

An Overview on Compact Routing (in Discrete Domains)

Cyril Gavoille

University of Bordeaux, France

12 October 2015

Searching and Routing in Discrete and Continuous Domains
CMO-BIRS Workshop, Oaxaca, Mexico

The Compact Routing Problem

Input: a network G (a weighted connected graph)

Output: a **routing scheme** for G

A ***routing scheme*** is a distributed algorithm that allows **any** source node to route messages to **any** destination node, given the destination's network identifier

The Compact Routing Problem

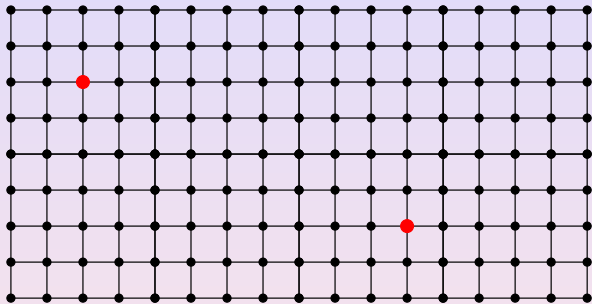
Input: a network G (a weighted connected graph)

Output: a **routing scheme** for G

A **routing scheme** is a distributed algorithm that allows **any** source node to route messages to **any** destination node, given the destination's network identifier

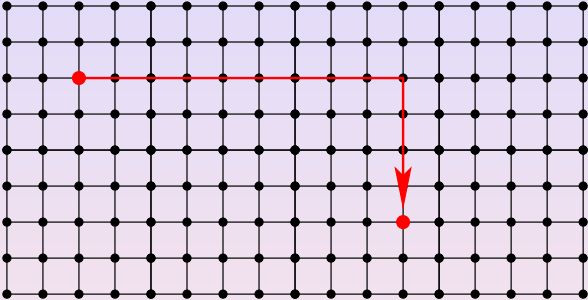
Goal: to minimize the size of the routing tables

Example: Grid with X,Y-coordinates



Routing algorithm: X,Y-routing

Example: Grid with X,Y-coordinates



Routing algorithm: X,Y-routing

Complexity Measures: Space & Stretch

Space = size of the largest local routing tables

Complexity Measures: Space & Stretch

Space = size of the largest local routing tables
(more precisely, size of the local routing algorithm
including all constants and data-structures)

In the grid example: space = $O(\log n)$ bits

Complexity Measures: Space & Stretch

Space = size of the largest local routing tables
(more precisely, size of the local routing algorithm including all constants and data-structures)

In the grid example: $\text{space} = O(\log n)$ bits

Stretch = ratio between length of the route and distance

$$|\text{route}(x, y)| \leq \text{stretch} \cdot \text{dist}(x, y)$$

In the grid example: $\text{stretch} = 1$ (shortest path)

Complexity Measures: Space & Stretch

Space = size of the largest local routing tables
(more precisely, size of the local routing algorithm including all constants and data-structures)

In the grid example: $\text{space} = O(\log n)$ bits

Stretch = ratio between length of the route and distance

$$|\text{route}(x, y)| \leq \text{stretch} \cdot \text{dist}(x, y)$$

In the grid example: $\text{stretch} = 1$ (shortest path)

Other: average space, time complexity of routing decision, time complexity for building routing tables, header size, number of header updates, ...

Complexity Measures: Space & Stretch

Space = size of the largest local routing tables
(more precisely, size of the local routing algorithm including all constants and data-structures)

In the grid example: $\text{space} = O(\log n)$ bits

Stretch = ratio between length of the route and distance

$$|\text{route}(x, y)| \leq \text{stretch} \cdot \text{dist}(x, y)$$

In the grid example: $\text{stretch} = 1$ (shortest path)

Question: for a given family of graphs, find routing schemes with best space-stretch trade-off

Two variants: Name-independent vs. Labeled

The destination enters the network with its **name**, which is determined by either the **designer** of the routing scheme (labeled), or an **adversary** (name-independent).

Labeled: the designer is free to name the nodes according to the topology and the edge weights of the graph

Name-independent: the input is a graph with fixed node names (more realistic IP addresses, cow-path problem with advice, ...)

Two variants: Name-independent vs. Labeled

The destination enters the network with its **name**, which is determined by either the **designer** of the routing scheme (labeled), or an **adversary** (name-independent).

Labeled: the designer is free to name the nodes according to the topology and the edge weights of the graph

Name-independent: the input is a graph with fixed node names (more realistic IP addresses, cow-path problem with advice, ...)

Other variants: random labeling, coordinate labeling based (geometric), ...

An overview: Labeled Model (1/2)

Labels and headers are of polylogarithmic size

$$\tilde{O}(f(n)) = f(n) \cdot \text{polylog}(n)$$

network	stretch	space/node (bits)
---------	---------	-------------------

arbitrary	1	$n \lceil \log n \rceil$	[folk]
-----------	---	--------------------------	--------

An overview: Labeled Model (1/2)

Labels and headers are of polylogarithmic size

$$\tilde{O}(f(n)) = f(n) \cdot \text{polylog}(n)$$

network	stretch	space/node (bits)	
arbitrary	1	$n \lceil \log n \rceil$	[folk]
$(2 \leq k \in \mathbb{N})$	$4k - 5$	$\tilde{O}(n^{1/k})$	[Thorup,Zwick]

An overview: Labeled Model (1/2)

Labels and headers are of polylogarithmic size

$$\tilde{O}(f(n)) = f(n) \cdot \text{polylog}(n)$$

network	stretch	space/node (bits)	
arbitrary	1	$n \lceil \log n \rceil$	[folk]
$(2 \leq k \in \mathbb{N})$	$4k - 5$	$\tilde{O}(n^{1/k})$	[Thorup,Zwick]
tree	1	$\tilde{O}(1)$	[TZ/Fraigniaud,G.]

An overview: Labeled Model (1/2)

Labels and headers are of polylogarithmic size

$$\tilde{O}(f(n)) = f(n) \cdot \text{polylog}(n)$$

network	stretch	space/node (bits)	
arbitrary	1	$n \lceil \log n \rceil$	[folk]
$(2 \leq k \in \mathbb{N})$	$4k - 5$	$\tilde{O}(n^{1/k})$	[Thorup,Zwick]
tree	1	$\tilde{O}(1)$	[TZ/Fraigniaud,G.]
doubling- α dim.	$1 + \varepsilon$	$\log \Delta$	[Talwar/Slivkins]
(Euclidian, bounded growth, ...)		$\tilde{O}(1)$	[Chan et al./Abraham,G. et al.]

An overview: Labeled Model (1/2)

Labels and headers are of polylogarithmic size

$$\tilde{O}(f(n)) = f(n) \cdot \text{polylog}(n)$$

network	stretch	space/node (bits)	
arbitrary	1	$n \lceil \log n \rceil$	[folk]
$(2 \leq k \in \mathbb{N})$	$4k - 5$	$\tilde{O}(n^{1/k})$	[Thorup,Zwick]
tree	1	$\tilde{O}(1)$	[TZ/Fraigniaud,G.]
doubling- α dim.	$1 + \varepsilon$	$\log \Delta$	[Talwar/Slivkins]
(Euclidian, bounded growth, ...)		$\tilde{O}(1)$	[Chan et al./Abraham,G. et al.]
planar	$1 + \varepsilon$	$\tilde{O}(1)$	[Thorup]

An overview: Labeled Model (1/2)

Labels and headers are of polylogarithmic size

$$\tilde{O}(f(n)) = f(n) \cdot \text{polylog}(n)$$

network	stretch	space/node (bits)	
arbitrary	1	$n \lceil \log n \rceil$	[folk]
$(2 \leq k \in \mathbb{N})$	$4k - 5$	$\tilde{O}(n^{1/k})$	[Thorup,Zwick]
tree	1	$\tilde{O}(1)$	[TZ/Fraigniaud,G.]
doubling- α dim.	$1 + \varepsilon$	$\log \Delta$	[Talwar/Slivkins]
(Euclidian, bounded growth, ...)		$\tilde{O}(1)$	[Chan et al./Abraham,G. et al.]
planar	$1 + \varepsilon$	$\tilde{O}(1)$	[Thorup]
H -minor-free	$1 + \varepsilon$	$\tilde{O}(1)$	[Abraham,G.]

An overview: Labeled Model (2/2)

network	stretch	space/node (bits)	
arbitrary	1	$n \log n$	[folk]
$(k \geq 2)$	$4k - 5$	$\tilde{O}(n^{1/k})$	[Thorup,Zwick'01]
$(k = 4)$	10.52	$\tilde{O}(n^{1/4} \log \Delta)$	[Chechik'13]

An overview: Labeled Model (2/2)

network	stretch	space/node (bits)	
arbitrary	1	$n \log n$	[folk]
$(k \geq 2)$	$4k - 5$	$\tilde{O}(n^{1/k})$	[Thorup,Zwick'01]
$(k = 4)$	10.52	$\tilde{O}(n^{1/4} \log \Delta)$	[Chechik'13]
$(k = 4)$	$9 + \varepsilon$	$\tilde{O}(n^{1/4} \log \Delta)$	[Roditty,Tov'15]

An overview: Labeled Model (2/2)

network	stretch	space/node (bits)	
arbitrary	1	$n \log n$	[folk]
$(k \geq 2)$	$4k - 5$	$\tilde{O}(n^{1/k})$	[Thorup,Zwick'01]
$(k = 4)$	10.52	$\tilde{O}(n^{1/4} \log \Delta)$	[Chechik'13]
$(k = 4)$	$9 + \varepsilon$	$\tilde{O}(n^{1/4} \log \Delta)$	[Roditty,Tov'15]
$(k \geq 3)$	$4k - 7 + \varepsilon$	$\tilde{O}(n^{1/k} \log \Delta)$	["]
$(k \geq 4)$	$3.68k + O(1)$	$\tilde{O}(n^{1/k} \log \Delta)$	[Chechik'13]

An overview: Labeled Model (2/2)

network	stretch	space/node (bits)	
arbitrary	1	$n \log n$	[folk]
$(k \geq 2)$	$4k - 5$	$\tilde{O}(n^{1/k})$	[Thorup,Zwick'01]
$(k = 4)$	10.52	$\tilde{O}(n^{1/4} \log \Delta)$	[Chechik'13]
$(k = 4)$	$9 + \varepsilon$	$\tilde{O}(n^{1/4} \log \Delta)$	[Roditty,Tov'15]
$(k \geq 3)$	$4k - 7 + \varepsilon$	$\tilde{O}(n^{1/k} \log \Delta)$	["]
$(k \geq 4)$	$3.68k + O(1)$	$\tilde{O}(n^{1/k} \log \Delta)$	[Chechik'13]
(unweighted)	$(2 \times, +1)$	$\tilde{O}(n^{3/4})$	[Abraham,G.'11]
(unweighted)	$((2 + \varepsilon) \times, +1)$	$\tilde{O}(n^{2/3})$	[Roditty,Tov'15]

An overview: Name-independent Model

network	stretch	space/node (bits)	
bounded growth	$1 + \varepsilon$	$\tilde{O}(1)$	[Abraham et al.]

An overview: Name-independent Model

network	stretch	space/node (bits)	
bounded growth	$1 + \varepsilon$	$\tilde{O}(1)$	[Abraham et al.]
doubling- α dim.	$9 + \varepsilon$	$\tilde{O}(1)$	[Konjevod et al./Abraham, G. et al.]

An overview: Name-independent Model

network	stretch	space/node (bits)	
bounded growth	$1 + \varepsilon$	$\tilde{O}(1)$	[Abraham et al.]
doubling- α dim.	$9 + \varepsilon$	$\tilde{O}(1)$	[Konjevod et al./Abraham, G. et al.]
H -minor-free (unweighted)	$O(1)$	$\tilde{O}(1)$	[Abraham, G.]

An overview: Name-independent Model

network	stretch	space/node (bits)	
bounded growth	$1 + \varepsilon$	$\tilde{O}(1)$	[Abraham et al.]
doubling- α dim.	$9 + \varepsilon$	$\tilde{O}(1)$	[Konjevod et al./Abraham, G. et al.]
H -minor-free (unweighted)	$O(1)$	$\tilde{O}(1)$	[Abraham, G.]
trees	$2^k - 1$	$\tilde{O}(n^{1/k})$	[Laing]

An overview: Name-independent Model

network	stretch	space/node (bits)	
bounded growth	$1 + \varepsilon$	$\tilde{O}(1)$	[Abraham et al.]
doubling- α dim.	$9 + \varepsilon$	$\tilde{O}(1)$	[Konjevod et al./Abraham, G. et al.]
H -minor-free (unweighted)	$O(1)$	$\tilde{O}(1)$	[Abraham, G.]
trees	$2^k - 1$	$\tilde{O}(n^{1/k})$	[Laing]
arbitrary	3	$\tilde{O}(\sqrt{n})$	[A., G., Malkhi, Nisan, Thorup]

An overview: Name-independent Model

network	stretch	space/node (bits)	
bounded growth	$1 + \varepsilon$	$\tilde{O}(1)$	[Abraham et al.]
doubling- α dim.	$9 + \varepsilon$	$\tilde{O}(1)$	[Konjevod et al./Abraham, G. et al.]
H -minor-free (unweighted)	$O(1)$	$\tilde{O}(1)$	[Abraham, G.]
trees	$2^k - 1$	$\tilde{O}(n^{1/k})$	[Laing]
arbitrary	3	$\tilde{O}(\sqrt{n})$	[A., G., Malkhi, Nisan, Thorup]
	$O(k^2 2^k)$	$\tilde{O}(n^{1/k})$	[Arias et al./Awerbuch, Peleg]
	$O(k)$	$\tilde{O}(n^{1/k})$	[Abraham, G. et al.]

Lower Bounds for Name-Independent

Rem: lower bound for labeled \Rightarrow lower bound for name-indep

Lower Bounds for Name-Independent

Rem: lower bound for labeled \Rightarrow lower bound for name-indep

network	stretch	space/node (bits)	
arbitrary	< 1.4	$\Omega(n \log n)$	[G., Pérennès.]
	< 3	$\Omega(n)$	[G., Gengler]
(only $k = 1, 2, 3, \times, 5$)	$< 2k + 1$	$\Omega(n^{1/k})$	[Thorup, Zwick]

Lower Bounds for Name-Independent

Rem: lower bound for labeled \Rightarrow lower bound for name-indep

network	stretch	space/node (bits)	
arbitrary	< 1.4	$\Omega(n \log n)$	[G., Pérennès.]
	< 3	$\Omega(n)$	[G., Gengler]
	(only $k = 1, 2, 3, \times, 5$)	$< 2k + 1$	$\Omega(n^{1/k})$ [Thorup, Zwick]
trees	≤ 3	$\Omega(\sqrt{n})$	[Laing, Rajaraman]
	$\leq 9 - \varepsilon$	$\Omega(n^{(\varepsilon/60)^2})$	[Konjevod et al.]

Lower Bounds for Name-Independent

Rem: lower bound for labeled \Rightarrow lower bound for name-indep

network	stretch	space/node (bits)	
arbitrary	< 1.4	$\Omega(n \log n)$	[G., Pérennès.]
	< 3	$\Omega(n)$	[G., Gengler]
	(only $k = 1, 2, 3, \times, 5$)	$< 2k + 1$	$\Omega(n^{1/k})$ [Thorup, Zwick]
trees	≤ 3	$\Omega(\sqrt{n})$	[Laing, Rajaraman]
	$\leq 9 - \varepsilon$	$\Omega(n^{(\varepsilon/60)^2})$	[Konjevod et al.]
for all $k \geq 1$	$< 2k + 1$	$\frac{1}{8}(n \log n)^{1/k}$	[Abraham, G. et al.]

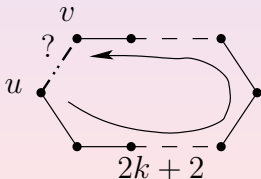
Theorem [Abraham, G., Malkhi]

- 1 Any name-indep. routing scheme using $< \frac{1}{8}(n \log n)^{1/k}$ bits/node has a **max stretch** $\geq 2k + 1$ for some graph.
- 2 Any name-indep. routing scheme using $< \frac{1}{8}(n/k)^{1/k}$ bits/node has an **average stretch** $\geq k/4$ for some graph.

Theorem [Abraham,G.,Malkhi]

- 1 Any name-indep. routing scheme using $< \frac{1}{8}(n \log n)^{1/k}$ bits/node has a **max stretch** $\geq 2k + 1$ for some graph.
- 2 Any name-indep. routing scheme using $< \frac{1}{8}(n/k)^{1/k}$ bits/node has an **average stretch** $\geq k/4$ for some graph.

Rem 1: All previous lower bounds for labeled case (Peleg,Upfal / G.,Pérennès / G.,Gengler / Kranakis,Krizanc / Thorup,Zwick) are based on the construction of **dense large girth** graphs



if $\text{stretch} < 2k + 1$, then u is forced to "know" the edge (u, v)

Theorem [Abraham, G., Malkhi]

- 1 Any name-indep. routing scheme using $< \frac{1}{8}(n \log n)^{1/k}$ bits/node has a **max stretch** $\geq 2k + 1$ for some graph.
- 2 Any name-indep. routing scheme using $< \frac{1}{8}(n/k)^{1/k}$ bits/node has an **average stretch** $\geq k/4$ for some graph.

Erdős-Simonovits Conjecture: \exists graph of girth $2k + 2$ with $\Omega(n^{1+1/k})$ edges (proved only for $k = 1, 2, 3, 5$). So, the extra $(\log n)^{1/k}$ term **cannot** be obtained with a girth approach.

Theorem [Abraham, G., Malkhi]

- 1 Any name-indep. routing scheme using $< \frac{1}{8}(n \log n)^{1/k}$ bits/node has a **max stretch** $\geq 2k + 1$ for some graph.
- 2 Any name-indep. routing scheme using $< \frac{1}{8}(n/k)^{1/k}$ bits/node has an **average stretch** $\geq k/4$ for some graph.

Rem 2: It makes a clear separation between labeled and name-independent routing, at least for the average stretch.

In the **labeled** model, schemes with $\text{polylog}(n)$ space and constant average stretch do exist for every graph! [Abraham, Bartal, Chan, Gupta, Kleinberg et al. (FOCS'05)]

In the **name-indep** model, if space is $\text{polylog}(n)$, then the average stretch must be $\Omega(\log n / \log \log n)$ for some graphs.

The Metric Model

A weaker model, but conceptually easier

Input: a metric space (V, d)

Output: an overlay network $G = (V, E)$, and a routing scheme for G

The Metric Model

A weaker model, but conceptually easier

Input: a metric space (V, d)

Output: an overlay network $G = (V, E)$, and a routing scheme for G

An extra complexity measure: the size $|E|$ of the overlay

The Metric Model

A weaker model, but conceptually easier

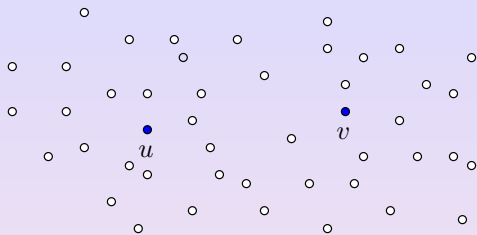
Input: a metric space (V, d)

Output: an overlay network $G = (V, E)$, and a routing scheme for G

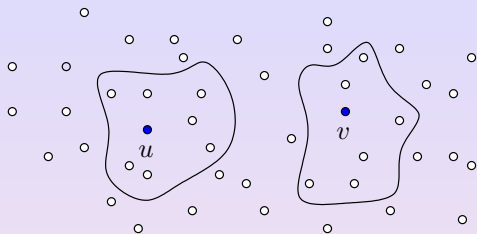
An extra complexity measure: the size $|E|$ of the overlay

Goal: to minimize the size of G , and the space for each node must be \approx the average degree of G

Example: Stretch-3 for Arbitrary Metric

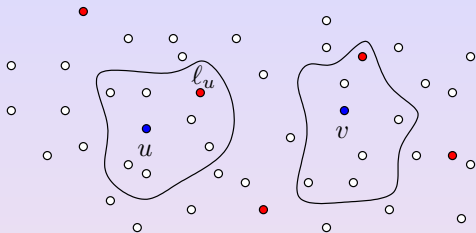


Example: Stretch-3 for Arbitrary Metric



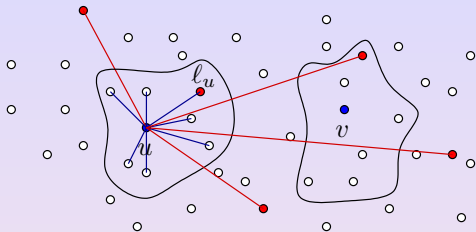
$B_u =$ the set of $\sqrt{n \ln n}$ closest nodes from u

Example: Stretch-3 for Arbitrary Metric



$B_u =$ the set of $\sqrt{n \ln n}$ closest nodes from u
 $L =$ hitting set of $\{B_u \mid u \in V\}$ of size $\leq \sqrt{n \ln n}$

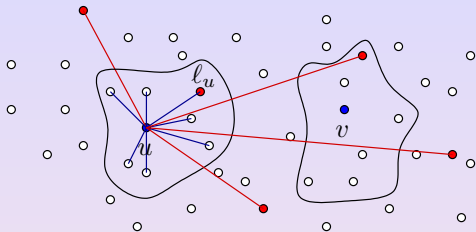
Example: Stretch-3 for Arbitrary Metric



$B_u =$ the set of $\sqrt{n \ln n}$ closest nodes from u
 $L =$ hitting set of $\{B_u \mid u \in V\}$ of size $\leq \sqrt{n \ln n}$

Overlay: $u \rightarrow w, \forall w \in B_u$ and $u \rightarrow \ell, \forall \ell \in L$

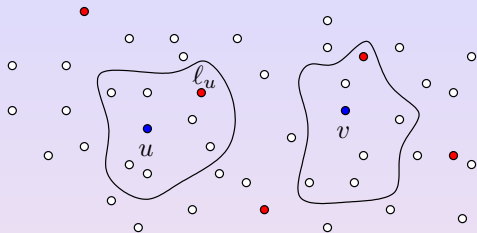
Example: Stretch-3 for Arbitrary Metric



B_u = the set of $\sqrt{n \ln n}$ closest nodes from u
 L = hitting set of $\{B_u \mid u \in V\}$ of size $\leq \sqrt{n \ln n}$

Overlay: $u \rightarrow w, \forall w \in B_u$ and $u \rightarrow \ell, \forall \ell \in L$
 $\Rightarrow |E| \leq \sum_u (|B_u| + |L|) \leq n^{3/2} \sqrt{\ln n}$

Example: Stretch-3 for Arbitrary Metric

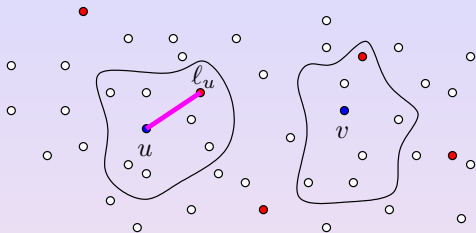


B_u = the set of $\sqrt{n \ln n}$ closest nodes from u
 L = hitting set of $\{B_u \mid u \in V\}$ of size $\leq \sqrt{n \ln n}$

Overlay: $u \rightarrow w, \forall w \in B_u$ and $u \rightarrow l, \forall l \in L$
 $\Rightarrow |E| \leq \sum_u (|B_u| + |L|) \leq n^{3/2} \sqrt{\ln n}$

Routing: If $v \in B_u$, route $u \rightsquigarrow v$, else $u \rightsquigarrow l_u \rightsquigarrow v$

Example: Stretch-3 for Arbitrary Metric

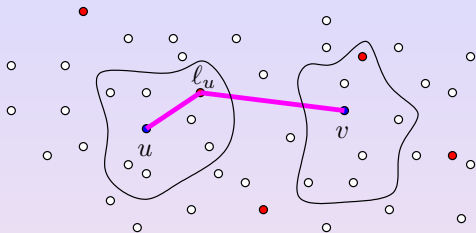


B_u = the set of $\sqrt{n \ln n}$ closest nodes from u
 L = hitting set of $\{B_u \mid u \in V\}$ of size $\leq \sqrt{n \ln n}$

Overlay: $u \rightarrow w, \forall w \in B_u$ and $u \rightarrow l, \forall l \in L$
 $\Rightarrow |E| \leq \sum_u (|B_u| + |L|) \leq n^{3/2} \sqrt{\ln n}$

Routing: If $v \in B_u$, route $u \rightsquigarrow v$, else $u \rightsquigarrow l_u \rightsquigarrow v$

Example: Stretch-3 for Arbitrary Metric



B_u = the set of $\sqrt{n \ln n}$ closest nodes from u
 L = hitting set of $\{B_u \mid u \in V\}$ of size $\leq \sqrt{n \ln n}$

Overlay: $u \rightarrow w, \forall w \in B_u$ and $u \rightarrow l, \forall l \in L$
 $\Rightarrow |E| \leq \sum_u (|B_u| + |L|) \leq n^{3/2} \sqrt{\ln n}$

Routing: If $v \in B_u$, route $u \rightsquigarrow v$, else $u \rightsquigarrow l_u \rightsquigarrow v$

Rem: $l_u \rightarrow v$ is not easy to implement in the graph model
(usually simulated with some tree routings)

Some Results in the Metric Model

Both labeled and name-independent variants exist ...

metric	stretch	average degree
--------	---------	----------------

Euclidian \mathbb{R}^d

$(d = 2)$	$O(1)$	$O(1)$	[Hassin, Peleg/Bose et al.]
(name-indep.)	$O(1)$	$O(1)$	[Abraham, Malkhi]
	$1 + \varepsilon$	$\tilde{O}(1)$	[Gottlieb, Roditty'08]

Some Results in the Metric Model

Both labeled and name-independent variants exist ...

metric	stretch	average degree	
Euclidian \mathbb{R}^d			
$(d = 2)$	$O(1)$	$O(1)$	[Hassin, Peleg/Bose et al.]
(name-indep.)	$O(1)$	$O(1)$	[Abraham, Malkhi]
	$1 + \varepsilon$	$\tilde{O}(1)$	[Gottlieb, Roditty'08]
doubling- α dim.			
	$1 + \varepsilon$	$\tilde{O}(\log \Delta)$	[Talwar/Chan et al./Slivkins]
	$1 + \varepsilon$	$\tilde{O}(1)$	[Abraham et al.'06]

Undirected vs. Directed

(graph model only!)

Problem: there is no stretch-space trade-off for routing in *directed* graphs! The stretch maybe not bounded if $o(n)$ bits of memory are used, even in strongly connected digraphs [Thorup,Zwick]

Undirected vs. Directed

(graph model only!)

Problem: there is no stretch-space trade-off for routing in *directed* graphs! The stretch maybe not bounded if $o(n)$ bits of memory are used, even in strongly connected digraphs [Thorup,Zwick]

New measure: roundtrip stretch factor

$$\text{stretch} = \frac{|\text{route}(u, v)| + |\text{route}(v, u)|}{\text{dist}(u, v) + \text{dist}(v, u)}$$

Rem: $\text{dist}(u, v) + \text{dist}(v, u)$ is now a distance function

Some Results for Arbitrary Digraphs

Labeled: [Roditty, Thorup, Zwick - SODA '02]

$$\text{stretch} = 4k + \varepsilon$$

$$\text{space} = \tilde{O}(\varepsilon^{-1} k n^{1/k} \log \Delta)$$

$$\text{labels} = o(\varepsilon^{-1} k \log^2 n \log \Delta)$$

$$\text{stretch} = 3$$

$$\text{space} = \tilde{O}(\sqrt{n} \log \Delta)$$

$$\text{labels} = o(\log^2 n \log \Delta)$$

Name-independent: [Arias, Cowen, Laing - PODC '03]

$$\text{stretch} = O(k^2)$$

$$\text{space} = \tilde{O}(\varepsilon^{-1} k n^{1/k} \log \Delta)$$

$$\text{labels} = o(\varepsilon^{-1} k^2 \log^2 n \log \Delta)$$

$$\text{stretch} = 6$$

$$\text{space} = \tilde{O}(\sqrt{n} \log \Delta)$$

$$\text{labels} = o(\log^2 n \log \Delta)$$

Lower bound: if stretch < 2 , then $\Omega(n)$ bits is required

Open Questions: Arbitrary Networks

Q1: Labeled: stretch $\leq 4k - O(1)$ for $\tilde{O}(n^{1/k})$ space. Optimal only for $k = 1, 2$. The lower bound on the stretch is $\leq 2k - 1$.

For $k = 4$, the lower bound is even not known to be $2k - 1$ because the Erdős-Simonovits conjecture is proved only for $k = 1, 2, 3, \infty, 5$.

Open Questions: Arbitrary Networks

Q1: Labeled: stretch $\leq 4k - O(1)$ for $\tilde{O}(n^{1/k})$ space. Optimal only for $k = 1, 2$. The lower bound on the stretch is $\leq 2k - 1$.

For $k = 4$, the lower bound is even not known to be $2k - 1$ because the Erdős-Simonovits conjecture is proved only for $k = 1, 2, 3, \infty, 5$.

Q2: Name-independent = labeled ???

For $k = 1, 2$, the same bounds hold.

Open Questions: Arbitrary Networks

Q1: Labeled: stretch $\leq 4k - O(1)$ for $\tilde{O}(n^{1/k})$ space. Optimal only for $k = 1, 2$. The lower bound on the stretch is $\leq 2k - 1$.

For $k = 4$, the lower bound is even not known to be $2k - 1$ because the Erdős-Simonovits conjecture is proved only for $k = 1, 2, 3, \infty, 5$.

Q2: Name-independent = labeled ???

For $k = 1, 2$, the same bounds hold.

Q3: Directed = Undirected???

Open Questions: Specific Networks

- Q4:** Name-indep. scheme for unweighted trees:
What's the best stretch if using polylog space?
Current best stretch: 17.

Open Questions: Specific Networks

- Q4:** Name-indep. scheme for unweighted trees:
What's the best stretch if using polylog space?
Current best stretch: 17.
- Q5:** Labeled scheme for weighted treewidth- k : Can we do shortest path with $o(k \log^2 n)$ -bit table & labels?
True for trees and weighted outerplanar (and even $K_{2,4}$ -minor free graphs): $\Theta(\log^2 n / \log \log n)$ bits are enough and necessary [Dieng,G.]. What about weighted series-parallel graphs?

Open Questions: Specific Networks

- Q4:** Name-indep. scheme for unweighted trees:
What's the best stretch if using polylog space?
Current best stretch: 17.
- Q5:** Labeled scheme for weighted treewidth- k : Can we do shortest path with $o(k \log^2 n)$ -bit table & labels?
True for trees and weighted outerplanar (and even $K_{2,4}$ -minor free graphs): $\Theta(\log^2 n / \log \log n)$ bits are enough and necessary [Dieng, G.]. What about weighted series-parallel graphs?
- Q6:** Shortest path in planar with polylog labels: space ranges between $\Omega(n^{1/3}) \dots O(n)$.
Current best bound: $7.18n$ bits [Lu'02].

Open Questions: Misc

- Q7:** $\tilde{O}(\deg(u))$ with constant stretch for general graphs? What about bounded degree graphs? Sparse graphs are known to be non-compact. But, bounded degree nodes “increase” distances, so stretch tends to 1. No lower bounds are known. Bounded degree includes expanders ...

Open Questions: Misc

- Q7:** $\tilde{O}(\deg(u))$ with constant stretch for general graphs? What about bounded degree graphs? Sparse graphs are known to be non-compact. But, bounded degree nodes “increase” distances, so stretch tends to 1. No lower bounds are known. Bounded degree includes expanders ...
- Q8:** Average stretch? ε -slack routing? Labeled and name-indep. differ. Average stretch & additive stretch are interesting in practice.

Open Questions: Misc

- Q7:** $\tilde{O}(\deg(u))$ with constant stretch for general graphs? What about bounded degree graphs? Sparse graphs are known to be non-compact. But, bounded degree nodes “increase” distances, so stretch tends to 1. No lower bounds are known. Bounded degree includes expanders ...
- Q8:** Average stretch? ϵ -slack routing? Labeled and name-indep. differ. Average stretch & additive stretch are interesting in practice.
- Q9:** Distributed algorithms for constructing tables? Known for some specific graphs (planar, [Frederickson'90]). Best $o(n^3)$ -message distributed implementation of stretch-3 space- \sqrt{n} name-indep. Abraham et al.'s scheme has stretch ... only 5.

Thank you!