

# Meta-Theorems for Cuttable Distributed Problems

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## Abstract

We prove that given any  $\alpha$ -approximation LOCAL algorithm for MINIMUM DOMINATING SET (MDS) on planar graphs, we can construct an  $f(g)$ -round  $(3\alpha + 1)$ -approximation LOCAL algorithm for MDS on graphs embeddable in a given Euler genus- $g$  surface. Heydt et al. [*European Journal of Combinatorics* (2025)] gave an algorithm with  $\alpha = 11 + \epsilon$ , from which we derive a  $(34 + \epsilon)$ -approximation algorithm for graphs of genus  $g$ , therefore improving upon the current state of the art of  $24g + O(1)$  due to Amiri et al. [*ACM Transactions on Algorithms* (2019)]. It also improves the approximation ratio of  $91 + \epsilon$  due to Czygrinow et al. [*Theoretical Computer Science* (2019)] in the particular case of orientable surfaces.

We generalize this result into two directions: (1) by considering other graph problems studied in Distributed Computing such as MINIMUM  $k$ -TUPLE DOMINATING SET, for which constant-round approximation algorithms were known for planar graphs, but not for graphs of bounded genus; and (2) by considering graph classes beyond bounded genus graphs, called *locally nice*, and relying on the asymptotic dimension of the class. We prove these results by a series of meta-theorems about *cuttable* minimization problems with constant-round approximation LOCAL algorithms. Roughly speaking, in cuttable problems, one can systematically extract small subgraphs whose solutions are in proportion to the global solution restricted to the neighbourhood of the subgraph.

## CCS Concepts

• **Theory of computation** → **Distributed computing models; Graph algorithms analysis.**

## Keywords

distributed algorithm, meta-theorem, LOCAL model, dominating set

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## 1 Introduction

MINIMUM DOMINATING SET (MDS) is a famous minimization problem on graphs, known to be NP-complete even in cubic planar graphs [18, 24]. The goal is to find a smallest subset of vertices of the input graph that intersects all radius-1 balls of the graph.

In this paper, we focus our attention on distributed algorithms that can approximate MDS on the graph underlying the topology of the network. Due to the covering property of a dominating set, the problem and its variants (like CONNECTED DOMINATING SET [37]) get increasingly more attention in Networking, in particular for mobile and ad-hoc networks. Not only are mobile networks important for point-to-point communications, but specialized ad-hoc networks, such as sensor networks, are important for environmental monitoring tasks. We refer to [27, 36] for extended discussions about the motivations of MDS for routing purposes of such networks.

In the LOCAL model, approximating MDS up to a constant factor in general  $n$ -vertex graphs is known to require  $\Omega(\sqrt{\log n / \log \log n})$  rounds [9, 26]. On the positive side, it is possible to  $(1 + \epsilon)$ -approximate MDS for any graph in  $\text{poly}(\epsilon^{-1} \log n)$  rounds by combining the techniques of [19] and of [34, Corollary 3.11].

For specific graphs, better round complexities can be achieved; cf. [30, 35] for a large collection of results (including unit-disk graphs, and planar graphs as a basis of the Gabriel graph model widely used in ad-hoc networks [38]). For instance,  $O(\log^* n)$  rounds suffice for planar graphs [11], or more generally for  $K_t$ -minor-free graphs [12] and for sub-logarithmic expansion graphs<sup>1</sup> [2], and  $o(\log^* n)$  rounds are not sufficient to get a  $(1 + \epsilon)$ -approximation of MDS on cycles [11] or unit-disk graphs [31]. So, achieving constant-round algorithms must be at the price of relaxing the  $(1 + \epsilon)$  approximation ratio.

For planar graphs, the quest for constant-approximation and constant-round algorithms seems to start with [29][28, Chp. 13][30], with a 130-approximation. Since then, a long line of research has

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<sup>1</sup>Roughly speaking, the expansion of a graph  $G$  is a function  $f$  that, for any  $r$ , bounds the maximum edge density of a depth- $r$  minor of  $G$  by  $f(r)$ , where a depth- $r$  minor is a minor of  $G$  where each branch set has radius at most  $r$ .

been aimed at improving this ratio. In short, the best approximation ratio to date is  $11 + \varepsilon$ , due to [22]. We refer to the nice survey of [22] for planar graphs and sub-families, including lower bounds.

In the meantime, the quest for constant-approximation ratio and constant-round distributed algorithms has been proposed for larger classes of graphs. However, very few examples are known of graph classes  $\mathcal{C}(p)$ , depending on some fixed parameter  $p$ , where MDS can be solved by a LOCAL algorithm with round complexity  $f(p)$  and truly-constant approximation ratio (independent of  $p$ ).

Larger graph classes that make good candidates for extending planar graphs are  $H$ -minor-free graphs, with some graph  $H$  not limited to  $K_5$  or  $K_{3,3}$ . For  $K_p$ -minor-free graphs, we only know of an exponential approximation ratio in  $p$  [22, 25], whereas a linear approximation ratio could be possible. For  $K_{3,p}$ -minor-free graphs [22], the dependency is much better (linear in  $p$ ), but still not truly constant. Very recently, it was shown [7] that, for  $K_{2,p}$ -minor-free graphs, the approximation ratio is 50 regardless of the value of  $p$ .

On the other end of the spectrum, the narrowest way to meaningfully extend planar graphs is through embeddable graphs<sup>2</sup> on surfaces<sup>3</sup> of Euler genus  $g$ . Such surfaces can be obtained from a sphere by adding  $g/2 \geq 0$  handles<sup>4</sup> (if they are orientable) or by adding  $g \geq 1$  cross-caps<sup>5</sup> (if they are nonorientable). The *Euler genus* of a graph  $G$  is the minimum number  $g$  such that  $G$  embeds on a surface of Euler genus  $g$ . In this context, [2, Theorem 3.11] proposed an  $(24g + O(1))$ -approximation algorithm with constant-round complexity (actually linear in  $g$ ). For graphs of orientable genus  $g$ , i.e., embeddable on an orientable surface of genus  $g$ , [13] designed a constant-round algorithm that returns a dominating set of size at most  $91M + 76g - 66$ , where  $M$  is the optimal size. By adding a simple extra brute-force step in their algorithm, we can convert it into a  $(91 + \varepsilon)$ -approximation (cf. Proposition D.2 in full version [6]), so with a ratio independent of  $g$ . However, the result of [13] does not transfer to graphs of bounded Euler genus, as for any  $g$ , there are graphs embeddable on the projective plane<sup>6</sup> (so of Euler genus 1) that cannot be embedded on any orientable surface of genus  $g$  (so of orientable genus at least  $g + 1$ ). So, for these graphs, the approximation ratio will be  $91 + \varepsilon$ , but with the number of rounds possibly depending on the number of vertices<sup>6</sup>.

Graphs of Euler genus  $g$  are included in  $K_{3,2g+3}$ -minor-free graphs, because the Euler genus of  $K_{3,2g+3}$  is at least  $g + 1$  (cf. [32, Theorem 4.4.7]) and the class of Euler genus- $g$  graphs is closed under taking minors. We observe that the approximation ratio for  $K_{3,p}$ -minor-free graphs, for  $p = 2g + 3$ , due to [22, Theorem 2.2] provides an approximation ratio of  $O(\sqrt{g})$ . Indeed, the ratio of their algorithm is precisely  $(2 + \varepsilon)(2\nabla_1 + 1)$ , where  $\nabla_1$  is the maximal edge density of a depth-1 minor of any graph of the class. As we can check (cf. Proposition D.3 in full version [6]) that  $\nabla_1 \leq \sqrt{3g/2} + 3$ , the approximation ratio is at most  $4\sqrt{3g/2} + 14 + \varepsilon$ . As it may occur

<sup>2</sup>I.e., that can be drawn on the surface without edge crossing, like planar graphs for the sphere.

<sup>3</sup>That are compact, connected 2-dimensional manifold without boundary.

<sup>4</sup>By adding a handle, we mean that two disjoint disks of the sphere are replaced by a cylinder.

<sup>5</sup>By adding a cross-cap, we mean that a disk of the sphere is replaced by a Möbius strip.

<sup>6</sup>For instance, a cycle with  $n = 2g + 6$  vertices where opposite vertices are connected by an extra edge as Euler genus 1 but orientable genus at least  $g + 1 = n/2 - 2$ , cf. [3].

that  $\nabla_1 \geq \sqrt{3g/2} - O(1)$  for some Euler genus- $g$  graph (cf. Proposition D.3 in full version [6]), the best known approximation ratio for MDS in Euler genus- $g$  graphs is  $\Theta(\sqrt{g})$ , and  $91 + \varepsilon$  for orientable genus- $g$  graphs.

It is important to note that no LOCAL algorithms for Euler genus- $g$  graphs can achieve simultaneously constant round complexity and constant approximation ratio. Indeed, by taking  $n = \Omega(\sqrt{g})$ , since there exists  $n$ -vertex graphs of Euler genus  $\Theta(n^2)$ , the  $\Omega(\sqrt{\log n / \log \log n})$  lower bound of [9, 26] yields a lower bound of  $\Omega(\sqrt{\log g / \log \log g})$ .

*Our Contributions.* In the remainder of the paper, by “LOCAL algorithm”, we mean deterministic distributed algorithm in the classical LOCAL model.

- We propose a  $(34 + \varepsilon)$ -approximation LOCAL algorithm for MDS in Euler genus- $g$  graphs whose round complexity depends only on  $g$  (cf. Corollary 4.3). This improves approximation ratios in both the orientable (previously  $91 + \varepsilon$ ) and non-orientable (previously  $24g + O(1)$ ) genus case. The analysis of the algorithm uses the fact that such graphs have asymptotic dimension at most two.
- This result follows directly from more general meta-theorems. More generally, we prove that any algorithm that is a constant-round  $\alpha$ -approximation for MDS on planar graphs can be turned into a  $f(g)$ -round  $(3\alpha + 1)$ -approximation algorithm for MDS on Euler genus- $g$  graphs, for some function  $f$ .
- We extend this result in two ways. First, we show that our meta-theorems hold for all local minimization problems that are well-behaved (including MDS and its variants). We provide a simple graph-theoretic property on the problem that is sufficient for our meta-algorithms to apply. This significantly simplifies proofs and circumvents having to reason about various algorithms.
- We apply our results on problems such as MINIMUM  $k$ -TUPLE DOMINATING SET (cf. Corollary 4.5), obtaining constant-ratio approximations on problems where none were previously known. We also obtain a ratio of  $O(\sqrt{g})$  for MINIMUM CONNECTED DOMINATING SET in Euler genus- $g$  graphs.
- Second, we prove that our meta-theorems hold on much broader “locally nice” graph classes, such as locally bounded-genus graphs, or locally bounded-genus graphs with a bounded number of apex vertices (whose unconstrained neighborhoods can break the surface embeddability of the graph).
- We prove three versions of our meta-theorem (see Meta-Theorem I, Meta-Theorem II and Meta-Theorem III), making it easily applicable and adapted to different situations.

Unlike similar notions, such as the classical network decomposition [4], no preliminary construction is required for any of our algorithms. Obviously, they depend on the parameters of the graph classes on which they are applied, like the Euler genus  $g$  and/or the asymptotic dimension of the class with its control function.

*Organization.* Basic notations and asymptotic dimension are introduced in Section 2. Section 3 derives a 616-approximation for MDS on bounded-genus graphs, gently introducing the ideas and motivations of the later meta-theorems. Finally, Section 4 presents

our different meta-theorems, and their applications to different problems of the literature.

## 2 Preliminaries

*Graphs and Domination.* A subset  $X \subseteq V(G)$  dominates  $S$  if  $S \subseteq N[X]$ . We denote by  $\text{MDS}(G, S)$  the minimum size of a subset of  $V(G)$  dominating  $S$ . Note that  $\text{MDS}(G, V(G))$ , denoted by  $\text{MDS}(G)$  for short, is the size of a minimum dominating set for  $G$ .

*Asymptotic Dimension.* In this section, we focus on defining asymptotic dimension<sup>7</sup> (a non-trivial task, as it happens) and explaining how to exploit it, in the hope that others may be able to exploit it in turn.

A graph  $G$  with a spanning subgraph  $H$  is  $k$ -colorable  $\delta$ -bounded in  $H$  if each vertex of  $G$  can be assigned a color from  $\{0, \dots, k-1\}$  so that the distance in  $H$  between any two vertices taken in a monochromatic path of  $G$  is at most  $\delta$ . In other words, all monochromatic connected components of  $G$  must have a weak diameter in  $H$  at most  $\delta$ . A proper  $k$ -coloring of  $G$  is nothing else than a  $k$ -coloring 0-bounded in  $H$ . If  $H$  has diameter  $D$ , then it is 1-colorable  $D$ -bounded in  $H$ .

The asymptotic dimension of a graph class  $\mathcal{C}$  is a non-negative integer denoted by  $\text{asdim}(\mathcal{C})$ , and related to the  $k$ -coloring boundedness of the  $r$ -th power of  $G$ . It is a notion closely related to *sparse partitions*<sup>8</sup> and *weak sparse covers* (see [5, §1.8] for precise connection between these notions). Recall that the  $r$ -th power of  $G$ , denoted by  $G^r$ , is the graph obtained from  $V(G)$  by adding edges between pairs of vertices at distance at most  $r$  in  $G$ .

Rather than giving its standard definition, we prefer the following equivalent one<sup>9</sup>:

**Proposition 2.1** ([5, Proposition 1.17]). *For every graph class  $\mathcal{C}$ ,  $\text{asdim}(\mathcal{C}) \leq d$  if and only if there exists a function  $f : \mathbb{N} \rightarrow \mathbb{N}$ , called control function, such that for every  $G \in \mathcal{C}$  and integer  $r \geq 1$ ,  $G^r$  is  $(d+1)$ -colorable  $f(r)$ -bounded in  $G$ .*

Any associated control function  $f$  that makes  $\text{asdim}(\mathcal{C}) = d$  is also called a  $d$ -dimensional control function of  $\mathcal{C}$ . The Assouad-Nagata dimension of  $\mathcal{C}$  has a similar definition, except that the control function must be linear. A crude example is that paths have Assouad-Nagata dimension 1, because, for each  $r \geq 1$ , it suffices to alternately color subsets of  $r$  consecutive vertices with two colors, see Fig. 1.

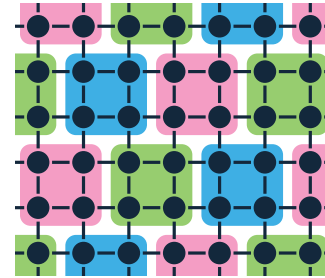
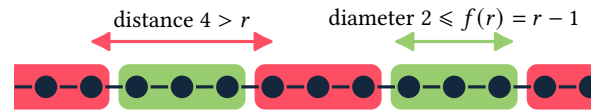
More generally,  $d$ -dimensional grids have a  $d$ -dimensional control function  $f(r) = d \cdot (r-1)$ .

Trees, and more generally graph classes of bounded treewidth (resp. layered treewidth) have asymptotic dimension 1 (resp. 2). Planar graphs, and more generally the classes of  $H$ -minor-free graphs (for any fixed  $H$ ) have asymptotic dimension 2 as shown by [5]: the dependency in  $H$  only shows in the control function  $f$ . Dense graph classes may also have small asymptotic dimension. For

<sup>7</sup>Asymptotic dimension was originally defined on metric spaces by Gromov [21] in the field of geometric group theory.

<sup>8</sup>However, those notions follow a different philosophy. In the study of asymptotic dimension, we want to minimize the dimension. In the study of sparse partitions and covers, the main goal is to minimize a function of the dimension and the diameter of the sets.

<sup>9</sup>In the original Proposition 1.17 of [5],  $G^r$  is  $(d+1)$ -colorable  $f(r)$ -bounded in  $G^r$ , instead of in  $G$ . By following this original definition, the function  $f$  is not a control function, whereas in Proposition 2.1 it is.



**Figure 1:** An example that shows that  $P^r$  is 2-colorable  $(r-1)$ -bounded in  $P$  for  $r = 3$  (on the left), and an example of a 3-coloring of grids for  $r = 2$  and  $f(r) = 2(r-1) = 2$  (on the right).

example, it is sufficient that there is a quasi-isometry into a class having small asymptotic dimension<sup>10</sup>. The dense class of chordal graphs, being quasi-isometric to the class of trees, has asymptotic dimension 1.

Intuitively, each connected component of a color class in  $G^r$  can be viewed as a “simple” component of the graph, that we already understand well. In the setting of LOCAL algorithms with constant round complexity  $r/2$ , each vertex can observe only its distance- $(r/2)$  neighborhood. If the underlying graph class has asymptotic dimension  $d$ , this neighborhood intersects at most  $d+1$  such simple parts. As a result, we can derive strong approximation guarantees, since each vertex’s view of the graph remains constrained and conceptually simple. We observe that all our algorithms make use of the control function  $f$ , but not for any particular pre-computing coloring or partitioning, so that the asymptotic dimension  $d$  is actually used only in analysis of the approximation ratio.

## 3 MDS on Bounded Euler Genus Graphs

This section explains how asymptotic dimension helps us devise approximation algorithms for MDS on Euler-genus- $g$  graphs. It also serves as an introduction to the method of cutting and motivates the later meta-theorems. The following algorithm for planar graphs will serve as a black-box building block for the more involved algorithm of Theorem 3.2.

**Proposition 3.1.** *There is a 205-approximation LOCAL algorithm  $A$  for MINIMUM DOMINATING SET in planar graphs with round complexity 6.*

To present the algorithm (cf. Algorithm 1) we first need to define the “best” minimum dominating sets of a labelled graph  $H$ . Among all dominating sets, we first discard those which contain some

<sup>10</sup>It is conjectured in [5] that, even more generally, any graph class forbidding some graph as a *fat* minor should also have asymptotic dimension at most 2.

$v$  for which there exists  $w$  with  $N[v] \subseteq N[w]$ , and among the remaining ones, we declare “best” the one lexicographically smallest considering the labels of the vertices.

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**Algorithm 1** The algorithm for dominating set on planar graphs.

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**Require:** A planar graph  $G$ .

**Ensure:** A dominating set  $D$  of  $G$  such that  $|D| \leq 205 \cdot \text{MDS}(G)$ .

- 1:  $D \leftarrow \emptyset$
  - 2: Each vertex  $u$  computes  $G[N^5[u]]$  and all minimum dominating sets of  $N^4[u]$  in  $G$ . Among those sets,  $u$  selects the “best” one as  $D_u$ .
  - 3: Each vertex  $u$  picks the vertex  $v_u$  of smallest label in  $D_u \cap N[u]$ , and adds it to  $D$ .
- 

By construction, Algorithm 1 outputs a dominating set  $D$  within 6 rounds. In fact, Algorithm 1 is very natural in the sense that in order to compute a global dominating set we use local dominating sets. The proof on the approximation factor is given in the full version [6, Appendix C.1]. We extend Algorithm 1 to genus  $g$  graphs by dealing separately with local subgraphs of  $G$  which are planar and which are not planar. For a graph  $G$ , the  $T$ -error set of  $G$  w.r.t. planarity is the vertex set defined by  $X = \{u \in V(G) \mid G[N^T[u]] \text{ is not planar}\}$ .

**THEOREM 3.2.** *There exists a function  $C$  such that for any  $g$ , there is a 616-approximation LOCAL algorithm for MINIMUM DOMINATING SET in Euler genus- $g$  graphs with round complexity  $C(g)$ .*

**PROOF.** Suppose  $G$  has Euler genus  $g$  and let  $f$  be a 2-dimensional control function of the class of Euler genus- $g$  graphs. A 2-hop in  $X \subset V(G)$  is a sequence of vertices  $x_1, \dots, x_k$  in  $X$  such that consecutive vertices are at distance at most two in  $G$ .

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**Algorithm 2** The algorithm for dominating set on bounded genus graphs.

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**Require:** A graph  $G$  of genus  $\leq g$ .

**Ensure:** A dominating set  $D$  of  $G$  such that  $|D| \leq 616 \cdot \text{MDS}(G)$ .

- 1: If  $N^{f(10)+5}[u]$  is planar,  $u$  runs Algorithm 1 on the corresponding locally planar subgraph.  
Let  $X$  be the set of undominated vertices after step 1.
  - 2: If  $u \in X$ ,  $u$  computes lexicographically smallest minimum set  $D_{X'}$  in  $G$  which dominates the set  $X' \subseteq X$  of vertices reachable from  $u$  with a 2-hop in  $X$ . It adds  $N[u] \cap D_{X'}$  to  $D$ .
- 

**Claim 3.3.** *The round complexity is some constant  $C(g)$  depending only on  $g$ .*

**PROOF OF CLAIM.** It suffices to show that there is no 2-hop of weak diameter  $\geq 2(f(10) + 6)g$  of vertices in  $X$ . Suppose such a 2-hop on vertices of  $X$  exists. Take two vertices  $x$  and  $y$  on the 2-hop which are at least  $2(f(10) + 6)g$  apart. Note that for any  $i < 2(f(10) + 6)g$  there exists a vertex on the 2-hop at distance  $i$  or  $i+1$  from  $x$ . Hence there exist  $u_1, \dots, u_{g+1} \in X$  where  $u_i$  is at distance  $2(f(10) + 6)(i - 1)$  or  $2(f(10) + 6)(i - 1) + 1$  from  $x$ . Therefore, the sets  $N^{f(10)+5}[u_1], \dots, N^{f(10)+5}[u_{g+1}]$  are pairwise disjoint and they are non-planar, so the genus of the graph is at least  $g + 1$  by

additivity (cf. [32, Theorem 4.4.3]). As the genus is at most  $g$ , this is a contradiction.  $\diamond$

The coloring of the asymptotic dimension gives us color classes  $C_1, C_2, C_3$ , such that:

- $C_i = B_1^i \cup \dots \cup B_{j_i}^i$  with<sup>11</sup>  $\text{diam}_G(B_j^i) \leq f(10)$  for all  $1 \leq j \leq j_i$ , and
- for any  $j \neq \ell$ , we have  $B_j^i \neq B_\ell^i$ . More precisely,  $B_j^i$  and  $B_\ell^i$  are at distance at least 11.

*The Cutting.* Let  $Y_G$  be a minimum dominating set of  $G$ . Let  $G' = G[N[Y']]$  where  $Y' = Y_G \cap N^5[B_j^i]$ . Note that  $G'$  contains  $N^4[B_j^i]$  and  $|\text{MDS}(G')| \leq |Y_G \cap N^5[B_j^i]|$ .

*The Analysis Outside of the Error Set.* Note that  $G'$  either contains only vertices from  $X$  or contains at least one vertex which is not in  $X$  and is therefore planar. Hence, vertices in  $B_j^i \setminus X$  choose at most  $205 \cdot |\text{MDS}(G')| \leq 205 \cdot |Y_G \cap N^5[B_j^i]|$  vertices as their neighbourhood of distance 5 in  $G$  and  $G'$  is the same. Vertices not in  $X$  choose at most  $205|Y_G|$  vertices in each color class. Across all color classes, this adds up to at most  $3 \cdot 205|Y_G|$ .

*The Analysis With Non-Empty Error Set.* Let  $X_1, \dots, X_k$  be the 2-hop components. Note that for two distinct 2-hop components  $X_i$  and  $X_j$ , the neighbourhoods  $N[X_i]$  and  $N[X_j]$  do not intersect. As  $|D_{X_i}| \leq |Y_G \cap N[X_i]|$ , the vertices in  $X$  choose at most  $|Y_G|$  vertices.

In total, at most  $3 \cdot 205|Y_G| + |Y_G| = 616 \cdot |\text{MDS}(G)|$  vertices are added to  $D$ : the algorithm is a 616-approximation.  $\square$

In fact, if Algorithm 1 gives an approximation ratio  $\alpha$ , then the above algorithm for genus  $g$  graphs is a  $(3\alpha + 1)$ -approximation algorithm. One might now ask whether any algorithm for MDS on planar graphs can be turned into one for genus  $g$  graphs. We show that this is indeed possible. In order to do so, we need to show that the cutting step and its analysis work for any MDS algorithm for planar graphs. We delve into this deeper in the next section.

## 4 Meta-Theorems

In this section, we present our meta-theorems, which take as input a LOCAL approximation algorithm for a graph class  $\mathcal{C}$  and returns a LOCAL approximation algorithm for a broader class  $\mathcal{D}$  that locally looks like graphs in  $\mathcal{C}$ , possibly with some local exceptions.

Meta-theorems using black-box LOCAL algorithms might be difficult to state as there are some subtle issues with vertex identifiers (see for instance [16, 17, 20]). The LOCAL model assumes the input graph  $G$  has unique  $O(\log n)$ -bit vertex labels, i.e., with identifiers polynomial in  $G$ . In particular, running a LOCAL algorithm  $A$  on only a small part  $H$  of  $G$  might fail because: (1) vertex labels in  $H$  may not be polynomial in  $H$  anymore, and (2) algorithm  $A$  may require polynomial-size identifiers to work properly. Thus, Meta-Theorem I assumes that the black-box algorithm  $A$  does not require polynomial identifiers. This captures all order-invariant algorithms<sup>12</sup>. In contrast, Meta-Theorem II and Meta-Theorem III allow  $A$  to use polynomial identifiers, but impose restrictions on the

<sup>11</sup>We denote by  $\text{diam}_G(X)$  the largest distance in  $G$  between any two vertices of  $X$ .

<sup>12</sup>A model where vertices have unique labels, but the output of  $A$  is invariant under relabelings preserving the label order [20].

optimization problem. Due to space constraints, Meta-Theorem II & III are presented in Section 5.

*From Locally- $\mathcal{C}$  to Locally Nice Graphs.* Given a graph class  $\mathcal{C}$ , we say that a graph class  $\mathcal{D}$  is *T-locally- $\mathcal{C}$*  if for all  $G \in \mathcal{D}$  and  $u \in V(G)$ ,  $G[N^T[u]] \in \mathcal{C}$ .

This definition can already capture many graph classes, such as graphs of girth at least  $2k+2$ : those are just graphs that are  $k$ -locally- $\mathcal{T}$ , where  $\mathcal{T}$  is the class of trees. Similarly, graphs embedded on a surface with edge-width<sup>13</sup> at least  $2k+2$  are  $k$ -locally- $\mathcal{P}$ , where  $\mathcal{P}$  is the class of planar graphs. However, our meta-theorem captures broader classes of graphs as it can support local exceptions.

Given a graph class  $\mathcal{D}$  and  $G \in \mathcal{D}$ , the *T-error set* of  $G$  w.r.t.  $\mathcal{C}$  is the vertex set defined by  $X = \{u \in V(G) \mid G[N^T[u]] \notin \mathcal{C}\}$ . In other words, this is the set of vertices that are witnesses for  $G$  not being *T-locally- $\mathcal{C}$* . Clearly, if  $G$  has no *T-errors*, that is  $X = \emptyset$ , then  $G$  is *T-locally- $\mathcal{C}$* . Given a  $\rho$ -local<sup>14</sup> problem  $\Pi$ , the class  $\mathcal{D}$  is *T-locally  $\delta$ -nice* w.r.t.  $\mathcal{C}$  and  $\Pi$ , if the maximum weak diameter in any  $G \in \mathcal{D}$  of any connected component of  $G[N^{2\rho}[X]]$  is at most  $\delta$ , where  $X$  is the *T-errors* of  $G$  w.r.t.  $\mathcal{C}$ . This means that the set of *T-errors* can be covered by far-away bounded-radius balls of  $G$ . Bounded Euler genus graphs form a locally nice class of graphs w.r.t.  $\mathcal{P}$  (see the full version for the proof [6, Proposition C.11]).

*Uniform Approximation.* Our meta-theorems are inspired by the intriguing property [7, Proposition 3.1] that any approximate algorithm  $A$  for MDS is going to perform well in a target graph class  $\mathcal{D}$  that is locally- $\mathcal{C}$ , as long as  $\mathcal{D}$  has small asymptotic dimension, and  $A$  makes "good local choices" in  $\mathcal{C}$ . More precisely, a LOCAL algorithm  $A$  is a *k-uniform  $\alpha$ -approximation* in  $\mathcal{C}$ , if for all  $G \in \mathcal{C}$  and  $S \subseteq V(G)$ ,  $|A(G) \cap S| \leq \alpha \cdot \text{MDS}(G, N^k[S])$ , where  $A(G)$  is the set returned by  $A$  when run on  $G$ , and  $\text{MDS}(G, X)$  is the minimum size of a subset of  $V(G)$  dominating vertices of  $X$  in  $G$ . Unless explicitly said, we will assume for convenience that  $k \geq 1$ , as otherwise only very restricted graph classes  $\mathcal{C}$  can support 0-uniform constant-approximation algorithms for MDS (cf. full version [6, Proposition D.1]).

We can now state properly the fact that any  $k$ -uniform approximate algorithm that performs well for  $\mathcal{C}$ , performs well for a locally- $\mathcal{C}$  class of bounded asymptotic dimension. It can be shown that the bounded asymptotic dimension assumption cannot be removed, even for MDS<sup>15</sup>.

**Proposition 4.1** ([7, Proposition 3.1]). *Let  $A$  be a  $k$ -uniform  $\alpha$ -approximation LOCAL algorithm for MINIMUM DOMINATING SET in a hereditary<sup>16</sup> class of graphs  $\mathcal{C}$  with round complexity  $r \geq 1$ . Let  $\mathcal{D}$  be a graph class with  $d$ -dimensional control function  $f$ , and that is<sup>17</sup>*

<sup>13</sup>The length of shortest noncontractible cycle in the embedded graph.

<sup>14</sup>A problem whose solution can be verified by checking all neighborhoods of radius  $\rho$ . See next page for a formal definition.

<sup>15</sup>This can be achieved by observing that a simple constant-round 3-approximation LOCAL algorithm exists for MDS in trees, but (high) girth- $g$  graphs do not admit a constant-round  $o(g)$ -factor approximation. The latter follows by combining Ramsey-theoretic arguments with the lower bound proof due to [20] based on  $2k$ -regular graphs of girth  $g$ , where vertices can be linearly ordered so that a positive fraction of them have pairwise isomorphic radius- $r$  neighbourhoods, for any  $r, k, g$ .

<sup>16</sup>A class of graphs closed under vertex deletion.

<sup>17</sup>The original statement claimed erroneously that  $\mathcal{D}$  being *t-locally- $\mathcal{C}$*  for  $t = f(2k+1)$  is enough, instead of the correct  $t = f(2k+2) + r$ . This has no impact, as in practice we use Proposition 4.1 in Corollary 4.4 only for  $k \leq r = O(1)$ .

*( $f(2k+2) + r$ )-locally- $\mathcal{C}$ . Then  $A$  is also an  $\alpha(d+1)$ -approximation algorithm on  $\mathcal{D}$ .*

We extend Proposition 4.1 in multiple ways. First, we allow  $\mathcal{D}$  to be locally nice, and secondly, we prove that this also applies to problems other than MDS. This means that, for example, any approximate algorithm  $A$  solving a well-behaved problem that performs well on planar graphs can be converted into an algorithm  $B$  that performs well on bounded genus graphs, as long as  $A$  is  $k$ -uniform on  $\mathcal{C}$ , and  $\mathcal{D}$  has small asymptotic dimension. Before giving the full statement of Meta-Theorem I, we define the class of problems we are interested in.

*LOCAL Optimization Problems.* As defined in [20], a *simple graph problem*  $\Pi$  is an optimization problem in which a feasible solution<sup>18</sup> is a subset of vertices or edges, and the goal is to optimize the size of a feasible solution. A simple graph problem  $\Pi$  is  *$\rho$ -local* if there is a LOCAL algorithm  $R$  with round complexity  $\rho$  that can recognize a feasible solution<sup>19</sup>. Algorithm  $R$  recognizes  $S$  as a feasible solution if and only if, whenever applied on  $G$  with  $S$  given,  $R$  returns true for all vertices of  $G$ . See Section B.1 of the full version [6] for more formal definitions. We are mainly interested in small locality problems, that is when  $\rho$  is constant, as clearly all simple graph problems are  $\rho$ -local for  $\rho$  large enough. A lot of well-known problems such as optimization variants of LCL problems [33]. This includes MINIMUM VERTEX COVER, MINIMUM DOMINATING SET, MAXIMUM INDEPENDENT SET, MAXIMAL MATCHING, and MINIMUM  $k$ -TUPLE DOMINATING SET. The distance- $d$  versions of these problems<sup>20</sup> are also  $d$ -local problems. However, MINIMUM CONNECTED DOMINATING SET is  $\Omega(n)$ -local<sup>21</sup>. Our framework includes problems such as RED-BLUE DOMINATING SET that can take vertex or edge input labels on the graph, and problems defined on arbitrarily large degree graphs.

In this work, we focus on subsets of vertices, but the techniques presented here extend to subsets of edges. In our setting, we restrict our attention to cases where the vertices do not have access to  $n$ , the size of the graph. For some problems, such as MINIMUM DOMINATING SET, this restriction can be relaxed<sup>22</sup>. However, we omit this case for brevity, since we are not aware of any constant-round algorithm for a  $\rho$ -local problem that requires  $n$ . As, in the LOCAL model with constant number of rounds, the literature mostly studies minimization problems, we also focus on those. Observe that many classically interesting problems like MAXIMUM INDEPENDENT

<sup>18</sup>Here, a feasible solution is any subset of vertices or edges that satisfies the constraints of the graph problem. Said differently, an admissible candidate solution.

<sup>19</sup>We do not require that the algorithm checks the optimality of the solution. If we require this, most interesting problems would require  $\rho = \Omega(n)$  to be  $\rho$ -local.

<sup>20</sup>E.g. for MINIMUM DOMINATING SET, every vertex has a dominating vertex in its distance- $d$  neighborhood.

<sup>21</sup>In a  $n$ -vertex cycle, no LOCAL algorithm  $R$  of round complexity  $< n/2$  can distinguish the situation where a feasible solution is  $S = V(G) \setminus \{u\}$  from the situation where  $S = V(G) \setminus \{u, v\}$  with two opposite vertices  $u, v$ .

<sup>22</sup>In full generality, this applies to all problems for which, for every  $n$ , there exists an  $n$ -vertex graph within the input graph class that has bounded diameter and admits a constant-size solution. For MDS on planar graphs, we can take a depth-1 tree.

SET and MAXIMUM MATCHING do not admit constant-time constant-ratio approximations (see [8] for references), even on the restricted graph class of paths<sup>23</sup>.

Given a  $\rho$ -local problem  $\Pi$ , a graph  $G$  and  $X \subseteq V(G)$ , we define  $\text{OPT}_{\Pi}(G, X)$  as the optimal size of a subset of vertices of  $G$  that is feasible on  $X$  (and not necessarily for vertices outside  $X$ ). We naturally extend the definition of  $k$ -uniform algorithm as follows: a LOCAL algorithm  $A$  is a  $k$ -uniform  $\alpha$ -approximation for  $\Pi$  in a class of graphs  $\mathcal{C}$ , if for all  $G \in \mathcal{C}$  and  $S \subseteq V(G)$ ,  $|A(G) \cap S| \leq \alpha \cdot \text{OPT}_{\Pi}(G, N^k[S])$ .

We can now state our strengthening of Proposition 4.1. Note that we further require (when the set of errors is not empty) that our problem is *additive*, that is, any superset of a solution feasible on  $X$  is still feasible on  $X$ . For instance, MDS is additive because adding vertices to any dominating set maintains a dominating set.

**THEOREM 4.2 (META-THEOREM I).** *Let  $A$  be some  $k$ -uniform  $\alpha$ -approximation LOCAL algorithm that does not require polynomial identifiers, for an additive  $\rho$ -local problem  $\Pi$  in a class of graphs  $\mathcal{C}$  with round complexity  $r$ , where  $\mathcal{C}$  is hereditary and stable by disjoint union. Let  $\mathcal{D}$  be a graph class of asymptotic dimension  $d$  with  $d$ -dimensional control function  $f$  that is  $T$ -locally  $\delta$ -nice w.r.t.  $\mathcal{C}$  and  $\Pi$ , where  $T = f(2k + 2\rho) + \max\{k + \rho, r\}$ . Then, there exists a  $\max\{k, \rho\}$ -uniform  $(\alpha(d + 1) + 1)$ -approximation algorithm  $B$  on  $\mathcal{D}$  with round complexity  $T + \delta + 2$ . Furthermore, if  $\mathcal{D}$  is  $T$ -locally  $\mathcal{C}$ , the approximation ratio is  $\alpha(d + 1)$  and the additivity condition can be dropped.*

*Cutability.* It may be difficult to prove that some algorithm is  $k$ -uniform. Instead, we provide a simple sufficient criterion for deciding if any constant-round constant-ratio approximation algorithm for  $\Pi$  is  $k$ -uniform.

A  $\rho$ -local problem  $\Pi$  is *cuttable* for a hereditary graph class  $\mathcal{C}$  with parameter  $\beta > 0$  if for all  $G \in \mathcal{C}$  and  $X \subseteq V(G)$ , there exists some  $Y \supseteq X$  such that  $\text{OPT}_{\Pi}(G[Y]) \leq \beta \cdot \text{OPT}_{\Pi}(G, X)$ . Informally, this asks that  $X$  can be augmented into a set  $Y$  (if not the whole vertex set) so that the graph induced by this bigger set  $Y$  has an optimal solution somewhat smaller than the optimal feasible solution for  $X$ .

For instance, MDS is cuttable with parameter  $\beta = 1$  (see Claim C.3 in the full version [6]), as is the distance- $d$  version of MDS (see full version [6, Proposition C.6]). We prove in the full version [6, Section C.6] the fundamental property that every constant-round constant-ratio approximation for a cuttable problem is also  $k$ -uniform (as long as the algorithm does not require polynomial identifiers). Therefore, MDS admits uniform approximation algorithms. This means we can actually use Meta-Theorem I for any cuttable problem without the  $k$ -uniform assumption. Actually, for MDS, we can show that this property holds even if the algorithm requires polynomial identifiers (cf. full version [6, Proposition C.2]).

Recall that Euler genus- $g$  graphs have asymptotic dimension at most 2, and are locally nice (cf. full version [6, Proposition

C.11]). Thus, Meta-Theorem I implies that any constant-round  $\alpha$ -approximation for MDS gives a  $C(g)$ -round  $(3\alpha + 1)$ -approximation for graphs of Euler genus  $g$ .

The best known ratio for MDS in planar graphs is  $\alpha = 11 + \varepsilon$  due to Heydt et al. [22]. Therefore,

**Corollary 4.3.** *For any  $\varepsilon > 0$ , for any  $g$ , there is a  $k(\varepsilon)$ -uniform  $(34 + \varepsilon)$ -approximation LOCAL algorithm for MINIMUM DOMINATING SET in Euler genus- $g$  graphs with round complexity  $C(\varepsilon, g)$  for some functions  $C$  and  $k$ .*

We can even go one step further by applying Meta-Theorem I to the uniform approximation LOCAL algorithm derived from Corollary 4.3, and the class  $\mathcal{C}$  of Euler genus- $g$  graphs. We immediately obtain the following, where 34.01 can be thought of as  $34 + \varepsilon$ .

**Corollary 4.4.** *Let  $\mathcal{C}$  be the class of Euler genus- $g$  plus graphs, and  $\mathcal{D}$  be a graph class of asymptotic dimension  $d$  that is  $r$ -locally- $\mathcal{C}$  for some  $r$  large enough. Then, there is a  $34.01(d + 1)$ -approximation LOCAL algorithm for MINIMUM DOMINATING SET in  $\mathcal{D}$  with round complexity  $C(d, g, r)$  for some function  $C$ .*

Actually, Corollaries 4.3 and 4.4 hold for the larger class of Euler genus- $g$  plus  $a$ -apex graphs, i.e. graphs obtained from a graph of Euler genus  $g$  where at most  $a$  vertices are allowed to have an arbitrary neighborhood. These graphs exclude  $K_{3+a, 2g+3+a}$  as minor (as, without apices, the graphs exclude  $K_{3, 2g+3}$ ), and thus have asymptotic dimension 2 as well. They are also locally nice, since  $a$  apices cannot create short-cuts longer than  $2a$ . So, the approximation ratio is preserved, whereas the round complexity depends on  $a$ .

Moreover, our results can be applied to get approximation algorithms on graphs of bounded Euler genus for *all* other minimization problems in the Distributed Computing literature that admit a constant-round, constant-ratio approximation on planar graphs (for a comprehensive list of such problems, see the survey of Feuilloley [15]).

For instance, let us focus on the MINIMUM  $k$ -TUPLE DOMINATING SET problem (studied extensively in [37]), a generalization of MINIMUM DOMINATING SET. In this variant, the goal is to find the smallest subset of vertices such that every vertex outside the set is adjacent to at least  $k$  vertices within it. The classical MDS problem corresponds to the special case  $k = 1$ . For planar graphs, this problem admits a constant-round  $k/(k - 2)$ -approximation algorithm [10] for  $k > 2$ , and a 6-approximation [14] for  $k = 2$ . Both algorithms do not require polynomial identifiers.

Using the fact that MINIMUM  $k$ -TUPLE DOMINATING SET is cuttable and additive (cf. full version [6, Proposition C.5]), that Euler genus- $g$  graphs are locally nice (cf. full version [6, Proposition C.11]), and by applying Meta-Theorem I (and Proposition C.7 of the full version [6]), we get the following:

**Corollary 4.5.** *For every  $k \geq 2$ , there exists a  $C_1(k)$ -uniform  $\alpha$ -approximation LOCAL algorithm for MINIMUM  $k$ -TUPLE DOMINATING SET in Euler genus- $g$  graphs with round complexity  $C_2(g, k)$ , for some functions  $C_1, C_2$ , where  $\alpha = 19$  if  $k = 2$  and  $\alpha \leq 3k/(k - 2) + 1$  otherwise.*

For MINIMUM CONNECTED DOMINATING SET [37], which is an example of non-local problem, we can obtain a constant-round approximation with ratio  $O(\sqrt{g})$  in Euler genus- $g$  graphs by applying the "connected" reduction from MDS in bounded expansion

<sup>23</sup>The rough intuition for this is that on paths, any constant-time algorithm run on a vertex far away from the end of the path sees the same neighborhood, and therefore will output the same result on every vertex or edge. The only way to comply with the problem description, unlike minimization problem like MDS, is to not take any vertex in the independent set (or any edge in the maximum matching), at least in the middle of the path. This leads to an arbitrarily bad approximation ratio.

graphs due to [1, Theorem 5.8]. This gives an approximation ratio of  $2\alpha\nabla_1 \leq 68\sqrt{3g/2} + O(1)$ , where  $\alpha = 34 + \varepsilon$  is the best approximation ratio for MDS in Euler genus- $g$  graphs (Corollary 4.3), and  $\nabla_1 \leq \sqrt{3g/2} + 3$  (see full version [6, Proposition D.3]) is the expansion of Euler genus- $g$  graphs. We leave open the question of improving the ratio to a constant independent of the genus.

## 5 When the Black-Box Algorithm Requires Polynomial Identifiers

One of the key propositions (see Proposition C.7 in the full version [6]) applies to lots of optimization problems. However, it does not fulfil the polynomial identifiers requirement of the LOCAL model. We would like to have the same statement to apply in the LOCAL model with polynomial identifiers with the same assumptions. However, assigning new identifiers to vertices while maintaining control of the output of the algorithm is very challenging. Ramsey-type techniques (for example, for finding lower bounds on the approximation ratio on particular graph classes [23]) can be useful for this, but only on very particular graphs: ones where each neighborhood is similar. In general graphs, to the best of our knowledge, we do not know how to adapt this technique. Thankfully, for locally- $\mathcal{C}$  classes, where the set of errors is empty, we can weaken the statement of Proposition C.7 (see full version [6]) in a way that we can still apply a variation of Meta-Theorem I afterwards. The trick is to modify the coloring given by the asymptotic dimension so that all color classes are almost balanced in size (see full version [6, Lemma C.12]). With this technique, we just need to apply uniformity on sets that are large. Formally, LOCAL algorithm A is an  $\varepsilon$ -weakly  $k$ -uniform  $\alpha$ -approximation for  $\Pi$  in a class of graphs  $\mathcal{C}$ , if for all  $G \in \mathcal{C}$  and  $S \subseteq V(G)$  such that  $|S| \geq \lfloor \varepsilon |V(G)| \rfloor$ , we have  $|A(G) \cap S| \leq \alpha \cdot \text{OPT}_\Pi(G, N^k[S])$ . We say that the  $d$ -local problem  $\Pi$  is  $\varepsilon$ -weakly uniformizable on  $\mathcal{C}$  if for every integer  $d > 0$  and  $r$  and any real  $\alpha$ , there exists some integers  $k, r'$  and real  $\beta$  such that for every  $r$ -round  $\alpha$ -approximation A of  $\Pi$  on  $\mathcal{C}$  in constant time, there exists some  $r'$ -round LOCAL algorithm B that is a  $\varepsilon$ -weakly  $k$ -uniform  $\beta$ -approximation of  $\Pi$  on  $\mathcal{C}$  in constant time. The function  $(r, \alpha) \mapsto (k, \beta)$  is called the (weak) *binding* function.

It is a corollary of one of the proofs (of Proposition C.7 in the full version [6]) that any cuttable problem is weakly uniformizable, in the LOCAL model this time (see full version [6, Corollary C.10]). Therefore, we only need to work (see Lemma C.12 in the full version [6]) on color classes of linear size (which preserves the polynomial-ness of identifiers), and prove the following.

**THEOREM 5.1 (META-THEOREM II).** *Let  $d, k$  be integers  $\alpha > 0$  a real, A be a  $\frac{1}{d+1}$ -weakly  $k$ -uniform  $\alpha$ -approximation LOCAL algorithm for a  $\rho$ -local problem  $\Pi$  in a class of graphs  $\mathcal{C}$  with round complexity  $r$ , where  $\mathcal{C}$  is hereditary and stable by disjoint union. Let  $\mathcal{D}$  be a graph class of asymptotic dimension  $d$  with  $d$ -dimensional control function  $f$ , that is  $T$ -locally- $\mathcal{C}$  for  $T = f(6k + 6\rho) + 4k + 4\rho + \max\{k + \rho, r\}$ . Then A is a  $k$ -uniform  $\alpha(d+1)$ -approximation algorithm on  $\mathcal{D}$ .*

The proof of this can be found in the full version [6, Section C.11].

When the error set is not empty, we require further assumptions. For a discussion on why those assumptions are required, we refer to the full version [6, Section B.4]. A problem  $\Pi$  is called  $\omega$ -paddable

if for any integer  $N$ , there exists some  $G \in \mathcal{C}$  on at least  $N$  vertices such that  $\text{OPT}_\Pi(G) \leq \omega$ . We also add a small property that the problem needs to satisfy: a  $d$ -local problem  $\Pi$  is *dense* in  $\mathcal{C}$  if there exists some unbounded function  $g$  that for any  $G \in \mathcal{C}$  of diameter  $D$ , we have  $\text{OPT}_\Pi(G) \geq \beta \cdot D$ . We require that<sup>24</sup>  $\liminf f = +\infty$ . This essentially mean that there are no large (in diameter) portion of the graph with no vertices of the solution; every vertex is close to a vertex of the solution. Under the both additional assumptions of paddability and density, we can prove an analogue of Meta-Theorem II. See Meta-Theorem III (Theorem C.13 in the full version [6]) for the full statement.

*Summary Table of the Results.* Here is a simplified table of the assumptions necessary to apply our meta-theorem that transform a constant-round constant-factor approximation algorithm A on the class  $\mathcal{C}$  to another constant-round constant-factor approximation algorithm on the class  $\mathcal{D}$ . We assume that  $\mathcal{C}$  is hereditary and stable by disjoint union, and that  $\mathcal{D}$  is of bounded asymptotic dimension.

	$T$ -error set is empty ( $\mathcal{D}$ is locally- $\mathcal{C}$ )	$T$ -error set is not empty ( $\mathcal{D}$ is locally nice)
<i>A does not assume polynomial IDs</i>	$\Pi$ is cuttable (Meta-Theorem I and Proposition C.7 [6])	$\Pi$ is cuttable and additive (Meta-Theorem I and Proposition C.7 [6])
<i>A assumes polynomial IDs</i>	$\Pi$ is cuttable (Meta-Theorem II and Corollary C.10 [6])	$\Pi$ is cuttable, additive, paddable and dense <sup>25</sup> (Meta-Theorem III and Corollary C.10 [6])

**Table 1: Simplified table of the assumptions necessary to apply Meta-Theorem I, Meta-Theorem II and Meta-Theorem III, along with their references.**

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<sup>24</sup>It is possible to get rid of this property if one adds that the graphs in the definition of  $\omega$ -paddable also have bounded diameter. See the full version for more details [6].

<sup>25</sup>It is possible to remove the dense condition, at the expense of making the paddable condition stricter. See Section 5.

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