

The Stretch Factor of L_1 - and L_∞ -Delaunay Triangulations

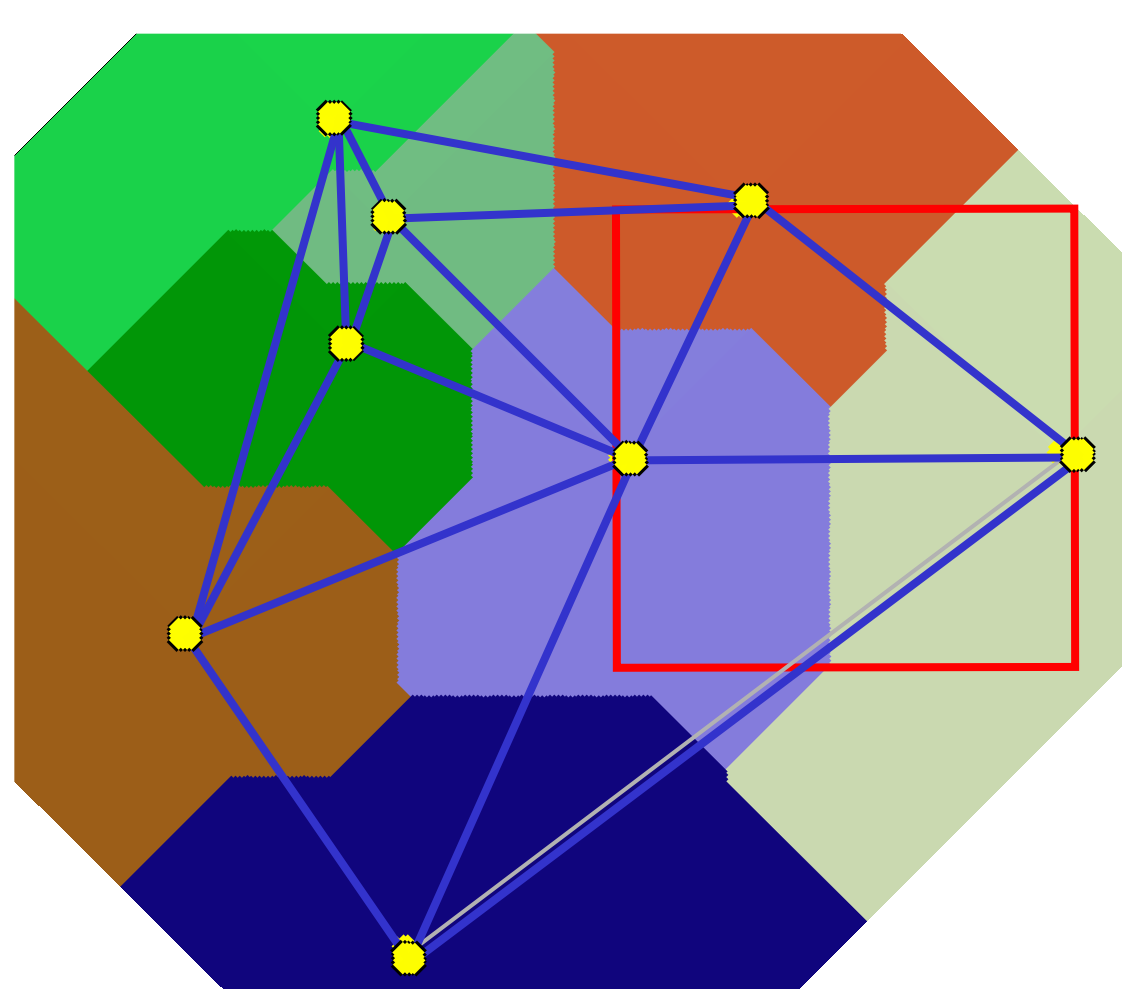
N. Bonichon, C. Gavoille, N. Hanusse, L. Perkovic
[bonichon, gavoille, hanusse]@labri.fr, lperkovic@cs.depaul.edu



L_∞ -Delaunay Triangulation

Definition : Given a point set S , the L_∞ -Delaunay Triangulation G of S is the Delaunay Triangulation based on the L_∞ norm.

In other words, there is an edge (u,v) in G iff there exists a square supporting u and v empty of points of S .



Stretch factor

Definition: Let S be a point set of the plane. Let $G=(S,E)$ be a weighted graph such that the weight of an edge (u,v) is $\|\vec{uv}\|$.

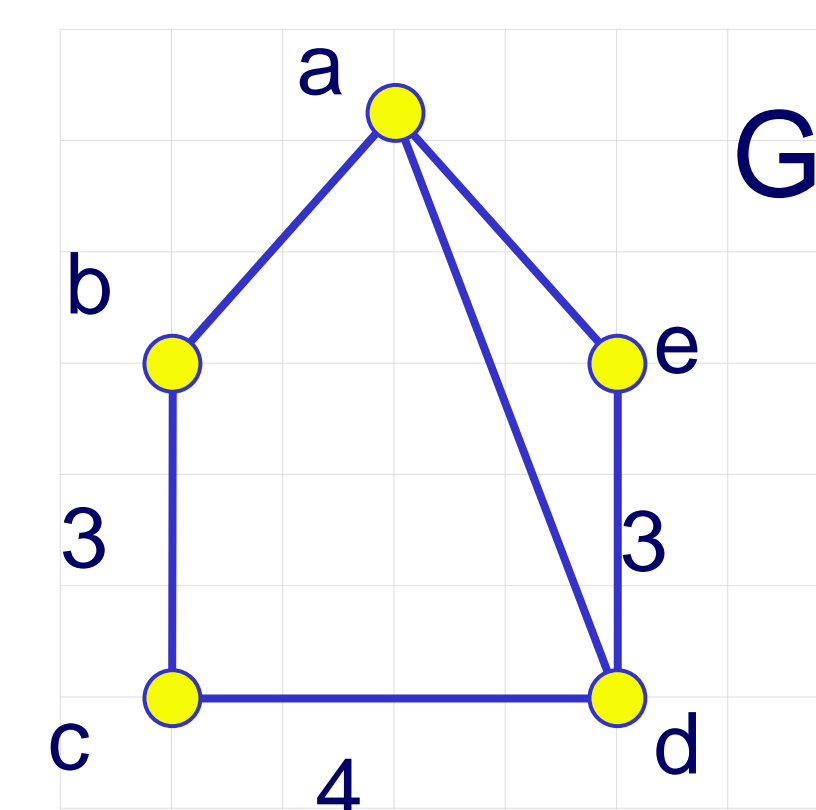
The **stretch factor** of G is

$$\max_{u \neq v \in S} \frac{d_G(u,v)}{\|\vec{uv}\|}$$

where $d_G(u,v)$ is the distance between u and v in the graph G .

In this example:

$d_G(b,e) = 4\sqrt{2}$ and $\|\vec{be}\| = 4$.
The stretch factor of G is $\sqrt{2}$.

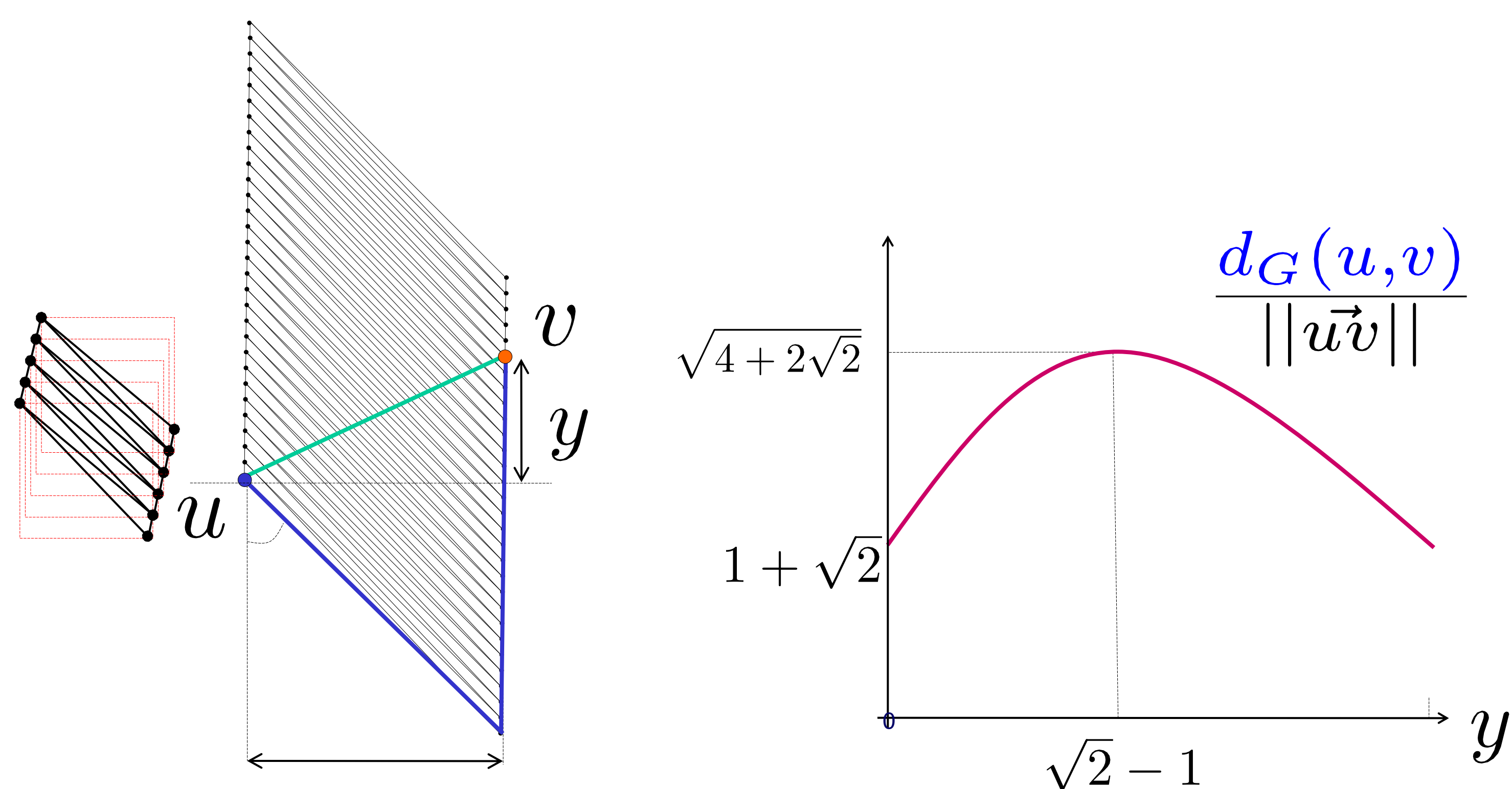


Main result

Theorem: The stretch factor of an L_∞ -Delaunay Triangulation is at most $\sqrt{4 + 2\sqrt{2}} \approx 2.61$. Moreover this bound is tight.

Best previous bound: $\sqrt{10}$.

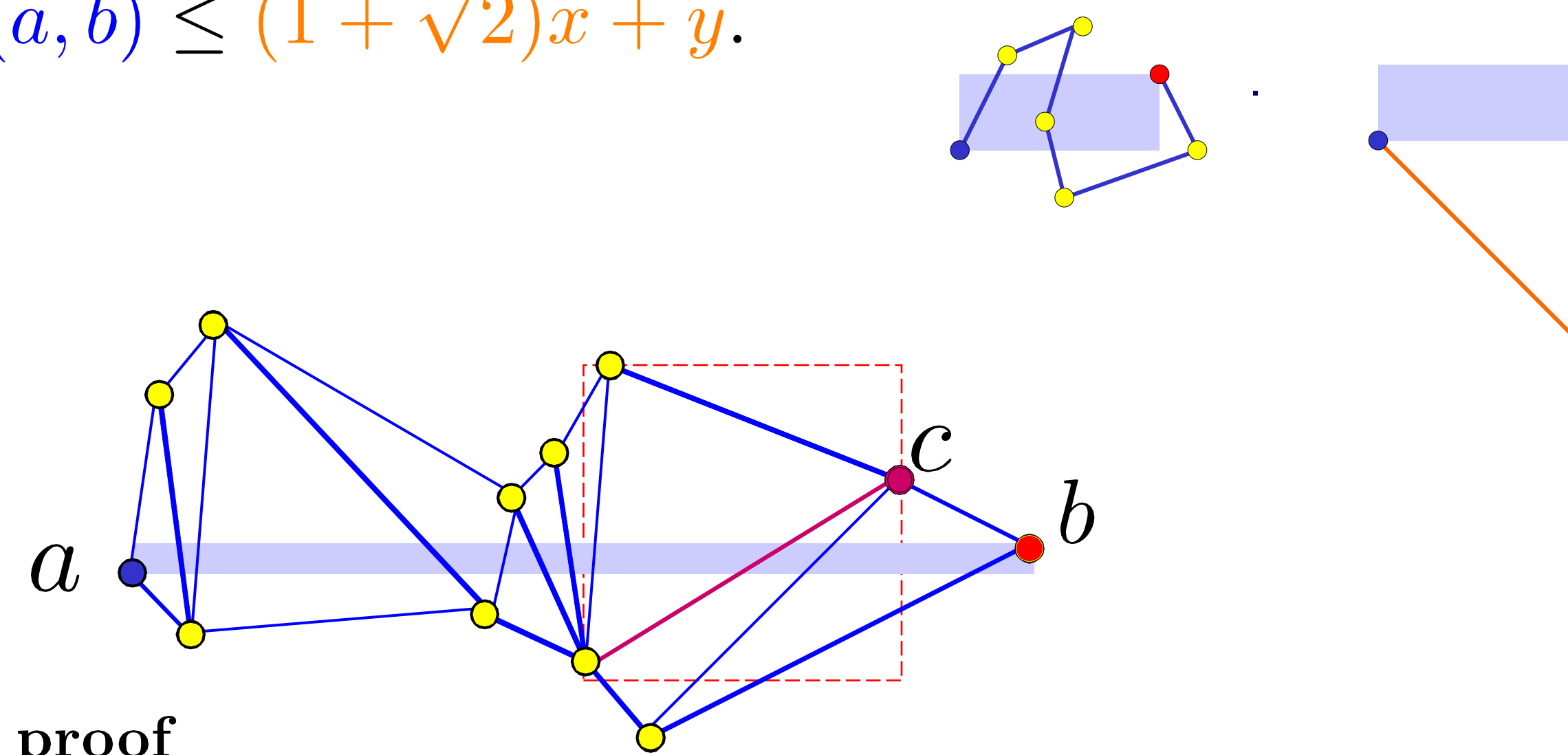
L_∞ -Delaunay Triangulation stretch factor: Lower bound



L_∞ -Delaunay Triangulation stretch factor: Upper bound

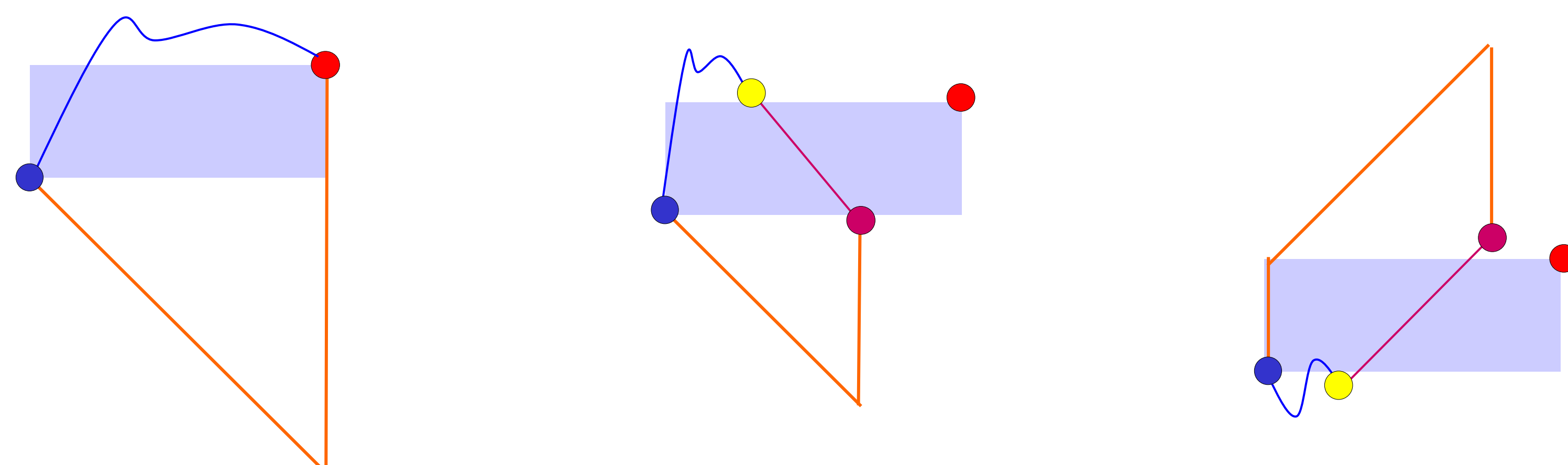
Lemma: Let G be the L_∞ -Delaunay triangulation of S . Let a and b be 2 points of S .

Let $x = d_x(a,b)$ and $y = d_y(a,b)$. If $y \leq x$,
 $d_G(a,b) \leq (1 + \sqrt{2})x + y$.



Sketch of proof

- We consider the sequence of faces that cross segment $[ab]$.
- A face is **inductive** if its second edge crossing $[ab]$ has a slope within ± 1 .
- We then show that
 - if there is no inductive face then $d_G(a,b) \leq (1 + \sqrt{2})x + y$.
 - otherwise let c the latest point of the first inductive face.
 - * if $y_c < y_b$ then $d_G(a,c) \leq (1 + \sqrt{2})x_c + y_c$
 - * if $y_c > y_b$ then $d_G(a,c) \leq (1 + \sqrt{2})x_c + 2y - y_c$



Perspectives

Stretch factor of Delaunay triangulations defined by arbitrary polygons:

p -gon	Lower bound	Upper bound
$p=3$	2 [Chew 89]	2 [Chew 89]
$p=4$	2.61	2.61
$p=5$	1.70	?
$p=6$	2	?
$p>6$	$\frac{1}{2}$ perimeter	?
$p=\infty$	$> \frac{1}{2}$ perimeter	1.998 [Xia 11]