

LOGICS

Some first-order theories

EG (Equality theory)

REF : $\forall x \ x = x$

SYM : $\forall x, y \ (x = y \rightarrow y = x)$

TRANS : $\forall x, y, z \ (x = y \wedge y = z) \rightarrow x = z)$

COMPF : $\forall \vec{x}, \vec{y} \ (\vec{x} = \vec{y} \rightarrow f(\vec{x}) = f(\vec{y}))$

COMPR : $\forall \vec{x}, \vec{y} \ (\vec{x} = \vec{y} \rightarrow R(\vec{x}) \rightarrow R(\vec{y}))$

P0 (Elementary arithmetics)

All the axioms from EG ;

A1 : $\forall x \ \neg S(x) = 0$

A2 : $\forall x \ (x = 0 \vee \exists y, x = S(y))$

A3 : $\forall x, y \ (S(x) = S(y) \rightarrow x = y)$

A4 : $\forall x \ (x + 0 = x)$

A5 : $\forall x, y \ (x + S(y) = S(x + y))$

A6 : $\forall x \ (x \times 0 = 0)$

A7 : $\forall x, y \ (x \times S(y) = x \times y + x)$

PA (Peano arithmetics)

All the axioms from P₀ ;

REC_Φ : $(\Phi(0) \wedge (\forall x(\Phi(x) \rightarrow \Phi(S(x)))) \rightarrow \forall x\Phi(x)$

for every formula $\Phi(x)$;

MO (Monoid theory)

All the axioms from EG ;

ASS : $\forall x, y, z \ x * (y * z) = (x * y) * z$

NE : $\forall x \ (x * e = x \ \wedge \ e * x = x)$

GR (Group theory)

All the axioms from MO ;

INV : $\forall x \ (x * I(x) = e \ \wedge \ I(x) * x = e)$