

# Routing in Networks with Low Doubling Dimension

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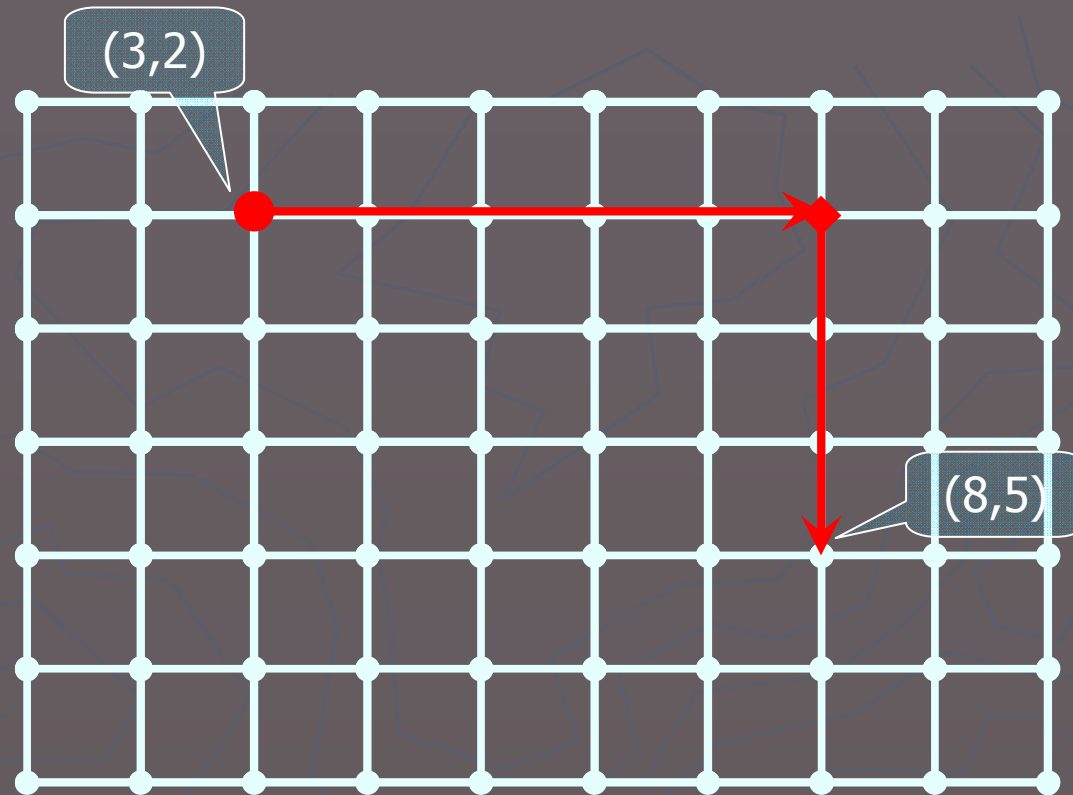
# The Compact Routing Problem

**Input:** a weighted network  $G$

**Output:** a *routing scheme* for  $G$

A routing scheme allows any source node to route messages to any destination node, given the destination's network identifier.

# Ex: Grid with X,Y-coordinates



Routes are constructed  
in a distributed manner

... according to some local  
routing tables (or routing algorithms)

# Quality & Complexity Measures

## Time vs. Space

- ▶ Near-shortest paths:

$$|\text{route}(x,y)| \leq \text{stretch} \cdot d_G(x,y)$$

- ▶ Size of headers and local routing tables
- ▶ **Goal:** constant stretch & polylog size tables

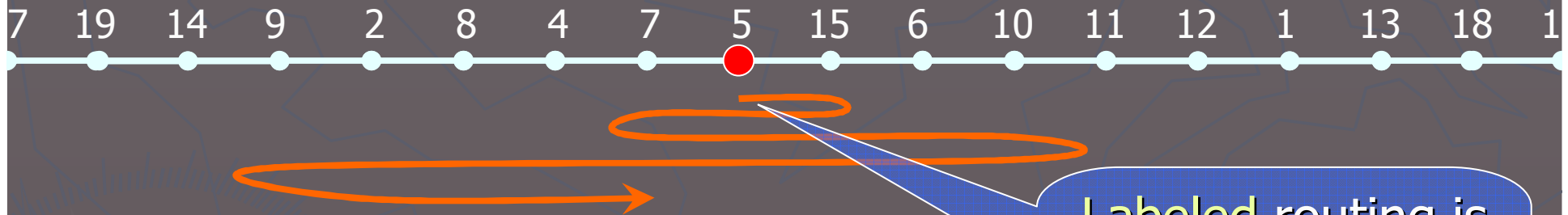
# Labeled vs. Name-independent Routing Schemes

- ▶ Name-independent: Node identifiers are chosen by an adversary (the input is a graph with the IDs)
- ▶ Labeled: Node IDs can be chosen by the designer of the scheme (as a routing *label* whose length is a parameter)

# ... in a Path

- ▶ Name-independent: Fixed IDs in  $\{1, \dots, n\}$

Routing from 5 to some target  $t$



Labeled routing is trivial! stretch 1 with  $O(1)$  space

- ▶ Stretch 9 with  $O(1)$  space [BYCROSS]
- ▶ Stretch  $1+\epsilon$  with  $\text{polylog}(n)$  space [AM05]
- ▶ Stretch 1 implies  $\Omega(n)$  bit space

# Overview of Compact Routing

**For arbitrary weighted networks**

stretch:  $\sim k$     space/node:  $\sim n^{1/k}$

( $n$ =#of nodes, integer parameter  $k > 0$ )

In more details: LABELED case

[Thorup-Zwick STOC+SPAA '01]

$\tilde{O}(n^{1/k})$  space for stretch  $4k-5$ ,  $k \geq 2$

(essentially optimal trade-off / stretch  $2k-1$ ?)

# Overview of Compact Routing

In more details: NAME-INDEPENDENT case  
(the most difficult variant)

[Awerbuch-Peleg FOCS '90]

$\tilde{O}(n^{1/k} \log \Delta)$  space for stretch  $O(k^2)$  where  $\Delta$   
is the aspect ratio of the network

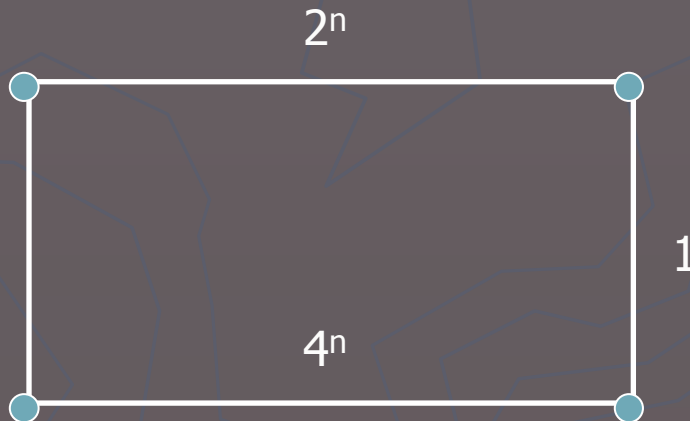
$$\Delta = \max d(u,v) / \min d(u,v) \quad (u \neq v)$$



# Overview of Compact Routing

Aspect ratio might be very large

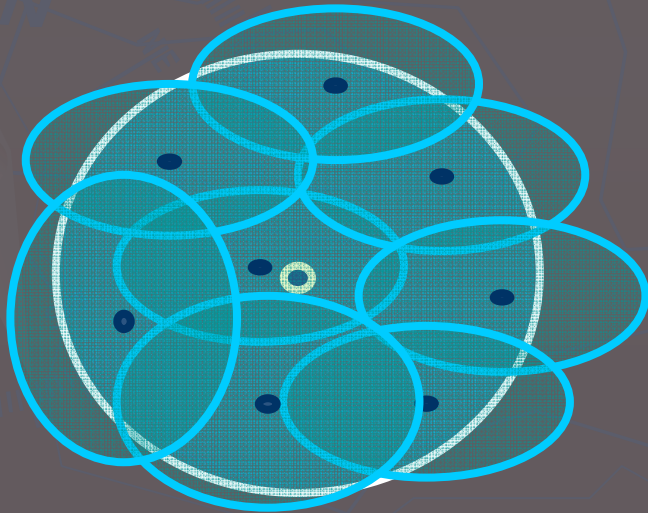
$\Delta = 2^n$  is possible!



**Scale-Free** routing scheme = size of tables and headers are independent of  $\Delta$

# Doubling Dimension

**[Definition]** A network (or a metric) has doubling dimension  $\alpha$  if every radius- $2r$  ball can be covered by  $\leq 2^\alpha$  radius- $r$  balls



- Euclidean networks have  $\alpha=O(1)$
- Include bounded growing networks
- Robust notion

# Doubling Dimension

## ► LABELED

Talwar (STOC '04)

stretch:  $1+\varepsilon$     space:  $O(1/\alpha\varepsilon)^\alpha \log^{2+\alpha} \Delta$

Chan et al. (SODA '05)

stretch:  $1+\varepsilon$     space:  $(\alpha/\varepsilon)^{o(\alpha)} \log n \log \Delta$

Slivkins (PODC '05)

stretch:  $1+\varepsilon$     space:  $(1/\varepsilon)^{o(\alpha)} \log n \log n \log \log \Delta$

**This paper**

stretch:  $1+\varepsilon$     space:  $(1/\varepsilon)^{o(\alpha)} \log^3 n$

# Doubling Dimension

## ▶ NAME-INDEPENDENT

Awerbuch-Peleg (FOCS '90)

stretch:  $O(\log^2 n)$     space:  $O(\log \Delta \log^2 n)$

Konjevod et al. (PODC '06)

stretch:  $9+\varepsilon$     space:  $(2+1/\varepsilon)^{o(\alpha)} \log^2 \Delta \log n$

**This paper**

stretch: 64    space:  $2^{O(\alpha)} \log^4 n$

# Doubling Dimension

- ▶ Also in this paper: Lower bounds

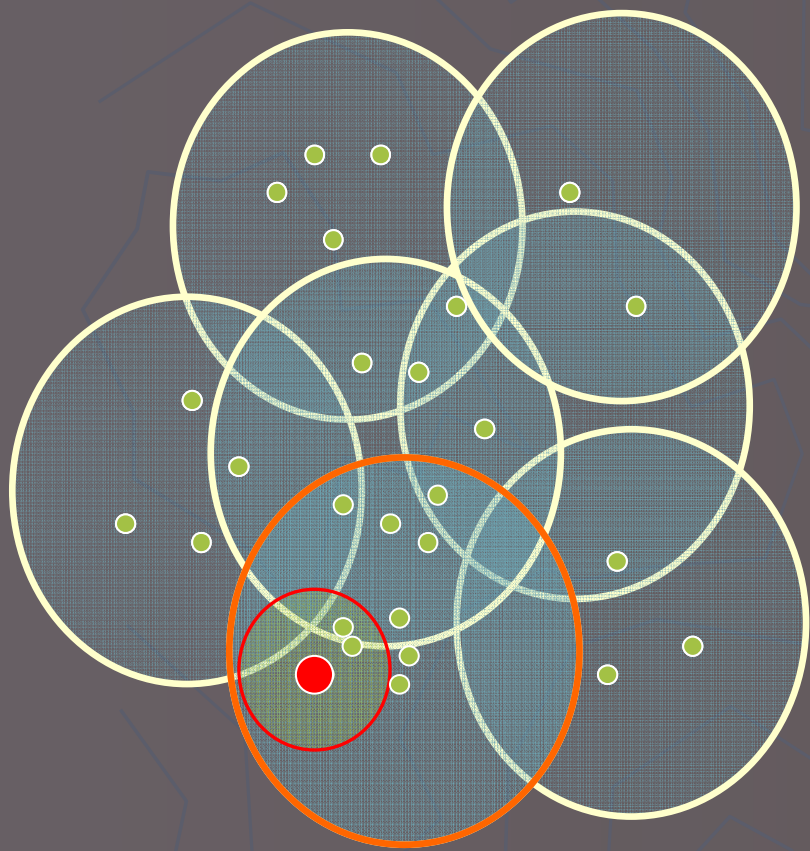
(Name-independent): If the stretch  $< 3$  then the space must be  $\Omega(\alpha n)$

(Labeled): If the stretch = 1 then the space must be  $\Omega(\sqrt{n})$  even for bounded growth networks

# Sparse Partition

**[Lemma]** For weighted network  $G$  of doubling dimension  $\alpha$  and every  $\rho > 0$ , one can construct (in polynomial time) a collection of “clusters”  $\mathcal{H}$  (connected subgraphs) of  $G$  such that:

1. [cover] Every radius- $\rho$  ball centered at  $u$  is contained in some cluster  $H \in \mathcal{H}$
2. [sparse] Every  $u$  belongs to at most  $4^\alpha$  clusters of  $\mathcal{H}$
3. [small radius]  $\forall H \in \mathcal{H}, \text{radius}(H) \leq 2\rho$



# Intuition: Try & Fail

Design a (name-independent) routing scheme for distance at most  $\rho$  nodes such that:

For any source  $s$  and target  $t \in G$

1. If  $t$  is at distance  $\leq \rho$  from  $s$ , then  $t$  is discovered after a route of length  $O(\rho)$
2. If  $t$  is at distance  $> \rho$  from  $s$ , a negative answer is reported back to  $s$  after a walk of length  $O(\rho)$

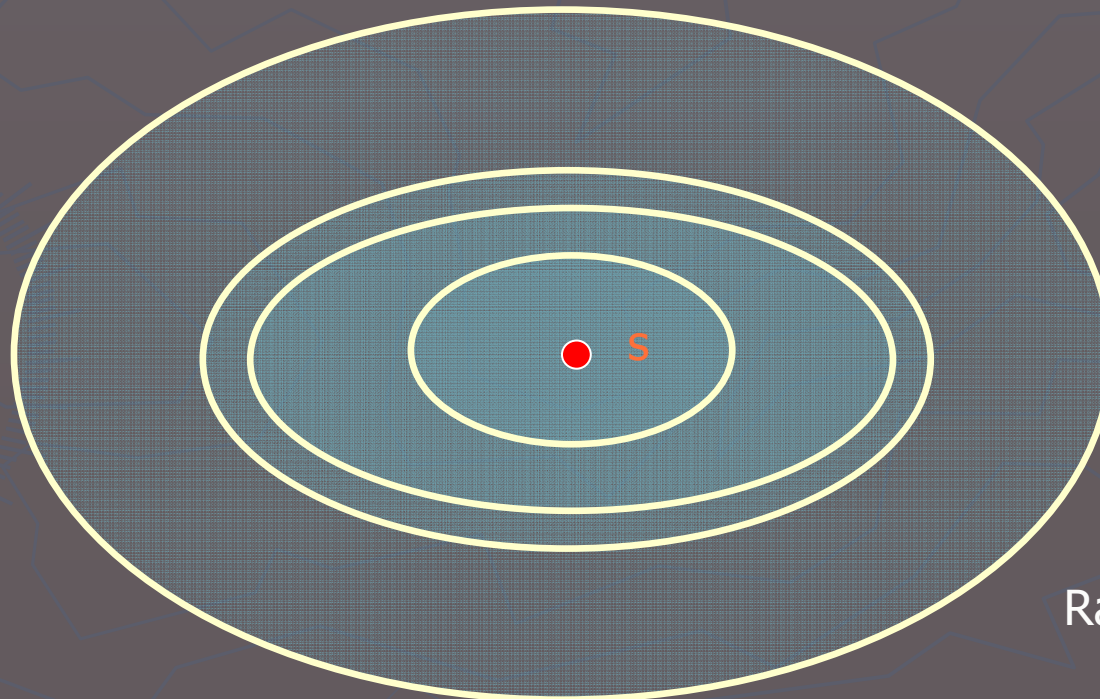
$\Rightarrow$  Trying with  $\rho = 1, 2, 4, \dots, 2^i \dots$ , any  $t$  will be found with a **constant** stretch factor and with an increasing factor of  $\log \Delta$  on the space.



# Removing the “ $\log \Delta$ ” factor

Construct a sparse partition as before, but store routing only for  $\log n$  levels instead of  $\log \Delta$

⇒ Dense-Sparse Partition

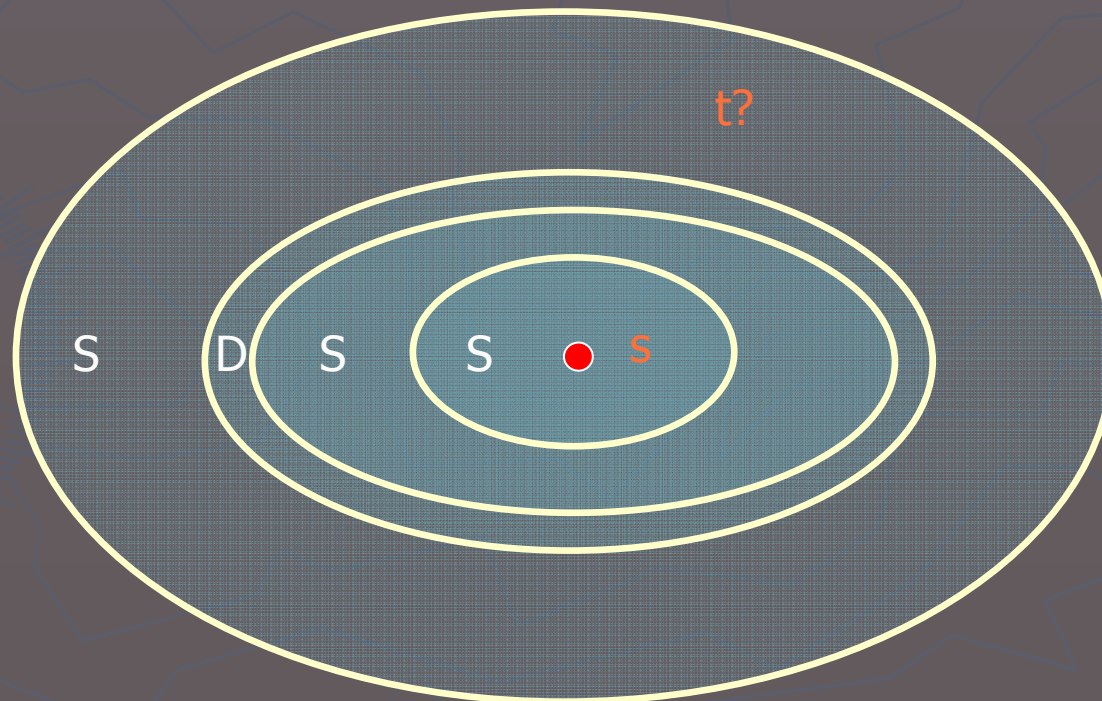


Radius- $2^i$  clusters  
for a node  $s$

# Removing the “log $\Delta$ ” factor

RANGE=levels that at least double the *volume* of the balls (at most logn ranges)

Dense range condition: if increasing the radius by a factor at most  $2^5$  the volume at least double



# Removing the “ $\log \Delta$ ” factor

Searching a target  $t$  from  $s$  in  $s'$  clusters:

FOR  $i=1$  to  $\log n$  DO:

- IF  $\text{range}(s,i)$  is dense, route with **dense**( $i$ ) routing.
- IF  $\text{range}(s,i)$  is sparse, route with **sparse**( $i$ ) routing.

**Dense**( $i$ ) routing: search in  $O(\text{radius}(\text{range}(s,i)))$  is OK.

**Sparse**( $i$ ) routing: negative answer reported faster (by sparsity stored information for that is small)

# Conclusion

- ▶ Recently, in PODC'06 Konjevod et al. prove that for name-independent routing if stretch  $< 9 - \varepsilon$  then space  $\Omega(n^{\varepsilon/16})$  for some networks of doubling dimension 10.
- ▶ Can be do stretch  $9 + \varepsilon$  and scale-free polylog routing? (best stretch is 64).