Self-stabilizing Systems in Spite of High Dynamics

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Self-stabilization, [Dijkstra, ACM Com., 74]





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Self-stabilizing Systems in Spite of High Dynamics

Some results:

Process Crashes: FTSS [Beauquier and Kekkonen-Moneta, PODC'97]
 Byzantine failures strict stabilization [Nesterenko and Arora, ICDCS'02]
 Intermittent Faults intermittent lost, duplication, or reordering of messages [Delaët and Tixeuil, JPDC, 2002]

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However: static communication networks

If topological are locally detected and infrequent, then a self-stabilizing algorithm designed for arbitrary topologies is well-suited.

Moreover:

Superstabilization: quick convergence after few topological changes from a legitimate configuration [Dolev and Herman, Chicago Journal of Theoretical Computer Science, 1997] Gradual Stabilization: a generalization of superstabilization [Altisen *et al.*, JPDC, 2019]

However: topological changes should be transient

Self-stabilization in Highly Dynamic Systems

Few results

Negative result: Even if the network is **always connected over the time**, silent self-stabilization is impossible! [Braud-Santoni *et al.*, IJNC, 2016]

Silence: converges within finite time to a configuration from which the values of the communication registers used by the algorithm remain fixed.

Self-stabilization in Highly Dynamic Systems

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Positive result: Self-stabilizing exploration of a highly dynamic ring by a cohort of synchronous robots [Bournat *et al.*, TCS, 2019]

Robot: visibility sensors, moving actuators, yet no communication capabilities.

However, only one edge maybe missing at a time (the network is always connected over the time)

- Self-stabilization in highly dynamic message-passing systems
 Dynamics modeled as Time-Varying Graphs (TVG)
 - [Casteigts *et al.*, IJPEDS, 2012]

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We look for (non-silent) self-stabilizing algorithm for general classes of TVGs

(e.g., we do not enforce the network to be in a particular topology at a given time)

- **TVG:** $\mathcal{G} = (V, E, \mathcal{T}, \rho)$
- V is a set of n nodes
- E is a set of arcs
- $lacksymbol{\,\,\,\,\,\,\,\,\,\,} {\mathcal T}$ is an interval over ${\mathbb N}^*$
- $\rho: E \times T \to \{0, 1\}$ is the *presence* function



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Snapshot of \mathcal{G} at time $t \in \mathcal{T}$:

 $G_t = (V, \{e \in E : \rho(e, t) = 1\})$



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Snapshot at 1: G₁



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Snapshot at 2: G₂



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Snapshot at 3: G₃



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Snapshot at 4: G₄



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Snapshot at 5: *G*₅

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Snapshot at 8: G₈

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Snapshot at 9: G₉



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Snapshot of \mathcal{G} at time $t \in \mathcal{T}$:

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Temporal Subgraph of \mathcal{G} for $[t, t'] \subseteq \mathcal{T}$:

$$\mathcal{G}_{[t,t']} = (V, E, [t,t'], \rho')$$

where ρ' is ρ restricted to [t, t'].





Journey: $\mathcal{J} = (e_1, t_1), (e_2, t_2), \dots, (e_k, t_k)$ such that $\forall i \in \{1, \dots, k\}$ **a** $e_i = (p_i, q_i) \in E$ **b** $\rho(e_i, t_i) = 1$ **c** $i < k \Rightarrow q_i = p_{i+1} \land t_i < t_{i+1}$

Temporal length: $t_k - t_1 + 1$



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Example:

((a, b), 1), ((b, c), 4), ((c, d), 5) is a journey of length 5 from a to d

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Snapshot at time 2: G₂



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Snapshot at time 3: G₃



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Snapshot at time 4: G_4



Journey: $\mathcal{J} = (e_1, t_1), (e_2, t_2), \dots, (e_k, t_k)$ such that $\forall i \in \{1, \dots, k\}$ **a** $e_i = (p_i, q_i) \in E$ **b** $\rho(e_i, t_i) = 1$ **c** $i < k \Rightarrow q_i = p_{i+1} \land t_i < t_{i+1}$

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Snapshot at time 5: G₅



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Snapshot at time 6: G₆



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Snapshot at time 8: G₈



Journey: $\mathcal{J} = (e_1, t_1), (e_2, t_2), \dots, (e_k, t_k)$ such that $\forall i \in \{1, \dots, k\}$ **a** $e_i = (p_i, q_i) \in E$ **b** $\rho(e_i, t_i) = 1$ **c** $i < k \Rightarrow q_i = p_{i+1} \land t_i < t_{i+1}$

Temporal length: $t_k - t_1 + 1$

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Snapshot at time 9: G₉



Considered Classes of TVGs

We only consider infinite TVG: $\mathcal{G} = (V, E, \mathcal{T}, \rho)$: \mathcal{T} is **right-open**.

Class $\mathcal{TC}^{\mathcal{B}}(\Delta)$ with $\Delta \in \mathbb{N}^*$ (Bounded Temporal Diameter): At any point in time, every node can reach all the others through a journey of temporal length at most Δ , *i.e.*, the temporal diameter is bounded by Δ , [Gómez-Calzado *et al.*, Euro-Par'15]

Class $\mathcal{TC}^{\mathcal{Q}}(\Delta)$ with $\Delta \in \mathbb{N}^*$ (Quasi Bounded Temporal Diameter): Every node can always eventually reach each other node through a journey of temporal length at most Δ . [New]

Class $\mathcal{TC}^{\mathcal{R}}$ (Recurrent Temporal Connectivity): At any point in time, every node can reach all the others through a journey, [Casteigts *et al.*, IJPEDS, 2012]

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$$\mathcal{TC}^{\mathcal{B}}(\Delta)\subseteq\mathcal{TC}^{\mathcal{Q}}(\Delta)\subseteq\mathcal{TC}^{\mathcal{R}}$$
Computational Model: Synchronous Rounds (1/2)

[Charron-Bost and Moran, STACS'18 / Barjon et al., CJ, 2019]

We model the dynamic network topology by a TVG $\mathcal{G} = (V, E, \mathcal{T}, \rho)$, where \mathcal{T} is **right-open**, *i.e.*, \mathcal{G} is infinite.

Let $o_{\mathcal{T}} = \min \mathcal{T}$ the first instant

The neighborhood of node p at Round i is

$$\mathcal{N}(p)^i = \{q \in V : \rho((p,q), o_{\mathcal{T}} + i - 1) = 1\}$$

(*i.e.*, neighbors of p in the snapshot G_{o_T+i-1})

 $\mathcal{N}(p)^i$ is unknown by all nodes

Computational Model: Synchronous Rounds (2/2)

[Charron-Bost and Moran, STACS'18 / Barjon et al., CJ, 2019]

Execution in \mathcal{G} : infinite sequence of configurations $\gamma_0, \gamma_1, \ldots$ such that

- γ_0 is arbitrary
- $\forall i > 0$, γ_i is obtained from γ_{i-1} as follows:
 - **1** Every node *p* sends a message consisting of all or a part of its local state in γ_{i-1} ,
 - **2** p receives all messages sent by nodes in $\mathcal{N}(p)^i$, and
 - **3** p computes its state in γ_i .

 $\forall i > 0$,

- γ_{i-1} is the configuration at the beginning of Round *i*
- γ_i is the configuration at the end of Round *i*



Self-stabilization in an Highly Dynamic Context

Adaptation of the definition in the book [Self-Stabilization, Dolev, 2000]

An algorithm \mathcal{A} is **self-stabilizing** for the specification SP on the TVG class \mathcal{C} if

there exists a non-empty subset of configurations \mathcal{L} , called the set of *legitimate configurations*, such that:

- I for every $\mathcal{G} \in \mathcal{C}$, for every configuration γ , every execution of \mathcal{A} in \mathcal{G} starting from γ contains a legitimate configuration $\gamma' \in \mathcal{L}$ (Convergence), and
- 2 for every G ∈ C, for every t ≥ o_T, for every legitimate configuration γ ∈ L, for every execution e in G_{[t,+∞)} starting from γ, SP(e) holds (Correctness).

Self-stabilization in Classes $\mathcal{TC}^{\mathcal{B}}(\Delta)$, $\mathcal{TC}^{\mathcal{Q}}(\Delta)$, and $\mathcal{TC}^{\mathcal{R}}$

These three classes are recurring in the sense that

$$\forall \mathcal{G} \in \mathcal{C}, \forall t \geq o_{\mathcal{T}}, \mathcal{G}_{[t,+\infty)} \in \mathcal{C}$$

In this case, the definition can be simplified as follows

An algorithm \mathcal{A} is **self-stabilizing** for the specification *SP* on the recurring TVG class \mathcal{C} if there exists a non-empty subset of legitimate configurations \mathcal{L} such that:

- 1 for every $\mathcal{G} \in \mathcal{C}$, for every configuration γ , every execution of \mathcal{A} in \mathcal{G} starting from γ contains a legitimate configuration $\gamma' \in \mathcal{L}$ (Convergence), and
- 2 for every G ∈ C, for every legitimate configuration γ ∈ L, for every execution e in G starting from γ, SP(e) holds (Correctness).



Self-stabilizing Algorithms for Highly Dynamic Systems (1/3)

Case-Study: Leader Election

In Classes:

Class $\mathcal{TC}^{\mathcal{B}}(\Delta)$ with $\Delta \in \mathbb{N}^*$: Bounded Temporal Diameter Class $\mathcal{TC}^{\mathcal{Q}}(\Delta)$ with $\Delta \in \mathbb{N}^*$: Quasi Bounded Temporal Diameter Class $\mathcal{TC}^{\mathcal{R}}$: Recurrent Temporal Connectivity

$$\mathcal{TC}^{\mathcal{B}}(\Delta) \subseteq \mathcal{TC}^{\mathcal{Q}}(\Delta) \subseteq \mathcal{TC}^{\mathcal{R}}$$



Self-stabilizing Algorithms for Highly Dynamic Systems (2/3)

Case-Study: Leader Election

In Class $\mathcal{TC}^{\mathcal{B}}(\Delta)$ with $\Delta \in \mathbb{N}^*$ (Bounded Temporal Diameter)

- Δ known
- Stabilization Time: at most 3∆ rounds
- Memory Requirement: $O(\log(n + \Delta))$ bits per node

Self-stabilizing Algorithms for Highly Dynamic Systems (3/3)

Case-Study: Leader Election

Class $\mathcal{TC}^{\mathcal{Q}}(\Delta)$ with $\Delta \in \mathbb{N}^*$ (Quasi Bounded Temporal Diameter)

- Δ and n known
- Memory Requirement: $O(n(\log(n + \Delta)))$ bits per node

Class $\mathcal{TC}^{\mathcal{R}}(\text{Recurrent Temporal Connectivity})$

- n known
- Memory Requirement: infinite

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Class $\mathcal{TC}^{\mathcal{R}}(\text{Recurrent Temporal Connectivity})$

- n known
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Stabilization time unboundable in those two classes, but ...

Speculation

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- the system satisfies its requirements for all executions,
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Idea: worst possible scenarios are often rare in practice.

A speculative algorithm self-adapts its performances w.r.t. the "quality" of the environment, *i.e.*, the more favorable the environment is, the better the complexity of the algorithm should be.

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In **Self-Stabilizing (Static) Systems:** a self-stabilizing mutual exclusion algorithm whose stabilization time is significantly better when the execution is synchronous. [Dubois and Guerraoui, PODC'13]

Speculative Self-stabilizing Algorithms for Highly Dynamic Systems

Our solutions

Class $\mathcal{TC}^{\mathcal{Q}}(\Delta)$ with $\Delta \in \mathbb{N}^*$ (Quasi Bounded Temporal Diameter)

- Δ and *n* known
- Memory Requirement: $O(n(\log(n + \Delta)))$ bits per node
- **Speculation:** stabilization time in $\mathcal{TC}^{\mathcal{B}}(\Delta) \subseteq \mathcal{TC}^{\mathcal{Q}}(\Delta)$ is at most 2Δ rounds

Class $\mathcal{TC}^{\mathcal{R}}(\text{Recurrent Temporal Connectivity})$

n known

- Memory Requirement: infinite
- Speculation: stabilization time in TC^B(△) ⊆ TC^R is at most △ + 1 rounds

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 $\forall v \in IDSET$,

- v is a real ID if $\exists p \in V, id(p) = v$,
- *v* is a fake ID otherwise

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Strategy:

- **1** First, **eliminate all fake IDs**, and then
- **2** Compute in all output variables the **minimum real ID**, **noted** $id(\ell)$.

Bounded Temporal Diameter, Δ known

Variables: $lid(p) \in IDSET$ and $t(p) \in \{0, ..., 2\Delta\}$



Bounded Temporal Diameter, Δ known

Variables: $lid(p) \in IDSET$ and $t(p) \in \{0, ..., 2\Delta\}$

For each node *p*, at each round:

1 *p* sends $\langle lid(p), t(p) \rangle$

2 If p receives some messages, then

- Let (x, t_x) the smallest received pair (lexicographic order)
- ▶ if $\langle x, t_x \rangle < \langle lid(p), t(p) \rangle$, then $\langle lid(p), t(p) \rangle := \langle x, t_x \rangle$

3 t(p) + +

4 if $lid(p) \ge id(p) \lor t \ge 2\Delta$, then $\langle lid(p), t(p) \rangle := \langle id(p), 0 \rangle$ (Reset)

Bounded Temporal Diameter, Δ known

Legitimate Configurations:

$$orall p \in V, \mathit{lid}(p) = \mathit{id}(\ell) \wedge t(p) \leq \Delta \wedge p = \ell \Rightarrow t(p) = 0$$

Bounded Temporal Diameter, Δ known

Legitimate Configurations:

$$\forall p \in V, \mathit{lid}(p) = \mathit{id}(\ell) \land t(p) \leq \Delta \land p = \ell \Rightarrow t(p) = 0$$

Correctness:

• No fake ID $\Rightarrow id(\ell)$ is the minimum value of *IDSET* in the network.

So, $lid(\ell) = id(\ell)$ and $t(\ell) = 0$ forever



Bounded Temporal Diameter, Δ known

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■ Bounded temporal diameter ⇒ no reset

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 The timestamps associated to each fake ID increment at each round until reaching 2Δ and so vanishing,

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At most Δ additional rounds are necessary to reach a configuration where ∀p ∈ V, lid(p) = id(ℓ) ∨ t(p) ≤ Δ

Hence, a stabilization time of at most 3Δ rounds.



Quasi Bounded Temporal Diameter, n and Δ known (1/3)

Variables:

- lid(p) \in IDSET
- members(p): queue of at most *n* pairs $\langle id, t \rangle \in IDSET \times \{0, \dots, \Delta\}$

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- lid(p) \in IDSET
- members(p): queue of at most *n* pairs $\langle id, t \rangle \in IDSET \times \{0, \dots, \Delta\}$

For each node *p*, at each round:

• p sends every pair $\langle x, y \rangle \in members(p)$ such that $y < \Delta$

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- For each received pairs $\langle x, y \rangle$
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- Increment all timestamps in members(p)

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- Update lid(p) with the smallest ID in members(p)



Quasi Bounded Temporal Diameter, *n* and Δ known (2/3)

Legitimate Configurations: $\forall p \in V$, $lid(p) = id(\ell) \land \{id : \langle id, _ \rangle \in members(p)\} = \{id(q) : q \in V\}$

Correctness: trivial since the set of legitimate configuration is closed

Quasi Bounded Temporal Diameter, *n* and Δ known (3/3)

Convergence:

The timestamps associated to each fake ID increment at each round until reaching Δ: after at most Δ rounds, no fake ID is sent and so no fake ID can be ever inserted in a *members* queue.

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Speculation:

Bounded Temporal Diameter \Rightarrow all real ID are inserted in each members queue in each period of Δ rounds

Hence, the stabilization time is at most 2Δ rounds in $\mathcal{TC}^{\mathcal{B}}(\Delta)$.

Self-stabilization in $\mathcal{TC}^{\mathcal{R}}$

Recurrent Temporal Connectivity, n known (1/3)

Variables:

- lid(p) \in IDSET
- **members**(p): map of at most *n* pairs $\langle id, t \rangle \in IDSET \times \mathbb{N}$
Recurrent Temporal Connectivity, n known (1/3)

Variables:

- lid(p) \in IDSET
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For each node *p*, at each round:

p sends members(p)

Recurrent Temporal Connectivity, n known (1/3)

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- members(p): map of at most *n* pairs $(id, t) \in IDSET \times \mathbb{N}$

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Recurrent Temporal Connectivity, n known (2/3)

Legitimate Configurations: $\forall p \in V$, $lid(p) = id(\ell) \land \{id : \langle id, _ \rangle \in members(p)\} = \{id(q) : q \in V\}$

Correctness: trivial since the set of legitimate configuration is closed

Recurrent Temporal Connectivity, n known (3/3)

Convergence: similar to the previous algorithm

Speculation: in $\mathcal{TC}^{\mathcal{B}}(\Delta)$,

- After the first round, at each round, each node p sends $\langle id(p), 0 \rangle$
- For every node q, a pair $\langle id(p), x_q \rangle$ with $x_q \leq \Delta$ reaches q within x rounds.
- At that time, all fake IDs have a timestamps > x_q.
 So, id(p) is inserted and never more removed.

Hence, a stabilization time in at most $\Delta + 1$ rounds in $\mathcal{TC}^{\mathcal{B}}(\Delta)$.

Conclusion

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Actually, we have even considered more general classes!

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Extensions:

- Even more general classes
- Expressiveness in particular TVG classes

Transformer (*e.g.*, [Katz and Perry, DC, 1993]): Propagation of Information with Feedback + Leader Election Karine Altisen, Stéphane Devismes, and Anaïs Durand, Colette Johnen, and Franck Petit. Self-stabilizing Systems in Spite of High Dynamics. Submitted to OPODIS'19.