# Path separability of Graphs

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# Outline

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# Motivations

- To separate graphs in order to apply "Divide and Conquer"

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- The notion of $k$-path separability defined by Abraham et al. (PODC'06), to solve "Objects Location Problem"
  - Compact routing with $O(k \log^2 n)$-bit tables
  - Distance labelling with $O(k \log n \log D)$-bit labels
  - Navigation in "Small-World" with $O(k^2 \log^2 n)$ hops

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Motivation

Definition

Results

What about the 1-path separable graphs

# $k$-path separable graphs.

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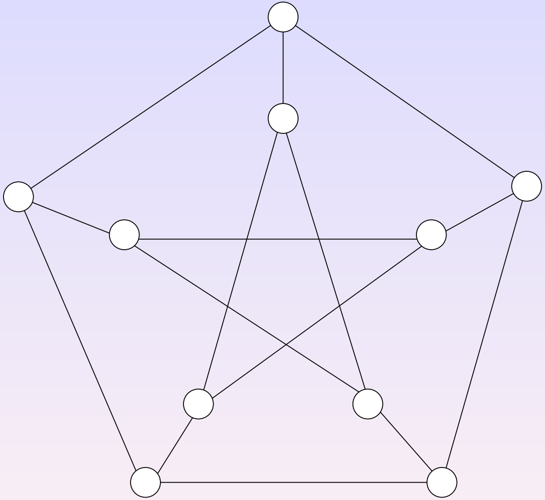
## Definition ($k$-path separability)

- $S = P_0 \cup P_1 \cup \ldots$, where each subgraph $P_i$ is the union of $k_i$ minimum cost paths in $G \setminus \bigcup_{j<i} P_j$ where $\sum_i k_i \leqslant k$ ; and
- every connected component of $G \setminus S$ (if any) is $k$-path separable and weigth at most $\omega(G)/2$.

# Example : Petersen Graph



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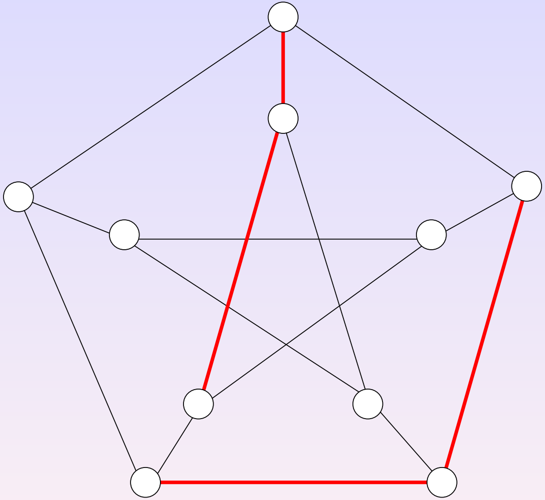

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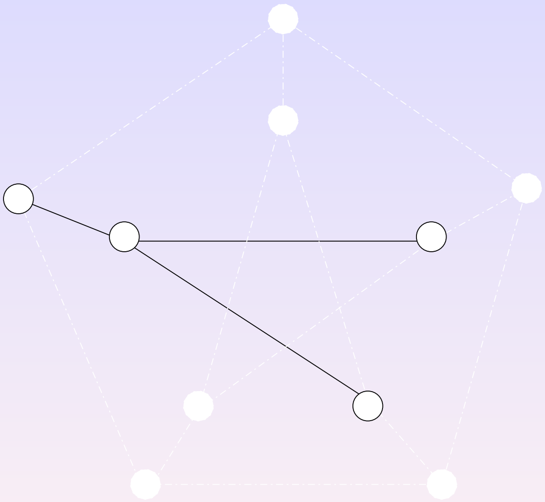

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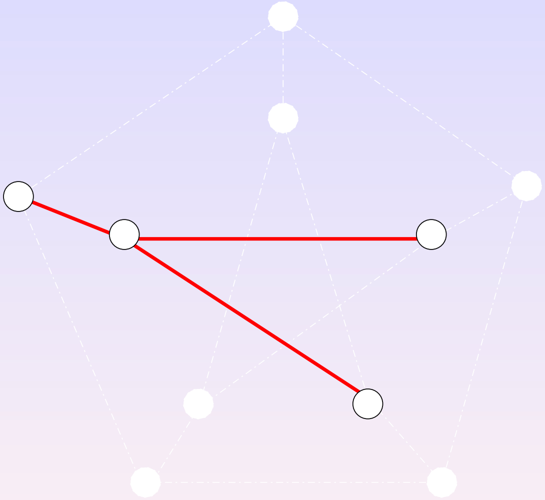

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$\Rightarrow 2$-path separable

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*Planar graphs are $3$-path separable.*

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**Theorem (Thorup - FOCS'01/JACM'04)**

*Planar graphs are $3$-path separable.*

**Theorem (Abraham and Gavoille - PODC '06)**

*$H$-minor free graphs are $f(H)$-path separable.*

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$PS_k$ is the family of graphs that are $k$-paths separable for every weight function.

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**Definition**

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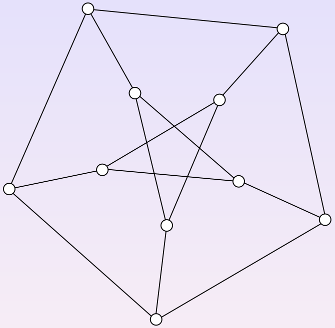
Trees $\subset PS_1$
Treewidth-3 $\subset PS_2$
Planar graphs $\subset PS_3$

# Minor graphs

A **minor** of $G$ is a subgraph of a graph obtained from $G$ by edge contraction.



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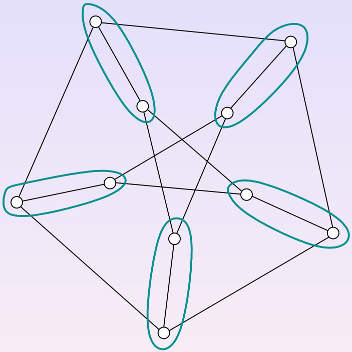

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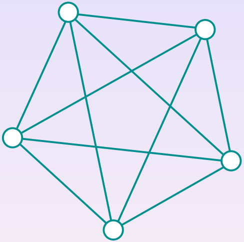

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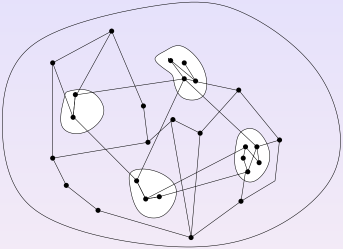
# Minor graphs

**Proposition**

*$PS_k$ is closed under minor taking.*

# Proof



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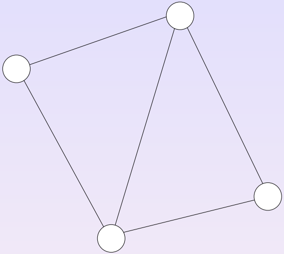

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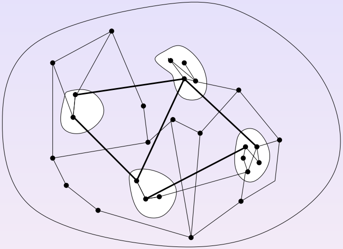

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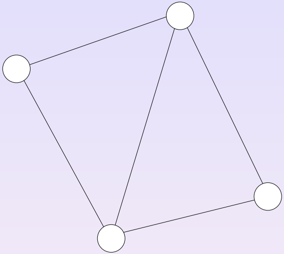

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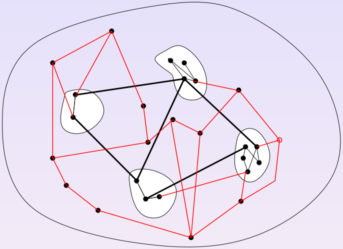

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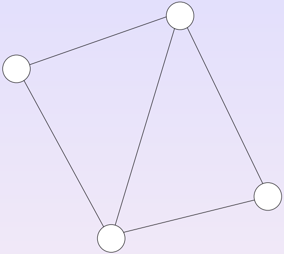

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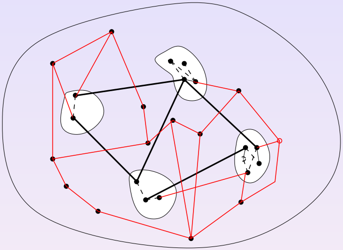

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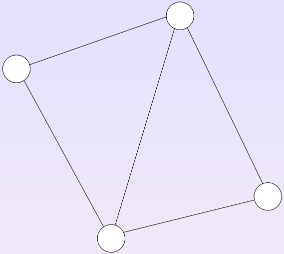

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# Forbidden minors

**Corollaire (Roberston & Seymour)**

*$G \in PS_k$ iff $G$ excludes a finite list of "forbidden" minors. Therefore, for constant $k$, membership for $PS_k$" can be tested in cubic time ... if the list is given.*

# Forbidden minors (at least 16)

The unique non-planar graph in $PS_1$ is $K_{3,3}$.

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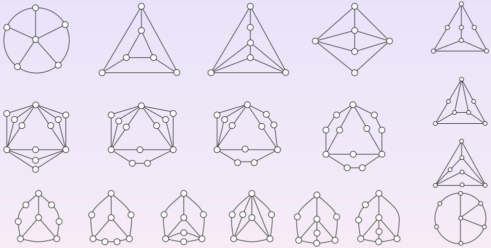
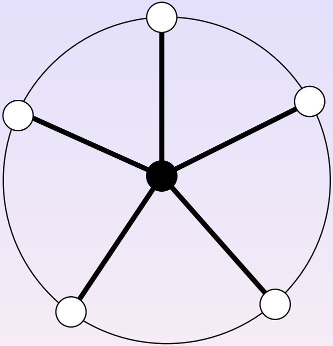
And for planar graphs :



# Particular example



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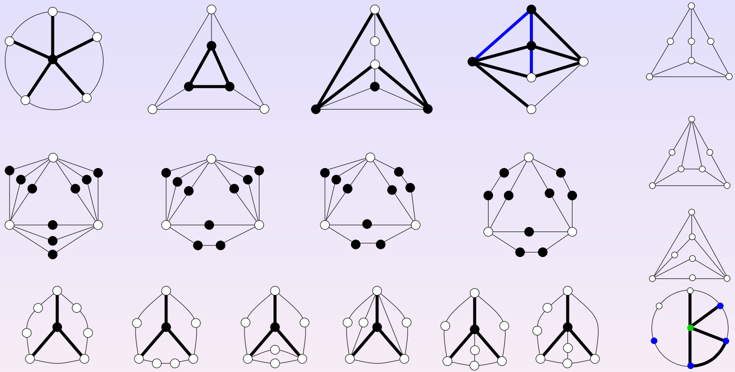

# Perspectives

- List all forbidden minors for $PS_1$.
- What about planar graphs ?

Thank you