d)] + [\lambda'(d)]^2}{[1 - \lambda(d)]^2} \quad (9)$$

It follows from Eq.(9) and $\lambda''(d) > 0$ (Assumption 1) that $g''(d) > 0$, and hence $g'(d)$ is a monotonic increasing function for d . We also have $d'(x) > 0$ and $d''(x) > 0$, so $d(x)$ and $d'(x)$ are both monotonic increasing functions for x . Thus, $g'[d(x)]d'(x)$ is a monotonic increasing function for x and $\gamma'(x) = 0$ only when $x = r/2$. From this and Eq.8, $\gamma(x)$ has an extremum at $x = r/2$. Since $g'[d(r)]d'(r) > 0$, we have $\gamma'(0) < 0$ and $\gamma'(r) > 0$. Thus the extremum at $x = r/2$ is a minimum. ■

Among all crossing paths, the path with the least amount of sampled points would be a line which is vertical to the connection line of these two sensors. We always consider this scenario to find the minimum detection gain. Any path with

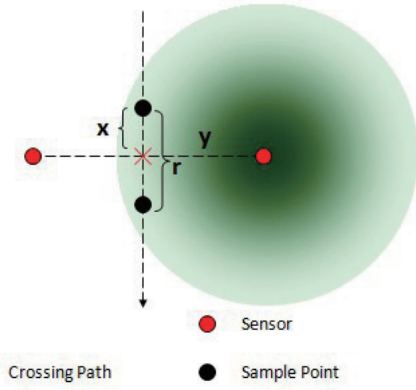


Fig. 2. The sampled points of a crossing path.

more sampled points should never be the path with minimum detection gain since we can always reduce the detection gain by decreasing the amount of sampled points.

Consider the line across two sensors which is vertical to the connection line of these two sensors in Fig. 2. At first, we only consider the sampled points of one sensor on the right side. Given the constant sampling frequency and maximum speed of targets, the distance between each two sampled points is constant. If we denote the distance by r , and denote the distance between the sensor and the path by y , we can know that $x = r/2$ in Fig. 2 leads to the least detection gain of the path according to Lemma 1. The conclusion is not related with y , so the analysis is the same for the sensor on the left side. We therefore have the Lemma 2.

Lemma 2: For a line across two sensors which is vertical to the connection line of these two sensors, the sampled points of these two sensors along the path are the same in the worst case.

Proof:

Assume the two sensors located at m and k . Consider the sensor located at m . Assume the detection gain is $\phi_m(P)$ and $\phi_m(Q)$ for points P and M on the path. Then the aggregate detection gain is $\phi_m(l) = g(\sqrt{x^2 + y^2}) + g(\sqrt{(r-x)^2 + y^2})$. The detection gain is minimum when $x = r/2$ according to Lemma 1. Similarly, the detection gain of sensor located at k is minimum when $x = r/2$. It is the same for sensors located at m and k in the worst case, where concludes the proof. ■

Thus, the sampled points of these two sensors along the path are the same in the worst case, which arranges symmetrically besides the connection line of these two sensors as shown in Fig. 4. When we consider the worst case, it appears that the sensors sample at the same time. Note that our result can be generally applied in the asynchronous case even we only consider the worst case.

Consider a sampled point P in Fig. 3. If the distance between the two sensors is denoted by r , and the distance between the sampled point and the connection line between these two sensors is denoted by y . x is marked in the figure. It can be inducted that $x = r/2$ in Fig. 3 leads to the least detection gain of point P according to Lemma 1, i.e., the point with the least detection gain should be on

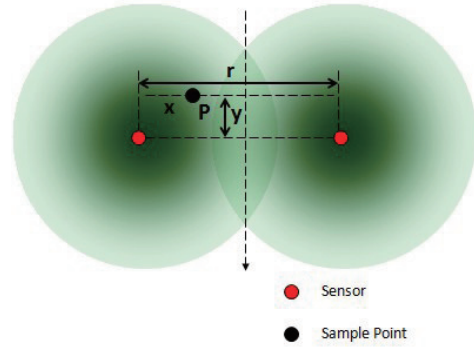


Fig. 3. The illustration of a sampled point.

the perpendicular bisector of the segment between these two sensors. We therefore have Lemma 3.

Lemma 3: Assume the distance between the sampled point and the connection line between these two sensors is denoted by y . For any fixed y , the point on the perpendicular bisector of the segment between these two sensors has the least detection gain.

Proof: Assume the detection gain of point P is $\phi(P)$. We have $\phi(P) = g(\sqrt{x^2 + y^2}) + g(\sqrt{(r-x)^2 + y^2})$. The detection gain is minimum when $x = r/2$ according to Lemma 1. If $x = r/2$, the point should be on the perpendicular bisector of the segment between these two sensors, where concludes the proof. ■

Now consider again two sensors at points m and k and a crossing path l . Let l' denote the perpendicular bisector of the segment between m and k . Each sampled point P on l has a corresponding point P' on l' with the same distance to the connection line between these two sensors. By the above lemma, $\phi(P) \geq \phi(P')$. The total detection gain of the sampled points P (on l) is no less than the total detection gain of their corresponding points P' (on l'). Since l' is a straight line, these points are apart with a distance less than or equal to V_{max}/f . If a target moves along l' at speed V_{max} , the distance between consecutive sampled points on l' will be V_{max}/f . So, the total detection gain of path l' is less than or equal to the total detection gain of all points P' , which in turn is less than or equal to the total gain of path l . Thus, we have the Theorem 1.

Theorem 1: The crossing path across two sensors with the minimum detection gain is along the perpendicular bisector of the segment between these two sensors.

The least detection probability path consists of a series of sampled points along the perpendicular bisector. Given the sampling frequency f and the maximum speed of intruders V_{max} , the distance between each two sampled points along the perpendicular bisector is V_{max}/f .

We only sum up the detection gain of points lie on the Voronoi edge which is a perpendicular bisector of each two sensors. For example, the detection gain of sensor A and B is calculated by the sampled points which lie on the red line as shown in Fig. 5. We denote $\psi(d)$ as the minimum detection gain of a pair of sensors which locate at point k and m respectively and d is the distance between k and m . Assume the set of sampled points on the corresponding Voronoi edge

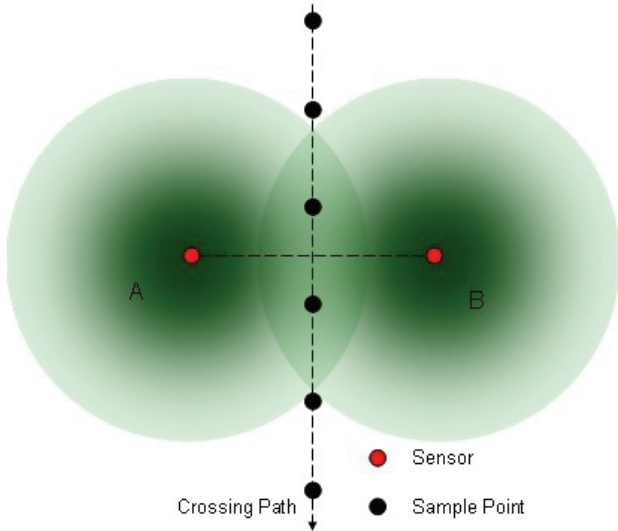


Fig. 4. The crossing path in worst detection case.

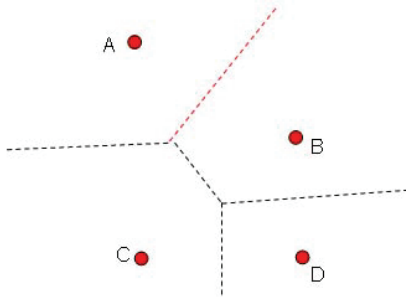


Fig. 5. The Voronoi graph of multiple sensors.

is Ω . We have

$$\psi(d) = 2 \sum_{j \in \Omega} \phi_k(j) \quad (10)$$

because $\phi_k(j) = \phi_m(j)$.

The minimum detection gain of each pair of sensors is determined by their locations. Since we aim to guarantee that the sum of detection gain is greater than aggregate detection gain threshold, we only consider the minimum detection gain of each pair of sensors in this paper and do not specifically distinguish between *minimum detection gain* and *detection gain* later. Given the linearity of detection gain, we can sum up the detection gain of each pair of sensors to extend the scenario to multiple sensors. If we want to calculate the detection gain to cross the barrier of sensors in Fig. 6, we just sum up the detection gain of each pair of connected sensors whose link line intersects with the crossing path l .

C. Flow Graph

According to the analysis of scenario of multiple sensors, we further build a **flow graph** to characterize the feature of the network. In the flow graph, sensors are viewed as nodes and the communication relationships between each two sensors are viewed as undirected edges, that is to say, two sensors within

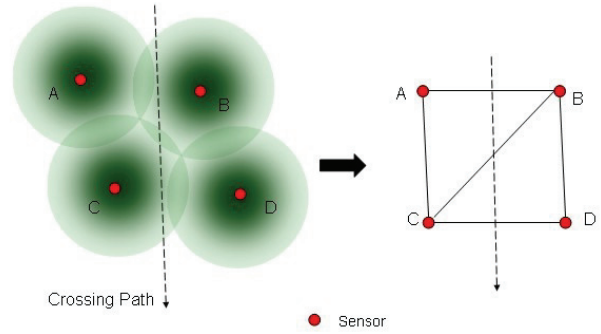


Fig. 6. The illustration of multiple sensors scenario. Each pair of sensors within the transmission range are connected. Sensor A and D are too far from each other and the detection gain is negligible, so there is no connection between them.

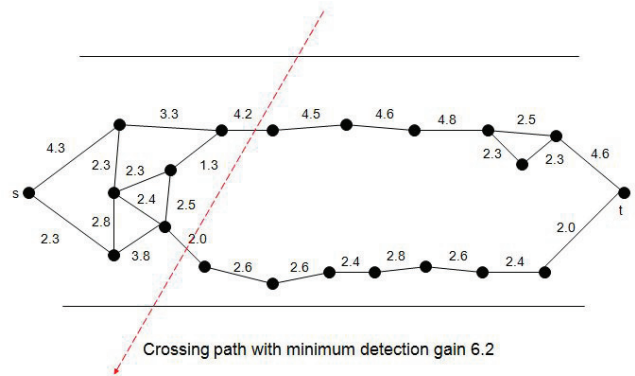


Fig. 7. An example of flow graph. The number above each edge denotes the capacity of the edge, which is determined by the distance between these two sensors. The arrow line is the crossing path with minimum detection gain with respect to the barrier of sensors.

the communication range R are linked with an edge, which can ensure the connectivity of the network. If the network can ensure barrier coverage, the connectivity of the network are also satisfied. Thus, the connectivity issue will not be discussed in the following part. The capacity of the edge is the detection gain which is determined uniquely by the locations of each two sensors.

Two virtual nodes s and t represent the left and right boundaries respectively. We consider sampled points along the boundaries when calculating the detection gain, which is similar to Eq.(10). But only one sensor contributes to the detection gain which is different from Eq.(10). An edge whose capacity is the detection gain exists between node i and s (or t) if the left boundary (or right boundary) of the region is within the communication range of sensor i .

Flow graph is an undirected graph. The example of flow graph is shown as Fig. 7.

A *cut* in Graph Theory is a set of edges without which the graph is separated. The edges which intersect with a crossing path is viewed as a *cut* with respect to the barrier according to the definition. Thus, the detection gain of a crossing path is the aggregate detection gain of edges it intersects with, i.e., the aggregate capacity of the edges in the cut set. According to the definition of ϵ -barrier coverage, the set of sensors forms

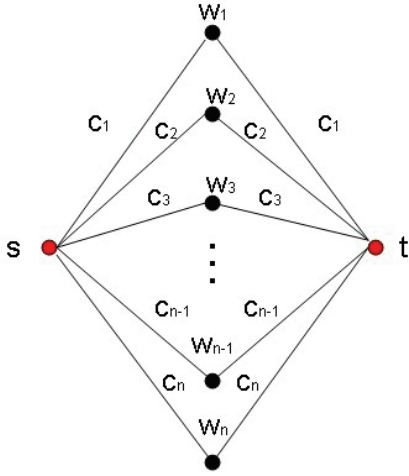


Fig. 8. A scenario to prove the NP-completeness of minimum weight barrier problem.

ϵ -barrier if and only if its minimum cut is greater than the gain threshold.

According to the classical Graph Theory, the minimum cut of network equals the maximum flow [28]. Therefore, the set of sensors is viewed as ϵ -barrier if its flow graph has a maximum flow over *aggregate gain threshold* $\varphi(\epsilon)$. The maximum flow defines the aggregate gain which the network can provide. Let $G = (V, E)$ denote the flow graph in region A where V consists of all activated sensors and E consists of all edges between nodes.

Minimum weight barrier problem can be represented as follows. Given a graph $G = (V, E)$ with the set of nodes and the edges with different capacity. Each node is assigned with a weight. Find a subset of nodes with minimum aggregate weight in V , which has a maximum flow greater than *aggregate gain threshold*.

V. ALGORITHM DESIGN

A. Minimum Weight Barrier Algorithm

We will prove that minimum weight barrier problem is NP-hard and propose an efficient bounded approximation algorithm in this section.

Theorem 2: The minimum weight barrier problem is NP-hard.

Proof: We consider a simplified version of minimum weight barrier problem. Assume the flow graph in this scenario, G is shown in Fig. 8.

The set of nodes with minimum aggregate weight in Fig. 8 which forms ϵ -barrier coverage can be found in polynomial time if minimum weight barrier problem is not NP-hard.

$$\begin{aligned} \min \quad & \sum_{i=1}^n w_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^n c_i x_i \geq \varphi(\epsilon) \end{aligned} \quad (11)$$

The deterministic version of this problem is a classic 0-1 integer programming problem which is proved to be NP-hard in [29]. Thus, minimum weight barrier problem is also NP-hard, where concludes the proof.

Since the problem is NP-hard, we can not find a polynomial algorithm to solve it unless $P = NP$. We will introduce an efficient approximation algorithm *Minimum Weight Barrier Algorithm* (MWBA) to find a *minimum weight ϵ -barrier*.

Note that the flow through each node is restrained to be not greater than $\varphi(\epsilon)$ in our algorithm because we only need to ensure that the flow of the whole network is greater or equal to $\varphi(\epsilon)$ and it will not contribute if the flow is greater than $\varphi(\epsilon)$.

Our aim is to find a set of nodes with minimum aggregate weight which can provide a max-flow over $\varphi(\epsilon)$ in V . The main idea of MWBA is removing nodes iteratively from initial node set V until the max-flow of remaining nodes is lower than $\varphi(\epsilon)$ to obtain an approximate minimum weight ϵ -barrier. Assume C is an empty set initially. We let Θ represent $V \cup C$ for simplicity in our description. E' denotes the set of edges between the connected nodes in Θ and G' is the flow graph of node set Θ and edge set E' , i.e., $G' = (\Theta, E')$. The major procedure of MWBA is shown as follows. Firstly, we adopt the Edmonds-Karp algorithm to determine the max-flow of $G = (V, E)$, $F(G)$, with time complexity $O(|V||E|^2)$ [28]. Secondly, we remove nodes successively from V . After removing each node, the max-flow of $G' = (\Theta, E')$, $F(G')$ is calculated once again. A node i removed from V is added to set C if the max-flow of $F(G')$ is less than $\varphi(\epsilon)$. The algorithm terminates when V is empty. Finally, C is the output of MWBA. It is easy to see that C is ϵ -barrier, because the max-flow of node set Θ is always greater than $\varphi(\epsilon)$ after every iteration of MWBA and C equals Θ when MWBA terminates.

To minimize the aggregate weight of nodes, we need to carefully design the removal strategy to achieve better performance. Edmonds-Karp algorithm is applied to determine not only the max-flow of the graph G' , but also the flow through each node in Θ when the max-flow is achieved. We let $F_{\Theta}(i)$ represent the flow through node i when the flow of G' is a max-flow. We argue that $F_{\Theta}(i)$ is an important factor because it depicts the contribution of a candidate node to the max-flow.

Suppose we have a graph G' and a node i in G' with a flow $F_{\Theta}(i)$ passing through. If we remove i , the instant flow reduces to $[F(G') - F_{\Theta}(i)]$. In fact, the max-flow of graph G' without node i is not less than the instant flow, which means that the decrement of max-flow caused by node i is absolutely not greater than $F_{\Theta}(i)$. Since $F_{\Theta}(i)$ restrains the decrement of max-flow when node i is removed from V , we can safely remove nodes with low $F_{\Theta}(i)$ whose effects on the max-flow of the graph are bounded. Thus, more nodes can be removed before the max-flow is lower than $\varphi(\epsilon)$. We adopt $F_{\Theta}(i)$ in MWBA as an important factor.

We consider to integrate the weights of nodes and $F_{\Theta}(i)$ together to determine which node is to be removed from V . Our goal is to remove as much weights as possible, which means the weight of node i is viewed as benefit and $F_{\Theta}(i)$ is the cost when removing node i . The integration function $M(i)$ in MWBA is the benefit over the cost, i.e.,

$$M(i) = w_i / F_{\Theta}(i) \quad (12)$$

We remove the node with the greatest $M(i)$ each time. In this way, nodes with less residual energy or lower upper

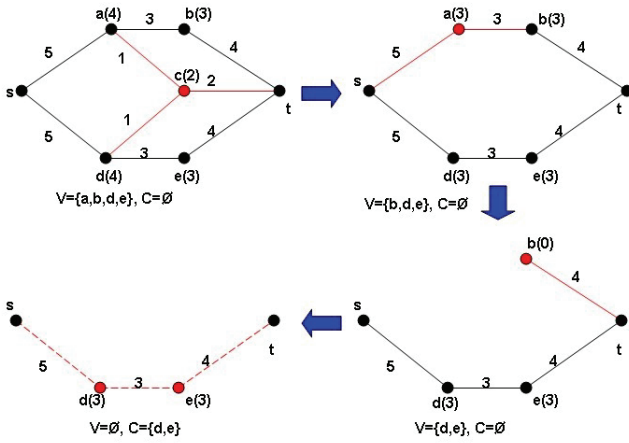


Fig. 9. An example of Algorithm 1.

bound of decrement of max-flow, i.e., $F_{\Theta}(i)$, are supposed to be removed from Θ with a higher priority. If w_i is infinite or $F_{\Theta}(i) = 0$, $M(i)$ is infinite which means node i has the highest priority to be removed. Every node i in set V are checked iteratively and added to C if the max-flow is lower than the threshold, while the nodes in Θ and edges are updated accordingly. MWBA terminates when V is empty. The remaining set C is the output of MWBA.

We use $M(i)$ as an important guidance to remove nodes but it is not always the best. In MWBA, we maintain a set of nodes Θ and a node is removed from Θ only if it will not cause violation against restraint of $\varphi(\epsilon)$. For example, if removing a node i with high $M(i)$ from V causes violation against restraint of $\varphi(\epsilon)$, we will add it into set C and thus it is not removed from Θ , which ensures that the nodes in Θ can always provide ϵ -barrier when running MWBA.

We give a simple example to illustrate MWBA in Fig. 9. Consider a flow graph $G = \{V, E\}$ which has 5 nodes with the same weight besides virtual nodes s and t , i.e., $V = \{a, b, c, d, e\}$. The capacity of each edge is also in Fig. 9. The number in brackets is the actual flow through each node. We assume that $\varphi(\epsilon) = 3$. At first, c with the minimum flow 2 is removed. The max-flow of $G' = \{\Theta, E'\}$ turns to be 6 after removing c . Then, nodes a, b, d, e have the same flow. Without the loss of generality, we remove a from V . The max-flow of G' turns to be 3 which is still providing ϵ -barrier coverage. Since there is no flow through node b , we remove it with no decrement of max-flow. If we remove node d or node e from V , the max-flow of G' reduces to 0. Thus, we add d and e to set C to keep the max-flow of G' is not lower than $\varphi(\epsilon)$. Finally, node set V is empty and the output $C = \{d, e\}$.

The algorithm is shown in Algorithm 1. The time complexity is polynomial since we only enumerate the elements in V once and each iteration cost $O(|V||E|^2)$. Thus, the total time complexity is $O(|V|^2|E|^2)$.

B. Algorithm Analysis

We analyze the performance of our proposed algorithm MWBA theoretically in this section. Let C denote the output of MWBA and OPT denote the optimum minimum weight ϵ -barrier, i.e., the optimum solution to the problem. Let w_{OPT}

Algorithm 1 Minimum Weight Barrier Algorithm

- 1) Calculate the max-flow of graph $G = (V, E)$. Let $C = \emptyset$.
- 2) For every node i in set V , calculate $M(i) = w_i/F_{\Theta}(i)$. If $V = \emptyset$, return ϵ -barrier C .
- 3) Find the node i with maximum $M(i)$ and remove i from V .
- 4) Update existing nodes in set Θ and edges E' accordingly.
- 5) Calculate the max-flow $F(G')$ of the graph $G' = (\Theta, E')$. If $F(G') < \varphi(\epsilon)$, then let $C = C \cup \{i\}$.
- 6) Back to step 2.

and w_C denote the aggregate weight of nodes in set OPT and C respectively. To provide an approximation ratio upper bound of MWBA, we have the following main result for MWBA, which theoretically guarantees the performance of MWBA even in the worst case.

Theorem 3: $w_C/w_{OPT} < \frac{N}{N+F(G)/\varphi(\epsilon)-1} \rho \theta \psi^{-1}[\varphi(\epsilon)]$.

As θ , ϵ and $F(G)$ are constants, the approximation ratio of MWBA is only related to the density ρ . Next, we will show how to derive the theorem.

Firstly, we will provide a lemma to give the ratio bound of aggregate weight between the output of MWBA, C , and the initial node set V . $F(G)$ is the max-flow of initial flow graph G and $F(G)/\varphi(\epsilon)$ is the redundancy factor which characterize the redundancy of sensor deployment.

Lemma 4: $w_C \leq \frac{N}{N+F(G)/\varphi(\epsilon)-1} w_V$.

Proof: Let $\bar{C} = V - C$ denote the set of nodes which are removed from initial set V by Algorithm 1. Suppose at the $(k+1)$ th iteration $F(G'_{k+1})$ is lower than $\varphi(\epsilon)$ for the first time. Let Q denote the set $V - V_k$, where V_k is V at the k th iteration. Thus, $C \subseteq V_k$. Obviously, $Q \subseteq \bar{C}$, which means $w_Q \leq w_{\bar{C}}$.

Since MWBA always selects to remove node i from V with maximum $M(i) = w_i/F_{\Theta}(i)$, we get that,

$$\begin{aligned} \frac{w_Q}{\sum_{i \in Q} F(i)} &\geq \max_{u \in C} \left\{ \frac{w_u}{F(u)} \right\} \\ &\geq \frac{w_C}{\sum_{u \in C} F(u)} \end{aligned} \quad (13)$$

where $F(i)$ denotes $F_{\Theta}(i)$ for simplicity in this proof section.

According to the definition of set Q and $F(i)$, we have

$$\sum_{i \in Q} F(i) \geq F(G) - \varphi(\epsilon) \quad (14)$$

With Eq.(13), (14),

$$\begin{aligned} \frac{w_{\bar{C}}}{w_C} &\geq \frac{w_Q}{w_C} \\ &\geq \frac{\sum_{i \in Q} F(i)}{\sum_{u \in C} F(u)} \\ &\geq \frac{F(G) - \varphi(\epsilon)}{\varphi(\epsilon)N} \end{aligned} \quad (15)$$

With the Eq. $w_V = w_{\bar{C}} + w_C$, we have

$$\frac{w_C}{w_V} \leq \frac{\varphi(\epsilon)N}{\varphi(\epsilon)N + F(G) - \varphi(\epsilon)} \quad (16)$$

$$w_C \leq \frac{N}{N + F(G)/\varphi(\epsilon) - 1} w_V \quad (17)$$

which concludes the proof. \blacksquare

Let N_1 denote the number of sensors in *OPT*. Obviously, it is optimal to deploy nodes in a straight line with a distance of d , where $\psi(d) = \varphi(\epsilon)$, to form an ϵ -barrier. $\psi^{-1}[\varphi(\epsilon)]$ is the minimum distance between two nodes to detect an intruder with a probability greater than ϵ . The minimum number of sensors which form an ϵ -barrier to A is $\frac{L}{\psi^{-1}[\varphi(\epsilon)]}$. We have

$$N_1 \geq \frac{L}{\psi^{-1}[\varphi(\epsilon)]}. \quad (18)$$

According to equation (1), the weight of energy-redundant node i , w_i satisfies that $\theta/B > w_i \geq 1/B$. Assume ρ is the line density of sensors scattered in the belt region, i.e., ρ is defined as the ratio of the number of sensors N to the length of belt region A . We have

$$w_{OPT} \geq N_1/B. \quad (19)$$

$$\begin{aligned} \frac{w_V}{w_{OPT}} &< \frac{N\theta/B}{N_1/B} \\ &\leq \theta N \frac{\psi^{-1}[\varphi(\epsilon)]}{L} \\ &= \rho \theta \psi^{-1}[\varphi(\epsilon)] \end{aligned} \quad (20)$$

Based on Lemma 4 and Eq.(20), we can derive the result in Theorem 3.

C. Algorithm Evaluation

We conduct extensive simulations to evaluate the performance of MWBA. The key factor of MWBA is $M(i) = w_i/F_\Theta(i)$ which means the benefit over the cost. We consider the weights of nodes and upper bound of decrement of max-flow, i.e., $F_\Theta(i)$, in an integrated way to determine which node is removed from Θ . If we only remove nodes with highest weight iteratively until the max-flow of graph is lower than $\varphi(\epsilon)$, we call the algorithm *Weight-Oriented*. If we only consider $F_\Theta(i)$ when removing, the algorithm is called *Flow-Oriented*.

The purpose of minimum weight ϵ -barrier problem is to minimize the aggregate weight of activated sensor set. We randomly assign weights to sensors and execute each algorithm hundreds of times in the same scenario. The aggregate weights of sensor set acquired by MWBA, Weight-Oriented and Flow-Oriented respectively are compared to prove the superiority of our algorithm design in Fig. 10. It can be seen that the aggregate weight of sensor set acquired by MWBA is much lower than others, which means that MWBA is much better than others to solve minimum weight ϵ -barrier problem. The effectiveness of $M(i)$, which integrates the weight and $F_\Theta(i)$, is validated in Fig. 10.

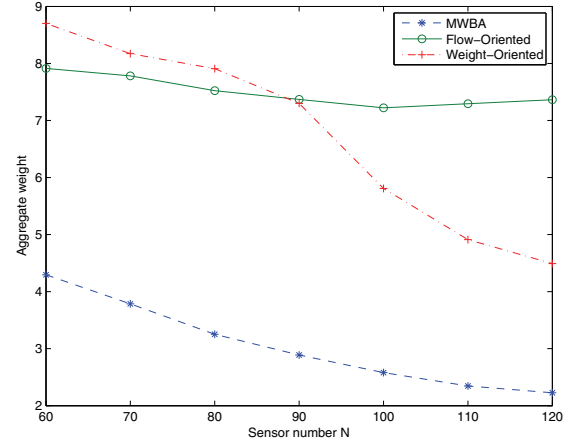


Fig. 10. Aggregate weights vs. N , the amount of deployed sensors, $\epsilon = 0.95$.

VI. LOCALIZED PROTOCOL

Though we have proposed an efficient approximation approach MWBA, it is a global solution that requires information from all other sensors with considerable communication costs, especially when the belt region is very long, such as the border line between two countries. In this section, we design a localized protocol for the minimum weight barrier problem. With our protocol, sensors do not need to communicate with all other sensors in the entire belt region globally. They only need to communicate with some sensors in a limited local region, which can save lots of communication costs and time, while still ensuring the global barrier coverage.

A. Challenges

We can not directly implement MWBA in a localized way. Actually designing a localized barrier coverage protocol is challenging. It has been shown that global optimal solution for localized barrier coverage protocol is infeasible in [5], [8]. By global optimal solution, we mean the optimal amount of sensors to provide global barrier coverage. However, it is still possible to design sub-optimal solutions for the problem. Since probabilistic sensing model is adopted, we have to tackle with a more general problem than that in [8]. We display two scenarios to demonstrate the difficulty of designing a localized protocol for barrier coverage.

First, consider Fig. 11. Suppose sensors in two neighboring regions marked as *local1* and *local2* make local decisions and guarantee ϵ -barrier coverage for their individual regions. As illustrated in the figure, there can still exist crossing paths with a detection probability lower than ϵ . If sensors only ensure that local network flow within each local region is greater than the threshold, the global network flow may still not be greater than the threshold. Local barrier coverage does not simply lead to global barrier coverage.

It is not sufficient to let sensors in neighboring regions share information and ensure ϵ -barrier coverage for a larger merged region. Fig. 12, borrowed from [8], illustrates this.

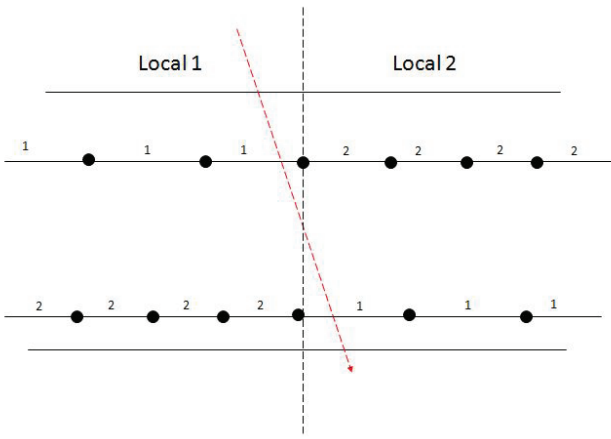


Fig. 11. Suppose the detection gain threshold is 3. Even two neighboring regions are both ϵ -barrier covered, there still exists a crossing path as the arrow line with a detection gain 2 which is lower than the threshold.



Fig. 12. Suppose the detection gain threshold is 1. Even all pairs of neighboring local regions ensure barrier coverage for their merged region, there still exists a crossing path with detection gain 0 as the arrow line.

B. Protocol Design

Even though a globally optimum solution cannot be obtained by a localized barrier coverage protocol, sub-optimal solutions are possible.

We first adopt the definition of L -zone from [8].

Definition 7: (L -zone) For a positive number L , an L -zone is a slice of the belt region of length L . Two of its edges coincide with the belt's two parallel boundaries, and the other two edges are orthogonal crossing lines separated by a distance of L .

Let the belt region be divided into L -zones and D -zones as illustrated in Fig. 13. We will refer to these D -zones as **body zones** and the zones joining them (namely L -zones) as **joint zones**.

We introduce two types of unit regions, called BJB-regions and JBJ-regions. A region of either type consists of three zones. As illustrated in 14, a BJB-region has two body zones and a joint zone in between, whereas a JBJ-region has two joint zones and a body zone. If we can ensure that every unit region is ϵ -barrier covered, then we can avoid the problem illustrated in Fig. 11. However, the problem shown in Fig. 12 still remains unsolved.

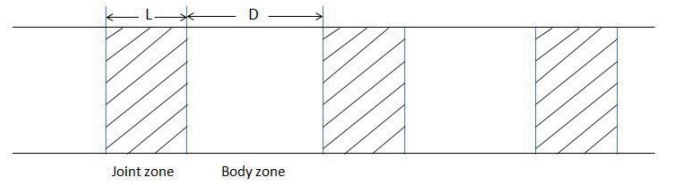
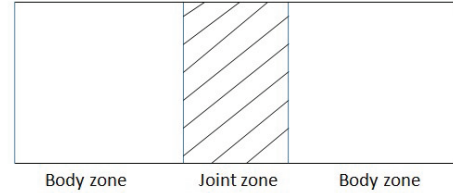
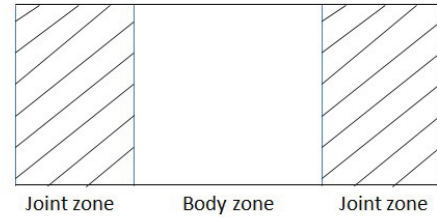


Fig. 13. Division of belt region into joint zones and body zones. The shaded zones are joint zones.



(a) Unit region 1 : B-J-B



(b) Unit region 2 : J-B-J

Fig. 14. Two types of unit regions.

To tackle the latter problem, we require each joint zone to be ϵ -blocking, which means that if a target travels from one body zone to another body zone (crossing at least one joint zone), the detection probability should be greater than or equal to ϵ . Thus, sensors in a joint zone not only connect body zones to form a barrier against crossing paths of the belt, but also form a barrier against paths between its two neighboring body zones. Paths across a joint zone from one body zone to another body zone are named *horizontal paths* as shown in Fig. 15. A problem like the one in Fig. 12 can be solved if every joint zone is ϵ -blocking. In summary, we have the following result.

Theorem 4: The belt region is globally ϵ -barrier covered if every joint zone is ϵ -blocking and every BJB- and JBJ-region is ϵ -barrier covered.

Proof: The theorem is obvious. If a crossing path intersects with no more than three zones, it is a crossing path within a BJB- and JBJ-region. Otherwise, it must cross a joint zone horizontally. In either case, it is detected with probability at least ϵ . ■

Now we discuss how to provide ϵ -barrier coverage to a unit region (BJB- or JBJ-region). Each sensor communicates with other sensors in its local zone (a joint zone or body zone). Sensors in body zones conduct MWBA to determine whether to stay active or switch to sleep. As mentioned earlier, local barrier coverage does not imply global barrier coverage because the global network flow may be less than the local network flow as shown in Fig. 11. To solve the problem, we let

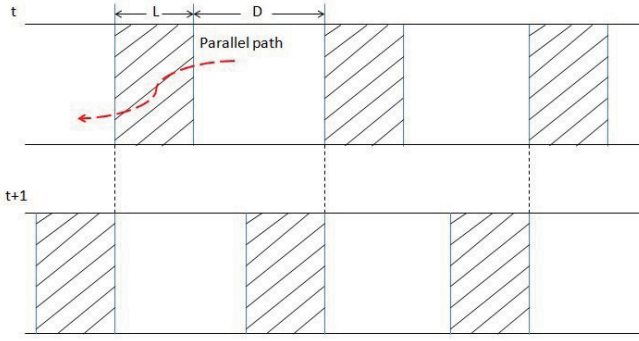


Fig. 15. Joint zone shifting per time slot.

sensors in each joint zone wait until their neighboring zones (body zones) have finished running MWBA. Sensors in joint zones are activated to connect the body zones to ensure that the global network flow is greater than the threshold.

After executing MWBA, find a flow of $\varphi(\epsilon)$ in the flow graph composed of activated sensors of each body zone. The flow is named **coordinated flow**. It is feasible because the flow graph can always provide a maximum flow over $\varphi(\epsilon)$ after running MWBA. Consider the activated sensors in a body zone which are connected with sensors in the neighboring joint zones. They know the magnitude of the coordinated flow through them and they can broadcast this information to the sensors in neighboring joint zones. We build a flow graph with all sensors in a joint zone and neighboring sensors in the two neighboring body zones, and introduce two virtual nodes s and t , as illustrated in Fig. 16. The capacity of each edge between virtual nodes and neighboring sensors in body zones is the coordinated flow through each neighboring sensor i . The flow graph is named **joint flow graph**. The process of constructing a joint flow graph is illustrated in Fig. 16, where the upper graph is composed of all sensors in a joint zone and several activated sensors in body zones which are connected with sensors in the joint zone. The coordinated flow through each sensor in the body zones is marked above the dashed lines. In the bottom graph, two virtual nodes s and t are added. We can build a joint flow graph for each joint zone.

The significance of joint flow graphs is due to the following lemma, by which we can locally activate sensors to provide ϵ -barrier coverage to a BJB-region.

Lemma 5: A BJB-region is ϵ -barrier covered if its joint flow graph has a flow of value equal to the *aggregate gain threshold* $\varphi(\epsilon)$.

Proof: According to our analysis in Section IV, the region composed of the joint zone and its two neighboring body zones is ϵ -barrier covered if the flow graph in the region has a maximum flow greater than or equal to *aggregate gain threshold* $\varphi(\epsilon)$.

The maximum flow of the joint flow graph can not be greater than $\varphi(\epsilon)$ since it has been restricted by the capacity of edges connected to virtual nodes. If there is a flow of value $\varphi(\epsilon)$ in the joint flow graph, then there is a flow of value $\varphi(\epsilon)$ from the entire flow graph of the three merged zones — the joint zone and its two neighboring body zones — one only needs to set the flow of the joint zone as the joint flow graph

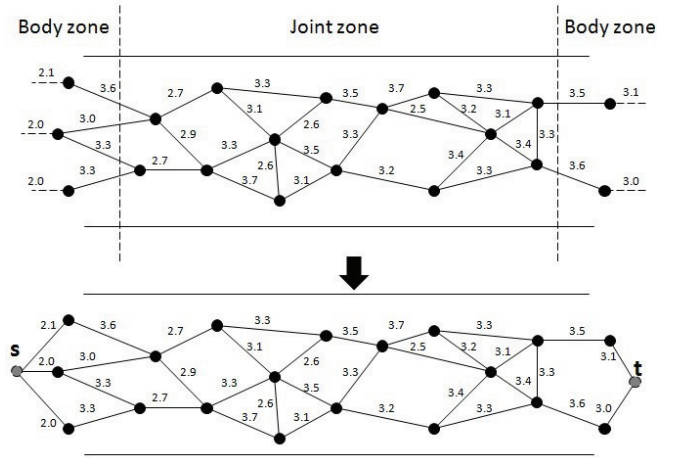


Fig. 16. Construct a joint flow graph in a joint zone.

and the flows of body zones as their coordinated flows. Thus, if the maximum flow of the joint flow graph equals $\varphi(\epsilon)$, then the maximum flow of the entire flow graph of the merged three zones will be greater than or equal to $\varphi(\epsilon)$ and guarantees that the three regions together is ϵ -barrier covered. ■

The above argument does not apply to a JBJ-region, for which we need to consider the two joint flow graphs defined by the two “J” (joint) zones.

Lemma 6: A JBJ-region is ϵ -barrier covered if the two joint flow graphs each have a flow of value $\varphi(\epsilon)$.

Proof: Assume the two joint flow graphs each have a flow of value $\varphi(\epsilon)$. Let the two flows be f_1 and f_2 . Since the two joint flow graphs share a common body zone with the same coordinated flow, it is easy to construct from f_1 and f_2 a flow of value $\varphi(\epsilon)$ for (the flow graph of) the JBJ-region if we set the flow in the body zone as its coordinated flow. ■

From Lemma 5 and 6, we immediately have the following theorem.

Theorem 5: If the joint flow graph of every BJB-region has a flow of value $\varphi(\epsilon)$, then every unit region, BJB or JBJ, is ϵ -barrier covered.

Theorem 5 takes care of one of the two required conditions in Theorem 4. Now consider the other condition: every joint zone be ϵ -blocking. This one is trivial. For ease of presentation, assume the original belt region is horizontal. If we view a horizontal joint zone as a vertical belt region, then it is ϵ -blocking as a (horizontal) joint zone iff it is ϵ -barrier covered as a (vertical) belt.

We design Joint Zone Algorithm (JZA) to activate sensors in each joint zone to guarantee the two required conditions in Theorem 4. JZA is executed after the running of MWBA in body zones. JZA is a centralized algorithm and each sensor in the zone needs to communicate with other sensors in each time slot to perform the calculation and determine whether to sleep in practical. Before running JZA, sensors in joint zone needs to collect the coordinated flow from neighboring sensors in neighboring body zones. Then, JZA always removes the sensor with greatest weight until the requirements of joint flow and blocking constraint can not be met and the remaining sensors are to be activated. The requirement of joint flow means the joint flow graph should have a flow of value which equals $\varphi(\epsilon)$.

Algorithm 2 Joint Zone Algorithm

Initially let C be the set of all sensors in the joint zone and sort sensors in C by weight;

while At least one sensors in C is unselected **do**

Select an unselected sensor j with greatest weight and mark it as *selected*;

if Set $C - \{j\}$ meets the requirements of joint flow and blocking constraints **then**

Remove sensor j from C ;

end if

end while

Activate sensors in C .

C. Localized Minimum Weight Barrier Protocol

We propose Localized Minimum Weight Barrier Protocol (LMWBP) as a localized protocol for global barrier coverage based on the analysis of MWBA and JZA.

As mentioned, the belt region is divided into body zones and joint zones alternately. The length of body zone is D and joint zone is L . The two ends of the belt region are both body zones with variable length. In each time slot, the sensors in body zones decide whether to sleep by running MWBA and those connected with sensors in joint zones among the active sensors in body zones broadcast the coordinated flow to their neighboring joint zone. After receiving all the messages containing flow information from both two neighboring body zones, sensors in the joint zone start to decide whether to sleep by running JZA. The coordinated flow information is broadcasted to sensors in joint zone, in a multi hop way. Each time slot, the sensors in each local zone perform MWBA or JZA to determine whether to sleep. In this way, sensors do not need to communicate with all other sensors in the entire belt region globally. They only need to communicate with some sensors in a small local region, which can save lots of communication cost and time.

We set a timeout threshold to wait for all coordinated flow information messages and the timeout threshold should be estimated by L , D and the line density of sensors. In fact, we can determine the threshold by experiments when deploying sensors in a real WSN since it is affected by many practical parameters. To be conservative, we can set the threshold to be relatively large if it is not convenient to conduct experiments.

The shortcoming of joint zone is that it consumes more energy than the body zone since it needs to be ϵ -blocking and thus more sensors are activated than optimal. To balance the additional cost, we shift the joint zone each slot as shown in Fig. 15. The two ends of the belt region are body zones with a length of $1.5D$ initially and the length varies during the shift of joint zones. The cycle of joint zone shifting is D/L , that is to say, when the joint zones have shifted for D/L time slots, the joint zones and body zones are set as initial state. Since the position of each sensor is known, every sensor is aware of which zone it belongs to if the cycle of zone shifting is fixed. The details of LMWBP are shown in Algorithm 3, where we omit the detail of coordinated flow and scheduling information broadcasting in a local zone.

Joint zone consumes more energy than body zone. That is

Algorithm 3 Localized Minimum Weight Barrier Protocol

1: At the beginning of each time slot, sensor i is aware of which zone it belongs to.

2: **if** i belongs to a body zone **then**

3: i communicates with all sensors within its own body zone;

4: Perform MWBA (Algorithm 1) to decide whether to sleep and find a coordinated flow;

5: Broadcast the coordinated flow;

6: **if** i is set to be active and i connects to sensors in joint zone **then**

7: Broadcast the coordinated flow to the joint zone;

8: **end if**

9: **else**

10: i communicates with all sensors within its own joint zone;

11: Forward the coordinated flow information from neighboring body zone in the joint zone when receiving;

12: **while** i has not received the coordinated flow information from both neighboring body zones **do**

13: Stay active and wait;

14: **if** Timeout **then**

15: Set the unknown coordinated flow to be zero.

16: Break;

17: **end if**

18: **end while**

19: Perform JZA (Algorithm 2) and decide whether to sleep;

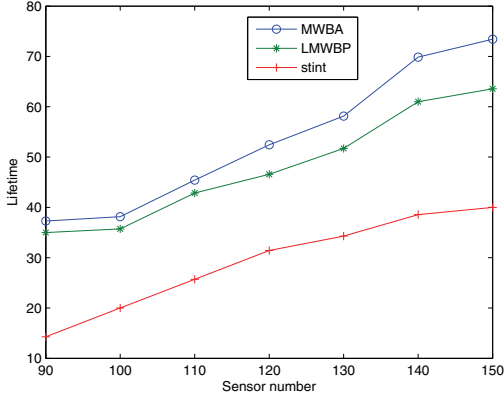
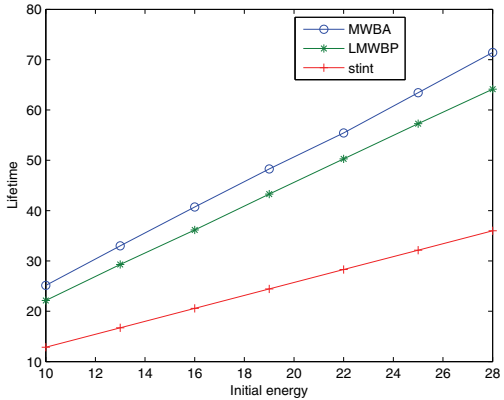
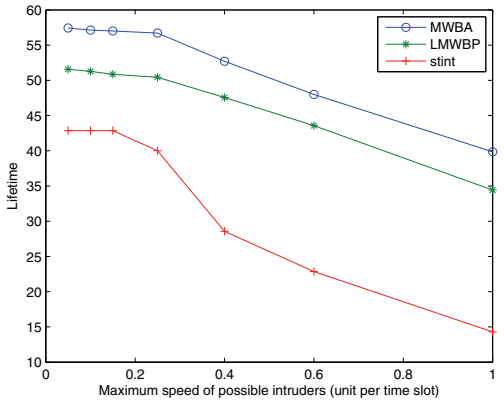
20: **end if**

the additional cost of localized protocol compared with global solution. We always choose L to be relatively much less than D to minimize the cost. In fact, if there exist no joint zone and only one body zone, the energy consumption is as efficient as the global solution MWBA while the approach deteriorate into a global one.

Note that our protocol can be easily applied to provide k -barrier coverage with the assumption of 0/1 sensing model, a classical problem discussed in [5], [8]. We can set the threshold $\varphi(\epsilon)$ to be k and if the sensing regions of each two sensors has overlapping, we add an edge with weight 1 between these two sensors in the flow graph. Then the localized protocol can be applied.

VII. SIMULATION RESULTS

We conduct several simulations to evaluate the performance of MWBA and the localized protocol LMWBP. The WSN in our simulations has N sensors, each with an initial energy of B units. The sensors are deployed randomly following uniform distribution in a belt region of 2×50 units². Active sensor in each time slot consumes 1 unit of energy. Note that in our simulations we do not consider communication energy cost in the initial setting phase at the beginning of each slot since it might be negligible compared with the energy cost during the time slot if the length of each time slot is considerable.

(a) Lifetimes vs. N , $\epsilon = 0.95, B = 20$ (b) Lifetimes vs. B , $\epsilon = 0.95, N = 100$ (c) Lifetimes vs. V_{max} , $\epsilon = 0.95, N = 100, B = 20$ Fig. 17. Lifetimes of MWBA, LMWBP and *Stint*.

The communication range R is set to be 4 units. The detection probability threshold is ϵ . We assume that the maximum speed of possible intruders is known as V_{max} .

We use the exponential attenuation probabilistic model introduced in [10] for our problem formulation here ¹.

The main purpose of energy-efficiently scheduling is to prolong the lifetime of WSN. We compare the performance in lifetime of WSN between MWBA, LMWBP and the state-of-the-art solution *Stint* in [6]. We set $D = 8$ and $L = 2$

¹The other probabilistic models can also be adopted in a similar way.

in LMWBP. Since LBCP in [8] can not ensure global barrier coverage, we do not take it for comparison in the simulation.

The simulations to acquire lifetime of network by MWBA and LMWBP are conducted mainly in following procedures. Firstly, at the beginning of each time slot, each sensor is assigned with a weight according to its residual energy. The sensor with more residual energy is assigned lower weight. Secondly, we employ MWBA and LMWBP to find ϵ -barrier and activate the sensors in the barrier during each time slot. Finally, the lifetime of network terminates if there exist no ϵ -barriers with redundant energy to provide ϵ -barrier coverage any more.

The best algorithm to maximize network lifetime in barrier coverage model, to our best knowledge, is the Algorithm *Stint* [6], which has been proved to achieved the optimal in boolean sensing model. There has not been any algorithm proposed for strong barrier coverage in probabilistic sensing model yet. *Stint* is designed as an optimal algorithm for the boolean disc sensing model in barrier coverage. To apply *Stint* here, we regard that two sensors have overlapping sensing regions if the detection gain between them is greater than $\varphi(\epsilon)$. *Stint* can not be directly applied in the probabilistic sensing model because it can not acquire the maximum disjoint paths through max-flow algorithm if edges with diverse capacities exist. Thus, we have to neglect the edges whose capacities are lower than $\varphi(\epsilon)$ in the flow graph.

The lifetimes achieved by *Stint*, *LMWBP* and *MWBA* versus different number of deployed sensors and initial energy are plotted in Fig. 17(a) and Fig. 17(b). The lifetime is lengthened if the number of deployed sensors N or the initial energy of each sensor B increases. The plots suggest that *MWBA* and *LMWBP* always have better performance of longevity compared with *Stint* in different scenarios. Since *MWBA* and *LMWBP* can utilize all detection gain between each pair of sensors within communication range, they outperform *Stint* naturally. The plots also confirm that *MWBA* as a global solution is more energy-efficient than *LMWBP* since it can activate less sensors each time slot, if communication cost is not considered.

We also compare the lifetimes versus different maximum speed of possible intruders in Fig. 17(c). If the maximum speed V_{max} decreases, the detection probability of a certain path increases correspondingly since the amount of sampling points along the path increases. Thus, less sensors are needed to perform sensing task, which leads to a longer lifetime. The sensing radius of boolean disc model r is chosen to ensure $\psi(2r) = \varphi(\epsilon)$ so that we can compare these two algorithms fairly. *Stint* also provides a longer lifetime when the maximum speed decreases due to the increasing of sensing radius. Besides the conclusion that *MWBA* and *LMWBP* always have better performance than *Stint*, the plots also show that the speed of possible intruders is an important factor when designing barrier systems. We may need to deploy more sensors to ensure the longevity of network if the speed of possible intruders is very high.

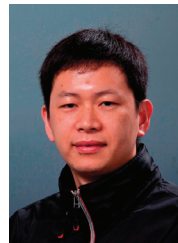
VIII. CONCLUSION

In this paper, we have analyzed the detection probability of moving path across the barrier theoretically based on

probabilistic sensing model. We construct the model of path detection and formulate the problem of scheduling the activation of sensors energy-efficiently which form ϵ -barrier in a randomly deployed sensor networks for the first time, based on the theoretical analysis of detection probability. In order to solve the problem, we analyze the scenario of multiple sensors performing sensing and introduce the concept of flow graph. We then propose an algorithm called MWBA, an efficient algorithm to solve the scheduling problem in polynomial time. The approximation bound of our algorithm is proved to be $O(\rho)$, where ρ is the line density of sensors scattered in the belt region, i.e., ρ is defined as the ratio of the number of sensors N to the length of belt region A . We also propose an energy-efficient localized protocol LMWBP which can be applied to solve a difficult problem that how to guarantee barrier coverage energy-efficiently in a localized way.

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