

Routing v.s. Spanners

Spanner et routage compact : similarités
et différences

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Outline

Spanners

Routing

The Question and the Answer

Outline

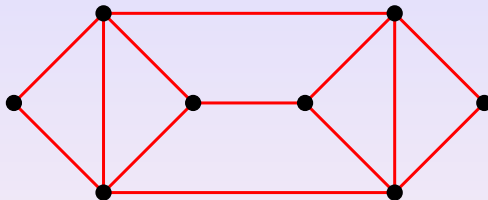
Spanners

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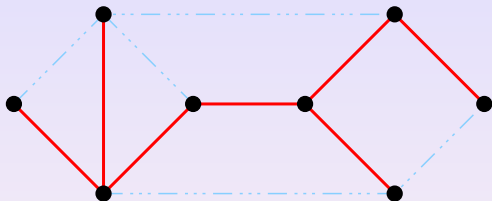
What is a Spanner?

A **spanner** of a graph G is a subgraph spanning $V(G)$



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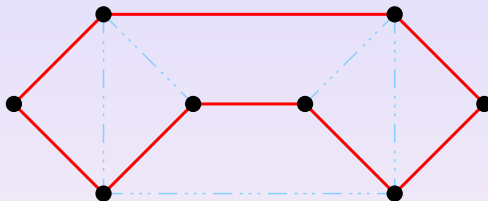
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- a spanning tree

What is a Spanner?

A **spanner** of a graph G is a subgraph spanning $V(G)$



- a spanning tree
- a Hamiltonian cycle
- ...

Approximate Distance Spanners

There are two “natural” criteria for a spanner of G :

- **size:** its number of edges.
- **stretch:** its maximum distance distortion from G .

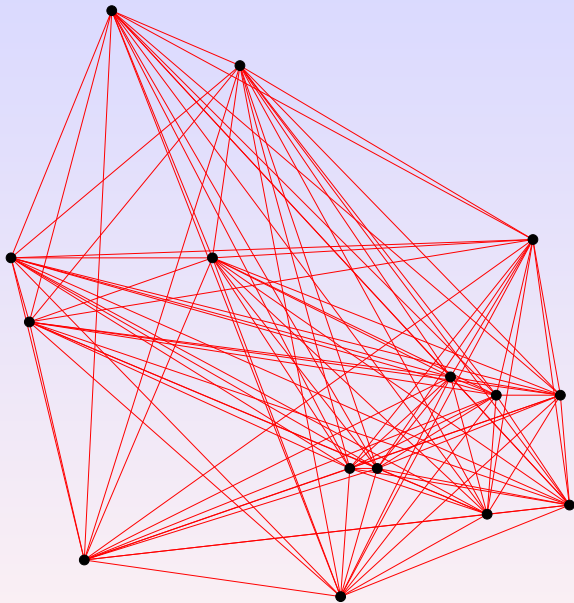
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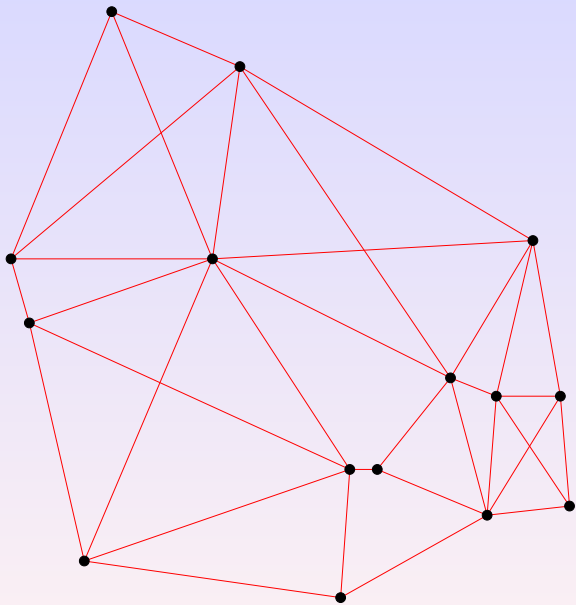
- **size:** its number of edges.
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Goals:

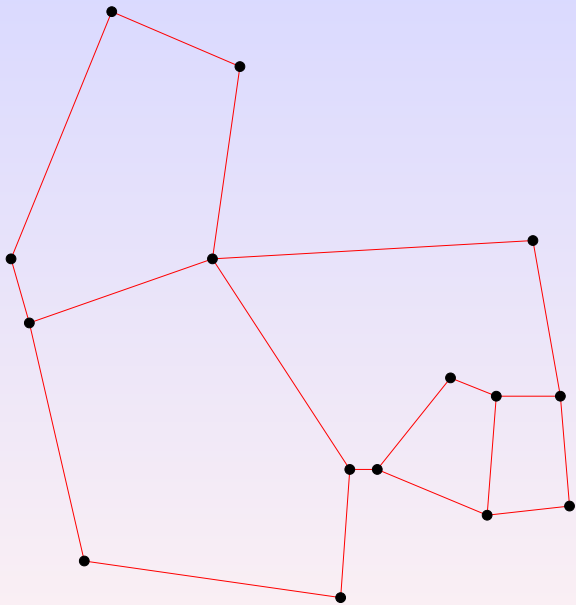
- find a good skeleton of the graph;
- decrease the size of the graph while preserving distances;
- optimize stretch-size tradeoffs.



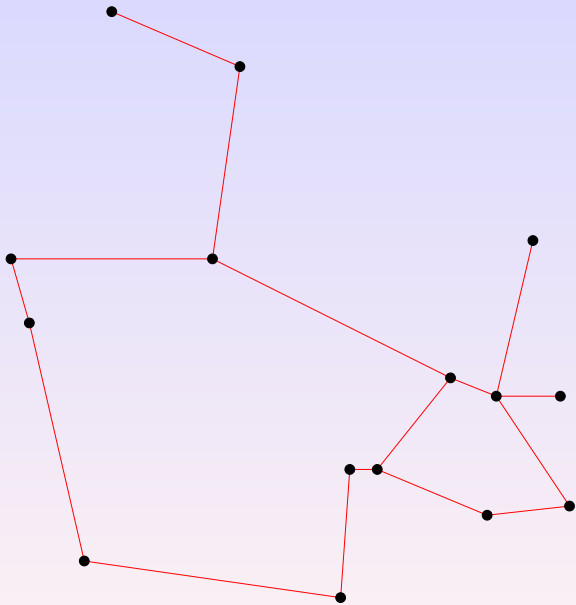
A complete Euclidian graph on 15 nodes



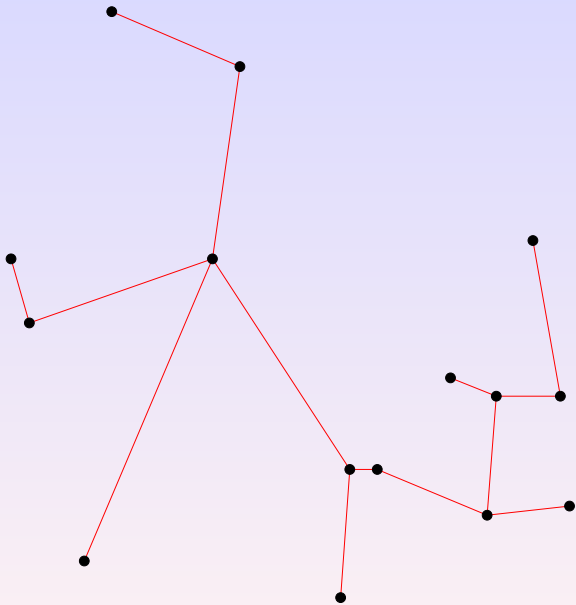
A (minimum cost) spanner with stretch 1.2



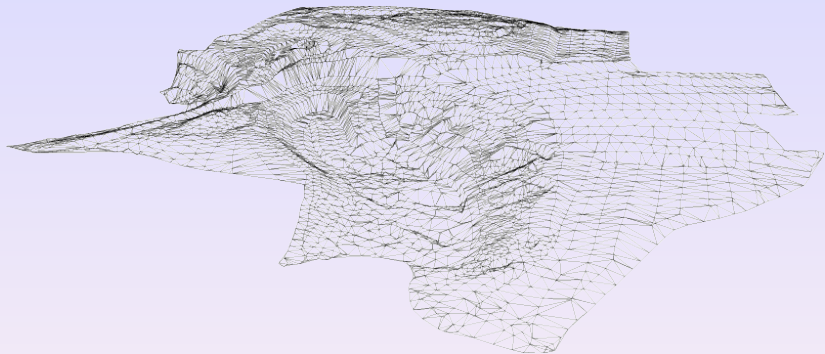
A (minimum cost) spanner with stretch 1.7



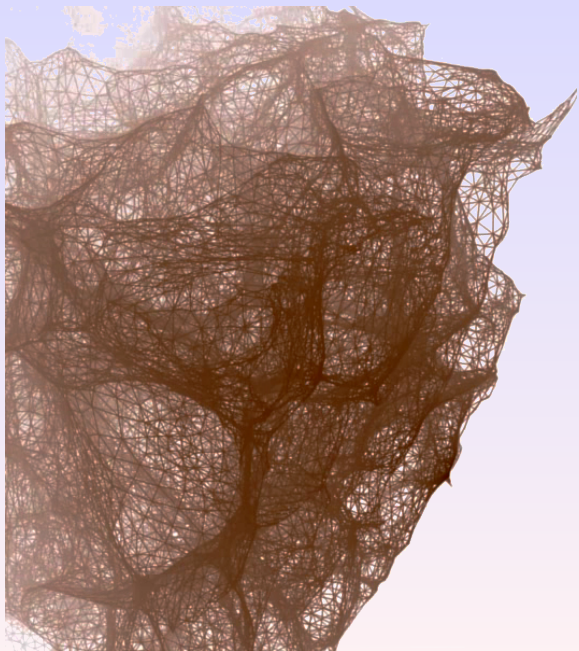
A (minimum cost) spanner with stretch 2.0



A (minimum cost) spanner with stretch 3.0



Delaunay triangulation



Spanner: Some Results (1/3)

For specific metrics, geometric spaces ...

- Every 2D Euclidean graph has a stretch-2 spanner of size $3n - 6$ [STOC '86] - Delaunay triangulations get stretch-2.42 (conjectured stretch-1.57)
- Every 2D Euclidean graph has a stretch- $(1 + \varepsilon)$ spanner of size $O(\varepsilon^{-1}n)$ [Yao- & Θ -Graphs]
- Generalization to doubling dimension and higher dimension spaces, also to hyperbolic spaces ...

Spanner: Some Results (2/3)

For general (connected) graphs ...

- Every weighted graph has a stretch- $(2k - 1)$ spanner of size $O(n^{1+1/k})$ [Greedy algorithm (Kruskal)]
- Every unweighted graph has a stretch $+2$ spanner of size $O(n^{1+1/2})$ [SICOMP'99], and a stretch $+6$ spanner of size $O(n^{1+1/3})$ [SODA'05]

Spanner: Some Results (3/3)

Recently ...

- Every unweighted stretch $(2k - 1)$ spanner must have $\Omega(\frac{1}{k}n^{1+1/k})$ edges in the worst-case [FOCS'06]
- Every unweighted graph has a f -fault-tolerant stretch- $(2k - 1)$ spanner of size $k^3 f^{k+3} \cdot \tilde{O}(n^{1+1/k})$ [STOC'09]
[a spanner for $G \setminus F$ for all subgraphs F with $|F| \leq f$]

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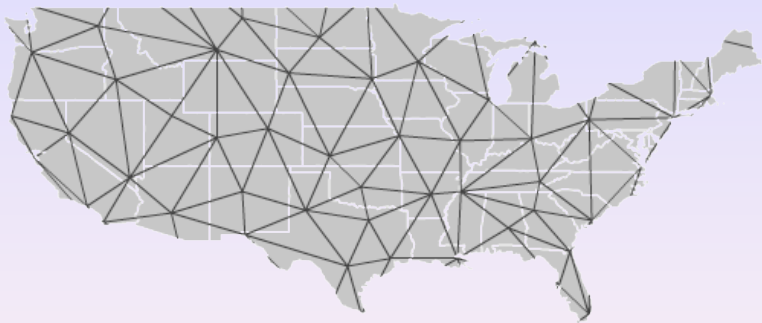
Spanners

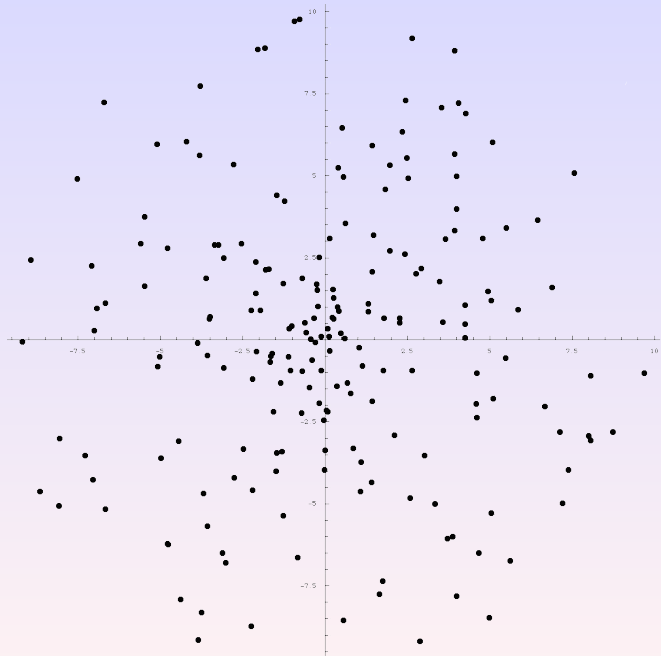
Routing

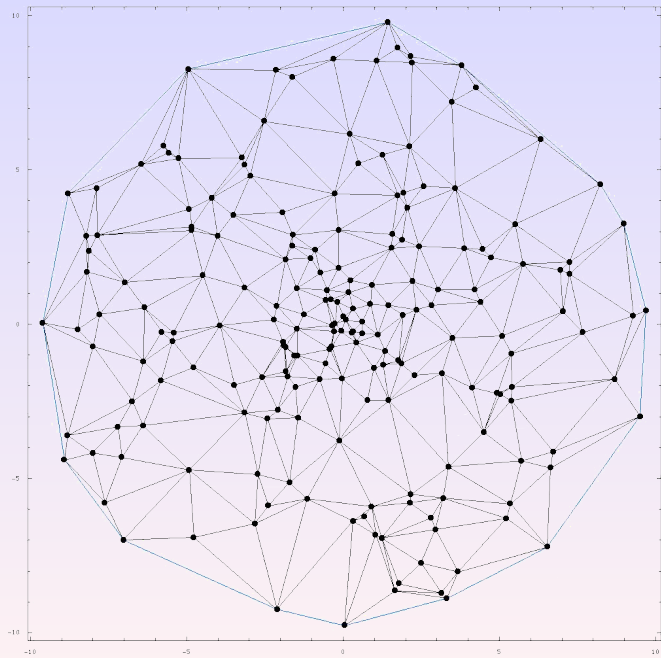
The Question and the Answer

Routing with Compact Tables









Compact Routing Schemes

Two “natural” criteria for designing a routing scheme for G :

- **size:** routing table size (and also address/header size)
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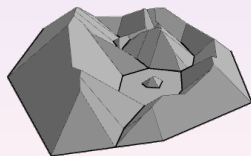
Goals:

- find an efficient routing scheme for G ;
- compress routing table size while preserving near-shortest routes;
- optimize stretch-size tradeoffs.

Routing: Some Results (1/3)

For metrics ...

- Every 2D Euclidean graph has a stretch-14.0 labeled routing scheme with $8 \log D$ -bit tables [Yao-Graph Y_7 + greedy routing]
- Every 2D Euclidean graph has a stretch- $(1 + \epsilon)$ labeled routing scheme with $O(\epsilon^{-1} \log D)$ -bit tables [cf. Abraham-Malkhi: PODC'04]
- Extension to 3D and more ...



Routing: Some Results (2/3)

For specific (connected) graphs ...

- Every weighted planar graph has a stretch- $(1 + \varepsilon)$ labeled routing scheme with $\tilde{O}(\varepsilon^{-1})$ -bit tables, addresses and headers [Thorup: J. ACM'05]



planar

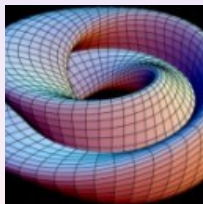
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For specific (connected) graphs ...

- Every weighted planar graph has a stretch- $(1 + \varepsilon)$ labeled routing scheme with $\tilde{O}(\varepsilon^{-1})$ -bit tables, addresses and headers [Thorup: J. ACM'05]
- Similar bounds for weighted H -minor free graphs [Abraham-G.: PODC'06]



planar

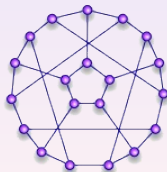


minor-free

Routing: Some Results (3/3)

For general (connected) graphs ...

- Every weighted graph has a stretch- $(4k - 5)$ labeled routing scheme with $\tilde{O}(n^{1/k})$ -bit tables and polylog addresses and headers [Thorup-Zwick: SPAA'01]
- Every weighted graph has a stretch- $O(k)$ *name-independent* routing scheme with $\tilde{O}(n^{1/k})$ -bit tables and polylog headers [Abraham et al.: SPAA'06]



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Summary

stretch	size	
$2k - 1$	$O(n^{1+1/k})$	Spanner: Greedy Algorithm
$2k - 1$	$O(kn^{1+1/k})$	Spanner: Tree Cover
$4k - 5$	$\tilde{O}(kn^{1/k})$	Routing: Tree Cover ++
+2	$O(n^{1+1/2})$	Spanner: Tree Cover
+6	$O(n^{1+1/3})$	Spanner: ad-hoc
$+f(k)$	$O(n^{1+1/k})$	Open for every $k > 3$

Fact: Spanner and Routing problems use similar techniques, and get similar bounds

Question(s)

Can we make additively stretched spanners routable?

[Baswana, Elkin, Pettie, ...]

—

Is there a routing scheme with sublinear space and additive stretch for all graphs?

Yes or No?

Yes or No?

PRO: Numerology!

Spanner: stretch-3 for size $O(n^{1+1/2})$

Routing: stretch-3 for size $\tilde{O}(n^{1/2})$

Spanner: stretch- $O(k)$ for size $O(n^{1+1/k})$

Routing: stretch- $O(k)$ for size $\tilde{O}(n^{1/k})$

Just a coincidence?

Yes or No?

PRO: Numerology!

Spanner: stretch-3 for size $O(n^{1+1/2})$

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Spanner: stretch- $O(k)$ for size $O(n^{1+1/k})$

Routing: stretch- $O(k)$ for size $\tilde{O}(n^{1/k})$

Just a coincidence?

- There exist spanners of size $o(n^2)$ with constant additive stretch (ex: size $n^{1+1/2}$ or $n^{1+1/3}$ for stretch +2 or +6).
- It should exist sublinear compact routing scheme with constant additive stretch!!!

Yes or No?

CON: Spanners do not tell us how to **route** on sparse graphs.

The problem is:

- Spanner: prove \exists a near-shortest path
- Routing: construct a near-shortest path

An Impossibility Result

There are graphs with sparse additive spanners but no additive compact routing schemes

Theorem (2009)

Every routing strategy providing, for each unweighted connected n -node graph, a labeled routing scheme with tables and addresses $\leq \mu$ bits, produces, for some graph of less than $2n$ edges, an additive stretch $\Omega(n^{1/3}/\mu^{2/3})$.

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Corollary: The additive stretch of every universal routing strategy with tables and addresses in $o(\sqrt{n})$ is unbounded.

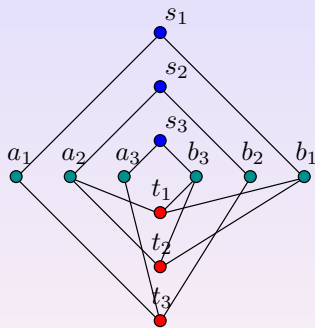
In particular, the stretch-7 routing scheme of [TZ] ($k = 3$) with $\tilde{O}(n^{1/3})$ -bit tables *must have* an additive stretch of $n^{1/9-o(1)}$.

THANK YOU!

The Graph Family $\mathcal{F} = \mathcal{F}(p, \delta)$

Graphs of \mathcal{F} are constructed from $p \times p$ boolean matrices.

Sets of p nodes: $S = \{s_i\}$, $A = \{a_i\}$, $B = \{b_i\}$, $T = \{t_j\}$.



$$M = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Connect a path of length δ between: $s_i \rightsquigarrow a_i$, $s_i \rightsquigarrow b_i$, and $t_j \rightsquigarrow a_i$ if $M[i, j] = 1$, and $t_j \rightsquigarrow b_i$ if $M[i, j] = 0$.