

Cursus: M1, computer-science
Code UE: JEIN8602
Subject: Formal languages theory
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Duration: 3H
 Documents: authorized
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The exercises are independant one from each other.
 It is *not* required to solve *all* the exercises. Every correct solution to an exercise will give from 4 to 6 marks (depending on the exercise).

Exercise 1 [/4] We consider the following regular expressions:

$$e_1 := (ab)^*c, \quad e_2 := (ba)^*, \quad e_3 := ((ab)^*c) \cup (ba)^*, \quad e_4 := (ab)^*(c \cup (ba)^*)$$

- 1- Give explicitly four words $u_1 \in L_{e_1}, u_2 \in L_{e_2}, u_3 \in L_{e_3}, u_4 \in L_{e_4}$.
- 2- Fill each cell of the following array with the answer “yes” (resp. “no”) if the word u of the row belongs to the language defining the column:

u	L_{e_1}	L_{e_2}	L_{e_3}	L_{e_4}
c	yes	no	yes	yes
abc				
ba				
$abba$				
ε				

(The first row is given as an example; no precise justification is required for this question).

- 3- Among the four following statements, which ones are true?

$$L_{e_3} = L_{e_4}, \quad L_{e_2} \subseteq L_{e_1}, \quad L_{e_2} \subseteq L_{e_3}, \quad L_{e_3} = L_{e_1} \cup L_{e_2}.$$

For the statements that are false, give a word that witnesses the falsity.

Exercise 2 [/4] Let us consider the regular expression:

$$e := (ab^*c)^*ab(a \cup b)^*$$

Construct, by Glushkov’s method, a finite automaton recognizing L_e .

Exercise 3 [/5] We consider the finite automaton \mathcal{A} described on figure 1. Note that 0 is

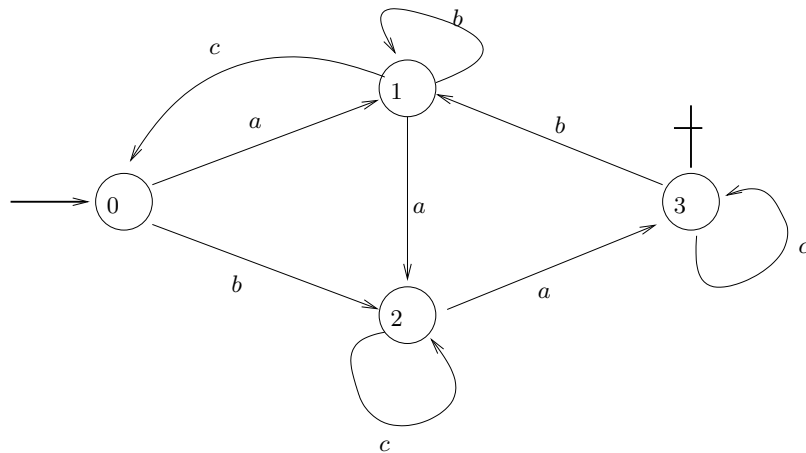


Figure 1: finite automaton \mathcal{A}

the only initial state and 3 is the only final state.

Construct a regular expression for the language $L_{\mathcal{A}}$ recognized by the automaton \mathcal{A} . Explain the successive steps of your construction.

Exercise 4 [/5] We consider the finite automaton \mathcal{B} described on figure 2. Note that 0 is

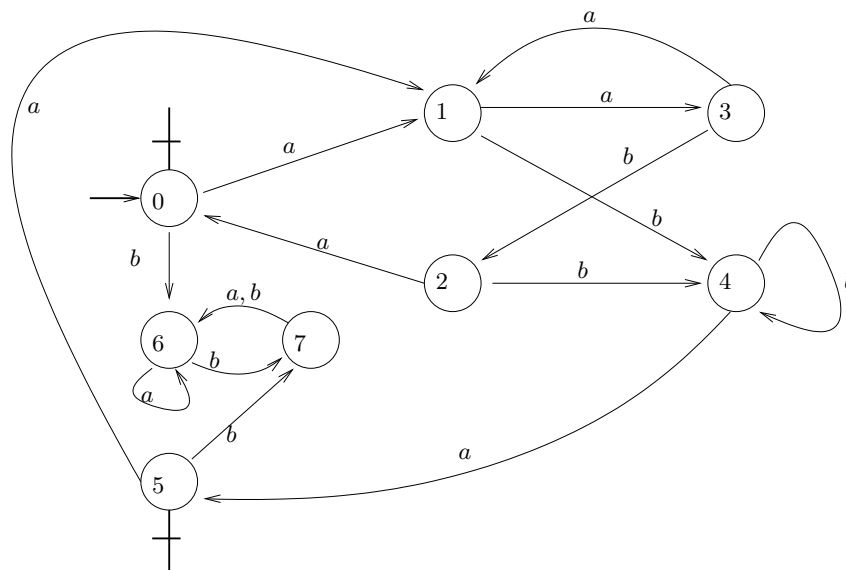


Figure 2: finite automaton \mathcal{B}

the initial state and 0, 5 are the final states.

1- Is \mathcal{B} *deterministic* ?

2- Is every state of \mathcal{B} *accessible* ?

3- Is \mathcal{B} *complete* ? Is \mathcal{B} *minimal* ? If not, transform \mathcal{B} into a minimal deterministic complete automaton \mathcal{C} that recognizes the same language.

Exercise 5 [/6] 1- For each of the following languages over the terminal alphabet $\{a, b, c, d\}$, construct a context-free grammar that generates the language L_i :

$$\begin{aligned} L_1 &:= \{(ab)^n \mid n \geq 0\} \\ L_2 &:= \{(ab)^n c^n \mid n \geq 0\} \\ L_3 &:= \{(ab)^n c^m \mid n \geq m \geq 0\} \\ L_4 &:= \{(ab)^p c^q d^r \mid q \geq 0, r \geq 0, p = q + r\} \\ L_5 &:= \{(ab)^p c^q d^r \mid q \geq 0, r \geq 0, p \geq q + r\} \end{aligned}$$

2- Construct a *non-ambiguous* context-free grammar generating the language L_5 .

Exercise 6 [/5] We consider the context-free grammar $G := (A, N, R)$ where $A = \{a, b\}$, $N = \{S, T, U\}$ and R consists of the following five rules:

r1: $S \rightarrow TU$

r2: $T \rightarrow aTT$

r3: $T \rightarrow b$

r4: $U \rightarrow aUT$

r5: $U \rightarrow bTT$

The start symbol of G is S .

1- Show that $aabbaabbb \in L(G, T)$. Give a leftmost derivation from T to $aabbaabbb$.

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2- Show that $aabbaabbbbbb \in L(G, S)$. Give a leftmost derivation from S to $aabbaabbbbbb$.

Give a derivation-tree for $aabbaabbbbbb$.

(The student can choose, among the various notions of syntax-tree, abstract syntax-tree, etc... his/her favorite notion and compute the corresponding tree).

3- What are the *productive* non-terminals of G ? the *useful* non-terminals of G ?

4- Is G a *simple* context-free grammar ? If not, transform G into a simple context-free grammar G' , generating the same language.