Dynamic Algorithms *via* Forbidden-Set Labeling

Cyril Gavoille
(LaBRI, University of Bordeaux)

Contents

1. Generalities on dynamic algorithms
2. Forbidden-set data-structures
3. Forbidden-set routing schemes

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Queries in Dynamic Graphs

Maintaining **data-structures** for dynamic graphs (node/edge addition/deletion) supporting queries like:

- Connectivity: $\text{Find}_G(u)$?
- Approximate distances: $d_G(s,t)$?
- Near-shortest path routing: $\text{Next-Hop}_G(s,t)$?
- …

Query (s,t), Update, Query(s',t'), Update, …

Goals for a Dynamic Scenario

- Fast **query** time (must be << time to answer the query in G without pre-processing)

Ex: $d_G(s,t)$. Instead of O(m+nlogn) time for Dijkstra, prefer $O(n^{\varepsilon})$ or even polylog(n) query time

- Fast **update** time (must be << pre-processing time)

Ex: instead of $O(n^3)$ time for an All-Shortest-Path-Pair algorithm, prefer $O(n^{\varepsilon})$ or polylog(n) update time

Observation

Fast update time ➔ **Small** data-structure

If the space is S(n), then amortized update time must be ≥ S(n)/n (starting for G=Ø and adding n nodes)

A dynamic scenario with low update time requires a "compact" data-structure solution in the static scenario.

Here compact does not means to store with `gzip`, but to only store what you need (possibly in a clever way).

An Algorithmic Challenge

- Optimal solutions exist … for trees **[Tarjan,Cole,…]**
- Connectivity is still open for dynamic general graphs
- Widely open for distance & routing queries

In general, node deletion is the most costly operation.
In this talk focus on scenario:

Q(s,t), Delete x, Q(s',t), Delete x', …

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Forbidden-Set Queries

Consider available a data-structure supporting query $Q^*(s,t,X)$, the query $Q(s,t)$ in the graph $G \setminus X$, for a static graph G and for any $\{s,t\} \cup X \subseteq V(G)$

We can solve the dynamic scenario using Q^* as follows:

- If node x is deleted, then just update the set X of forbidden nodes
- If ask for $Q(s,t)$ in $G \setminus X$, then use $Q^*(s,t,X)$ on G
- If query time is too big, then recompute a static data-structure for Q^* for the new static graph $G' = G \setminus X$

Low Amortized Updates

Assume G is a sparse graph, has n nodes, and:

- Pre-processing time for Q* in G is …… nlogn
- query time of Q*(s,t,X) is …… |X|·logn

Then, recompute Q* whenever $|X| \geq \sqrt{(2n)}$

- Query time is $\leq \sqrt{(2n)} \cdot \log n$
- Amortized update time for n operations:
  $((1+2+\ldots+|X|) \cdot \log n + n\log n)/|X| \approx \sqrt{(2n)} \cdot \log n$
  Sublinear!

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The Compact Routing Problem

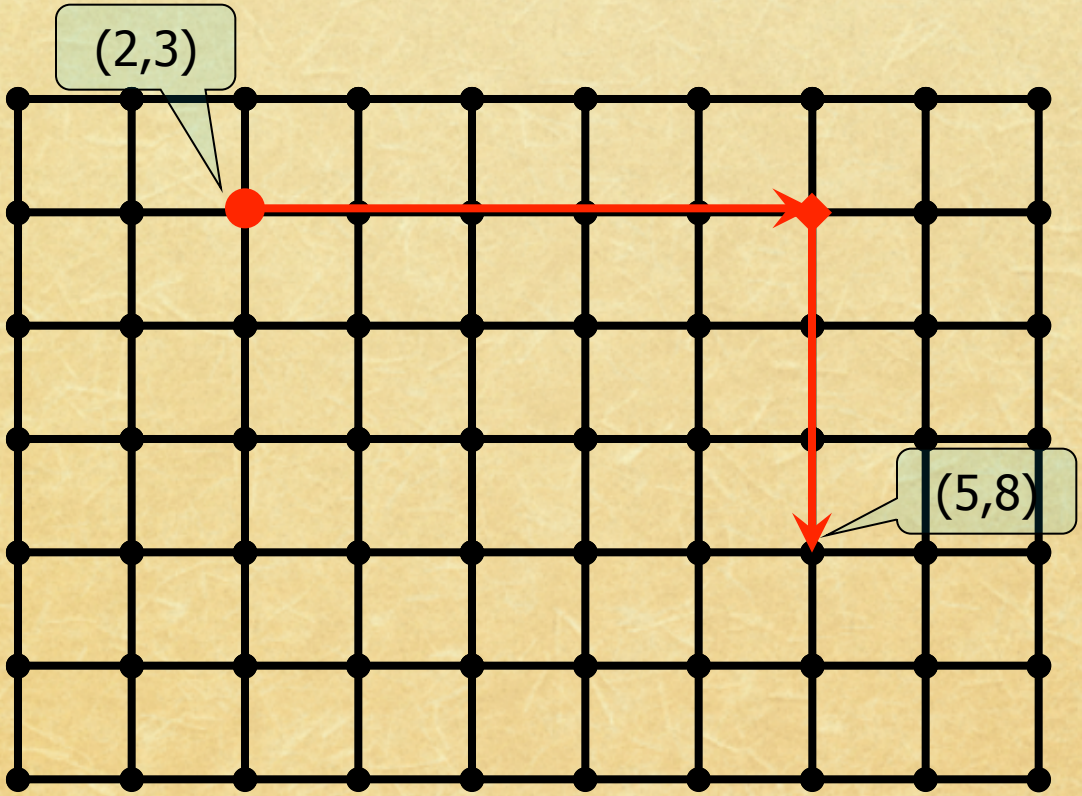
Input: a network **G** (a connected graph)

Output: a *routing scheme* for **G**

A *routing scheme* allows *any* source node to route messages to *any* destination node, given the destination's network identifier.

Node identifiers can be chosen by the designer of the scheme as a routing *label* whose length is a parameter.

Ex: Grid with X,Y-coordinates

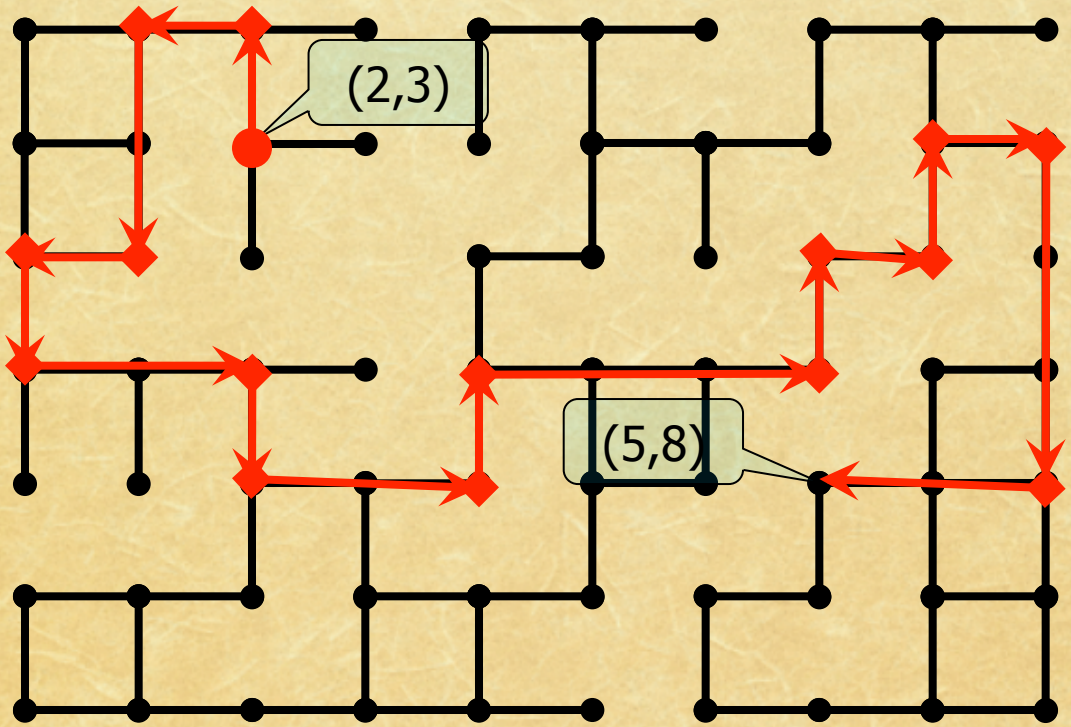


Routes are constructed
in a distributed manner

… according to some local
routing tables (or routing algorithms)

… and subgraphs of the grid?

(x,y)-coordinates no longer sufficient; routing in planar graphs…



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Quality & Complexity Measures

- Near-shortest paths: $|\text{route}(s,t)| \leq \textbf{stretch} \cdot d_G(s,t)$
- Size of the labels and routing tables
- **Goal**: constant stretch & compact (polylog) tables/labels

Trivial upper bound: O(nlogn) bits, each node stores the neighbour on the next-hop towards each destination

Routing in Static Planar Graphs

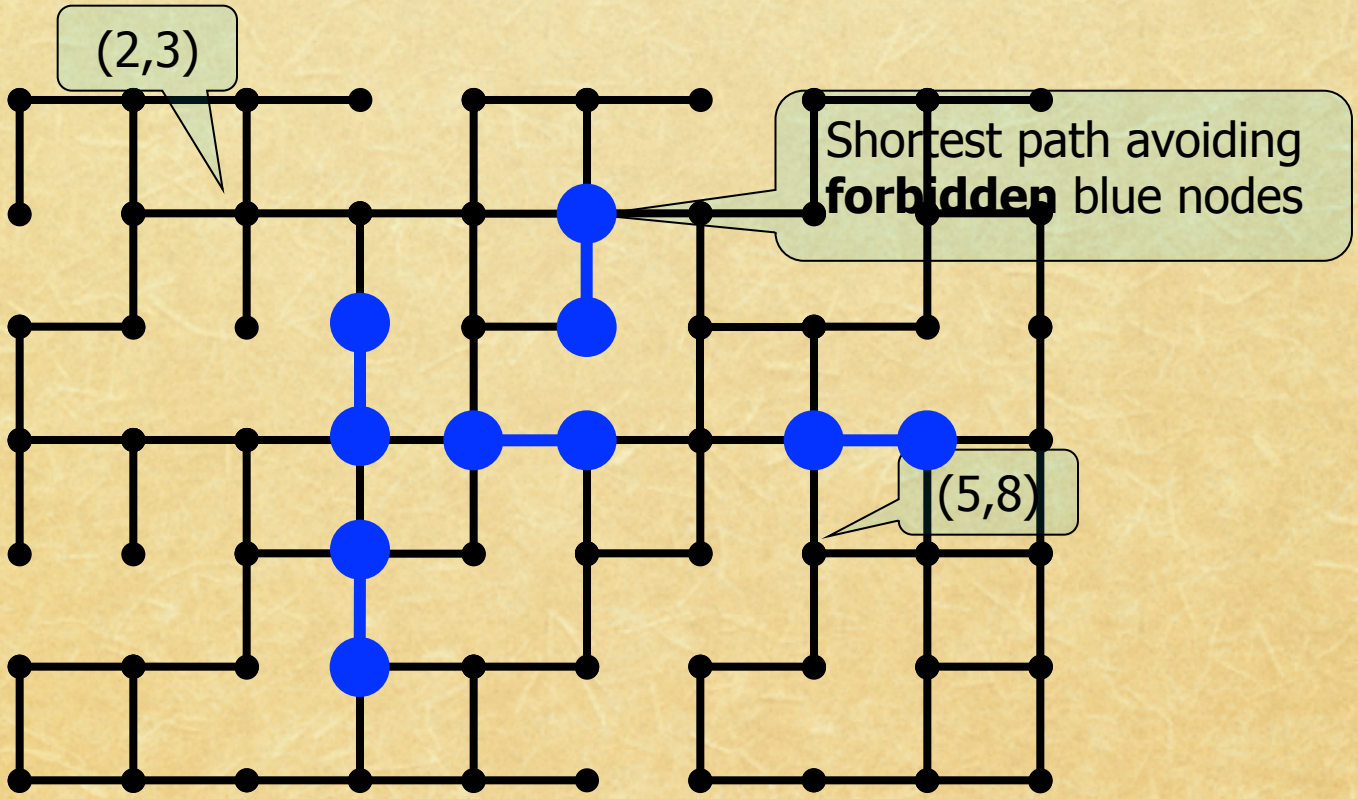
Stretch-1 [G. et al., J. Algorithms' 04]

Shortest-path routing on weighted planar graphs requires labels of $\Omega(\sqrt{n})$ bits. Treewidth-k graphs have stretch-1 routing schemes with $O(k \log^2 n)$-bit labels. For planar, $k=\sqrt{n}$.

Stretch > 1 [Thorup, JACM' 04]

Weighted planar graphs have $(1+\varepsilon)$-stretch routing schemes with $(1/\varepsilon) \cdot O(\log^2 n)$-bit labels.

Forbidden-Set Routing



Forbidden-Set Routing

Input: a network **G**

Output: a *forbidden-set routing scheme* for **G**

A *forbidden-set routing scheme* allows *any* source node s to route messages to *any* destination node t, *avoiding any set X of forbidden nodes or edges*, given the local table of s, the identifier of t and the identifiers of nodes/edges in X.

Motivations

Routing around failures

- Routing schemes are generally static; recomputation of labels/routing tables is costly.
- The set X can be a set of failed nodes/edges
- Best known techniques only handle single failures e.g. “fast reroute”, Cisco not-via

Internet routing

- ASes want control over where their packets travel; shortest-path routing not expressive enough
- BGP allows AS *i* to specify that its packets avoid AS *j*

Forbidden-Set Routing

[Upper bounds]

O(nlogn) no longer trivial!

The trivial upper bound is to store the entire graph at each node ➔ $O(n^2)$ bits/node.

[Lower bounds]

Static scenario applies (take X=Ø), i.e., Ω(n) for general graphs, and Ω(√n) for planar.

Known Results on Forbidden-Set Labeling

[Courcelle-Twigg, STACS' 07]

Stretch-1 forbidden-set distance and routing in treewidth-k graphs with $O(k^2 \log^2 n)$-bit labels.

[Chechik-G.-Peleg, PODC' 10]

Stretch-$(1+\varepsilon)$ forbidden-set distance and routing in unweighted graphs of doubling dimension with labels of $(1/\varepsilon)$·polylog(n)-bit labels.

New Results

[Abraham-Chechik-G., 2011]

Stretch-$(1+\varepsilon)$ forbidden-set distance and routing in weighted planar graphs with $(1/\varepsilon) \cdot \text{polylog}(n)$-bit labels.

➔ ***Corollary:*** $\sqrt{n} \cdot \text{polylog}(n)$ worst-case query and update time for $(1+\varepsilon)$-approximated distance oracle in dynamic planar graphs.

Previous bound: $n^{2/3}$ **[Klein et al., Algorithmica'98]**

Conclusion

A proof of concept:

Forbidden-Set Labeling Schemes with short labels (i.e., local & compact data-structures) indeed **do help** for the design of efficient data-structures in dynamic graphs.

Thank you!