

# Multimodal and multi-criteria shortest paths in transportation networks

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Shortest path problem on a road network

Let  $G$  be a graph with  $n$  vertices and  $m$  edges.

## Direct methods

- Dijkstra:  $\mathcal{O}(m \cdot \log n)$
- Bellman Ford:  $\mathcal{O}(m \cdot n)$

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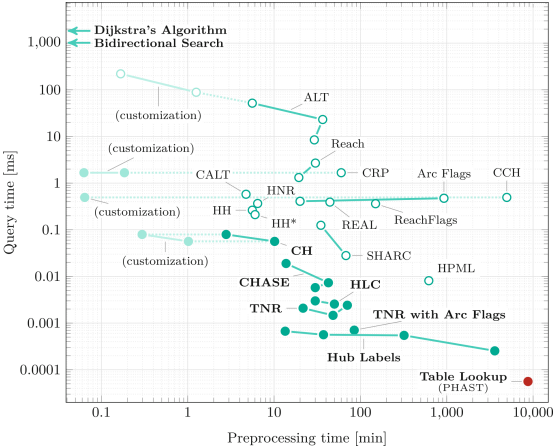
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## Speed-up

- landmarks: compute shortest distances from everyone to a well chosen subset of vertices
- separators: efficient for planar graphs
- contraction hierarchies: efficient for low tree-width graphs
- and many others.

# Classical methods



Comparison of different methods [Bast et al., 2016]

Western Europe: 18.0 millions vertices and 42.5 millions edges

## Multimodal multi-criteria shortest paths

We want to take into account:

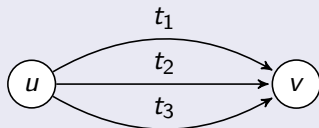
- other transportation means (foot, bicycle, bus, train, plane, ...)
- individual preferences (fast, cheap, easy, ...)

This emphasizes several issues:

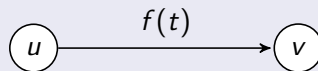
- huge graphs
  - data are collected by different providers: trade-off queries/uncertainty.
- ↪ distributed algorithms

## Temporal graphs

- Edges available at precise times ! Two ways to represent it:



One-time edges



Dependent-time edges

- Dijkstra-like algorithms [Dibbelt et al., 2013]
- Same speed-up technics, but less convincing improvements.



## Temporal graphs

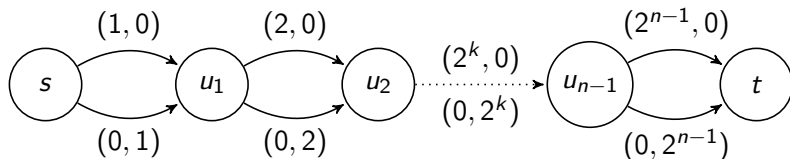
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  - one-time edges,
  - dependent-time edges.
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## Merging graphs

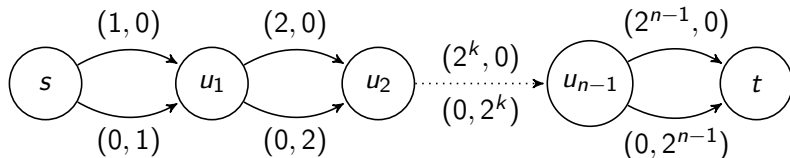
Two types of transportation: time dependent and time independent.  
Problem: how to modelize a "union graph" ? No consensus

Exemple: public transportation graph with small foot complete graphs  
[Wagner and Zündorf, 2017]

- Compare time, costs, transfert number, ...  $\implies d$  dimensions,
- Goal: obtain non dominated solutions (Pareto Set),
- Problem: too many solutions ! If  $\Delta$  is the maximum degree, possibly  $\Delta^n$  solutions.



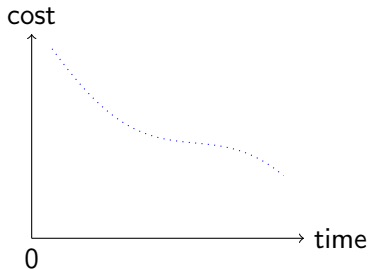
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$\rightarrow$  Approximation: cover the solution set.

# Multi-criteria

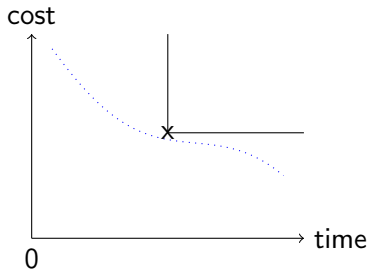
→ Polynomial cover set exists [Papadimitriou and Yannakakis, 2000]



Solution set with two dimensions

# Multi-criteria

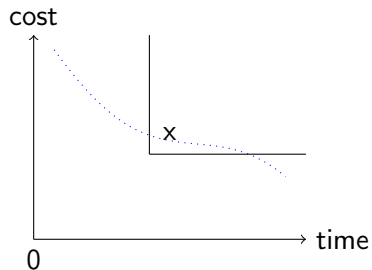
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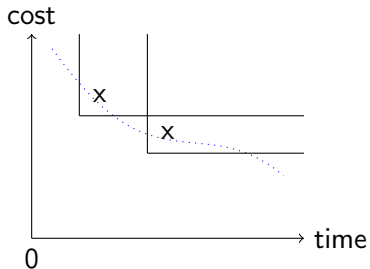
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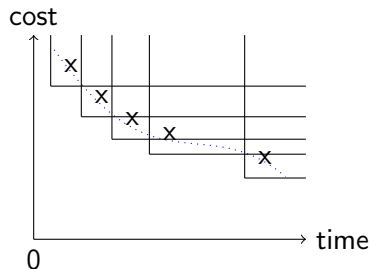
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# Multi-criteria

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Problem: for an  $(1 + \epsilon)$  precision, existing algorithms are exponential with respect to  $d$ :

$$\mathcal{O} \left( n^d \cdot m \left( \frac{\log(nK)}{\epsilon} \right)^{d-1} \right) \quad [\text{Tsaggouris and Zaroliagis, 2009}]$$



## Current work

We are trying to find a fully polynomial algorithm.  
Ideas so far: either correct or exponential.

## Future work

- distributed aspects,
- uncertainty

Thanks for your attention !

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- [2] Julian Dibbelt, Thomas Pajor, Ben Strasser, and Dorothea Wagner. Intriguingly simple and fast transit routing. In *International Symposium on Experimental Algorithms*, pages 43–54. Springer, 2013.
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- [4] George Tsaggouris and Christos Zaroliagis. Multiobjective optimization: Improved fptas for shortest paths and non-linear objectives with applications. *Theory of Computing Systems*, 45(1):162–186, 2009.
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