An Overview on Compact Routing

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The Compact Routing Problem

Input: a network G (a weighted connected graph)

Ouput: a **routing scheme** for G

A *routing scheme* is a distributed algorithm that allows **any** source node to route messages to **any** destination node, given the destination's network identifier

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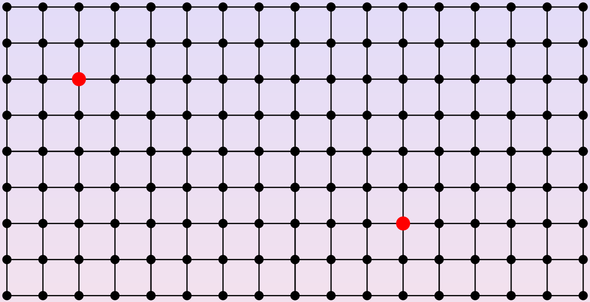
Goal: to minimize the size of the routing tables

Example: Grid with X,Y-coordinates



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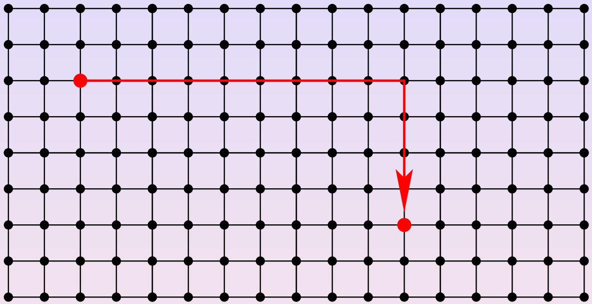

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Question: for a given family of graphs, find the best space-stretch trade-off

Two variants: Name-independent vs. Labeled

The destination enters the network with its **name**, which is determined by either the designer of the routing scheme (labeled), or an advesary (name-independent).

Labeled: the designer is free to name the nodes according to the topology and the edge weights of the graph

Name-independent: the input is a graph with fixed node manes

An overview: Labeled Model

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$\tilde{O}(f(n)) = f(n) \cdot \mathrm{polylog}(n)$

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	$O(k^2 2^k)$	$\tilde{O}(n^{1/k})$	[Arias et al./Awerbuch,Peleg]
	$O(k)$	$\tilde{O}(n^{1/k})$	[Abraham et al.]

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Theorem [Abraham,G.,Malkhi]

1. Any name-indep. routing scheme using $< (n \log n)^{1/k}$ bits/node has a **max stretch** $\geqslant 2k+1$ for some graph.
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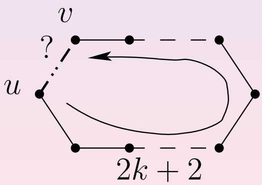
Rem 1: All previous lower bounds for labeled case (Peleg,Upfal / G.,Pérennès / G.,Gengler / Kranakis,Krizanc / Thorup,Zwick) are based on the construction of **dense large girth** graphs



if stretch$< 2k+1$, then u is forced to "know" the edge (u,v)

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Erdös Conjecture: $\exists$ graph of girth $2k+2$ with $\Omega(n^{1+1/k})$ edges (proved only for $k = 1, 2, 3, 5$). So, the extra $(\log n)^{1/k}$ term **cannot** be obtained with a girth approach.

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Rem 2: It makes a clear separation between labeled and name-independent routing, at least for the average stretch.

In the **labelel** model, $O(\mathrm{polylog}(n))$ space and $O(1)$ average stretch exsits for every graph! [Abraham, Bartal, Chan, Gupta, Kleinberg et al. (FOCS05)]

In the **name-indep** model, if space is $O(\mathrm{polylog}(n))$, then the average stretch must be $\Omega(\log n / \log\log n)$ for some graphs.

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A weaker model, but conceptually easier

Input: a metric space (V, d)

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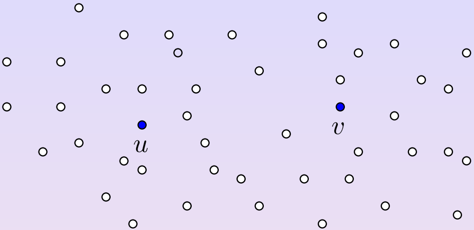
Goal: to minimize the size of G, and the space for each node must be $\approx$ the average degree of G

Example: Stretch-3 for Arbitrary Metric



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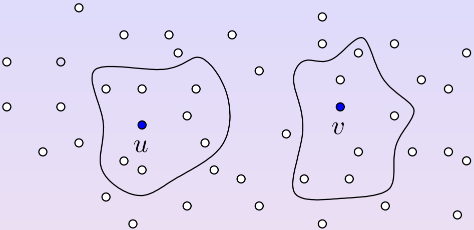

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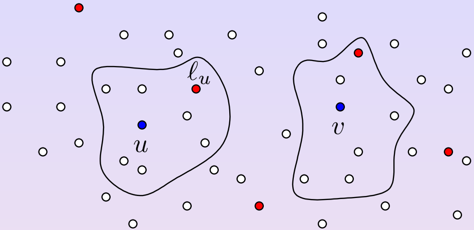

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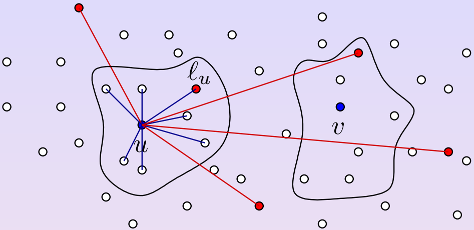

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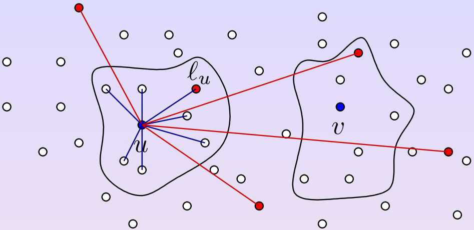

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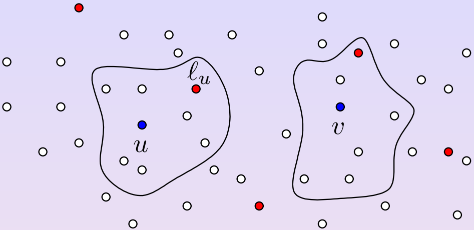

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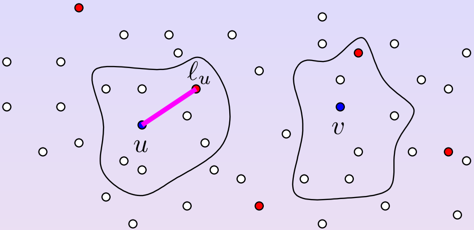

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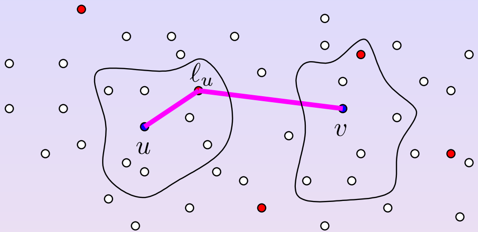

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Rem: $\ell_u \to v$ is not necessarily easy to implement in the graph model (usually simulated with some tree routings)

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Both labeled and name-independent variants exist ...

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Undirected vs. Directed

(graph model only!)

Problem: there is no stretch-space trade-off for routing in *directed* graphs! The stretch maybe not bounded if $o(n)$ bits of memory are used, even in strongly connected digraphs [Thorup,Zwick]

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New measure: roundtrip stretch factor

$$\text{stretch} \;=\; \frac{|\text{route}(u,v)| + |\text{route}(v,u)|}{\text{dist}(u,v) + \text{dist}(v,u)}$$

Rem: $\text{dist}(u,v) + \text{dist}(v,u)$ is now a distance function

Some Results for Arbitrary Digraphs

Labeled: [Roditty,Thorup,Zwick - SODA '02]

stretch$=4k+\varepsilon$	stretch$=3$
space$=\tilde{O}(\varepsilon^{-1}kn^{1/k}\log\Delta)$	space$=\tilde{O}(\sqrt{n}\,)$
labels$=o(\varepsilon^{-1}k\log^2 n\log\Delta)$	labels$=o(\log^2 n)$

Name-independent: [Arias,Cowen,Laing - PODC '03]

stretch$=O(k^2)$	stretch$=6$
space$=\tilde{O}(\varepsilon^{-1}kn^{1/k}\log\Delta)$	space$=\tilde{O}(\sqrt{n}\,)$
labels$=o(\varepsilon^{-1}k^2\log^2 n\log\Delta)$	labels$=o(\log^2 n)$

Lower bound: if stretch <2, then $\Omega(n)$ bits is required

Open Questions: For Arbitrary Networks

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Q3: Directed = Undirected???

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Q6: Shortest path in planar with $\tilde{O}(1)$ labels: $\Omega(n^{1/3})$ ... $O(n)$ (currently $7.18n$ bits [Lu '02])

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W3: Routing with additive stretch?
(initial works in random power law networks [Brady,Cowen '06]. The addtive stretch and the polylog labels depend on the graph parameter only. Works well in practice. Connection with distance labeling)

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W6: Distributed algorithms for constructing tables? Yes [Frederickson'90] for some speficic graphs (planar). Distributed implementation is possible but ... complicated!

Thank you!