

LOGICS

Natural deduction-NK

1-Axioms

$$\frac{}{\Gamma, A \vdash A} \text{ax}$$

2-Structural rules

$$\frac{\Gamma \vdash A}{\Gamma, B \vdash A} \text{wkn}$$

3-Connector rules

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \wedge^{\ell} \text{elim}$$

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \wedge^r \text{elim}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge \text{intro}$$

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \vee \text{elim}$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee^{\ell} \text{intro}$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee^r \text{intro}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash A \rightarrow B}{\Gamma \vdash B} \rightarrow \text{elim}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \rightarrow \text{intro}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash \neg A}{\Gamma \vdash \perp} \neg \text{elim}$$

$$\frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A} \neg \text{intro}$$

$$\frac{\Gamma, \neg A \vdash \perp}{\Gamma \vdash A} \perp \text{classic}$$

4-Quantifier rules

$$\frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[x:=t]} \forall \text{elim}$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash \forall x A} \forall \text{intro} \text{ (if } x \notin \text{FV}(\Gamma)\text{)}$$

$$\frac{\Gamma \vdash \exists x A \quad \Gamma, A \vdash B}{\Gamma \vdash B} \exists \text{elim} \text{ (if } x \notin \text{FV}(\Gamma, B)\text{)}$$

$$\frac{\Gamma \vdash A[x:=t]}{\Gamma \vdash \exists x A} \exists \text{intro}$$

Intuitionistic natural deduction- NJ

1-Axioms

$$\frac{}{\Gamma, A \vdash A} \text{ax}$$

2-Structural rules

$$\frac{\Gamma \vdash A}{\Gamma, B \vdash A} \text{wkn}$$

3-Connector rules

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \wedge^{\ell} \text{elim} \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \wedge^r \text{elim} \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge \text{intro}$$

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \vee \text{elim} \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee^{\ell} \text{intro} \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee^r \text{intro}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash A \rightarrow B}{\Gamma \vdash B} \rightarrow \text{elim} \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \rightarrow \text{intro}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash \neg A}{\Gamma \vdash \perp} \neg \text{elim} \qquad \frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A} \neg \text{intro}$$

$$\frac{\Gamma \vdash \perp}{\Gamma \vdash A} \perp \text{elim}$$

4-Quantifier rules

$$\frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[x:=t]} \forall \text{elim} \qquad \frac{\Gamma \vdash A}{\Gamma \vdash \forall x A} \forall \text{intro} \text{ (if } x \notin \text{FV}(\Gamma))$$

$$\frac{\Gamma \vdash \exists x A \quad \Gamma, A \vdash B}{\Gamma \vdash B} \exists \text{elim} \text{ (if } x \notin \text{FV}(\Gamma, B)) \qquad \frac{\Gamma \vdash A[x:=t]}{\Gamma \vdash \exists x A} \exists \text{intro}$$