

UE: ALGORITHMIQUE APPLIQUÉE - 4TIN903U

(final test – 1h time)

– *Authorised course document* –

Museum under control

The building of a museum is modeled by a graph where the nodes represent the rooms and the edges the galleries, i.e., the corridors between the rooms. For simplicity, we assume the galleries are thin and straight.

The goal is to control all the galleries of the museum by adding fully rotative cameras. It is assumed that the cameras can be placed at the rooms only. By making a 360 degree turn, a camera can control all galleries connected to the room where the camera is located.

For instance, if the museum graph is a square (a cycle of 4 nodes and 4 edges), then two cameras suffice (placed at two non-adjacent nodes).

Question 1. *Give a graph problem whose solution allows us to determine the minimum number of cameras needed to control a museum.*

Question 2. *Give the minimum number of cameras to control a museum which is a complete graph with n rooms?*

A fast cubic algorithm

A computer scientist just designs a nice algorithm for graphs with n nodes. Its theoretical complexity is cubic in the number of edges of the graph. She decides to program it to test its performances on a 1 GHz-clock computer which is able to run 1 billion of instructions per seconde.

Question 3. *How long shall her program run for an input graph having $n = 1\,000$ nodes? (approximately in the worst-case).*

Your answer should be on the form: x "unit(s)" where x is an integer < 10 , and where "unit" is a time unit chosen among: seconde, minute, hour, day, week, month or year.

Circle cover

The consider the following problem:

Input: A set P of n points in the plane, and an integer k .

Output: Is there a set of at most k circles that contains all the points of P .

Note that a point (x, y) that is inside of the inner disk defined by a circle C is not considered as to be "contained" by C . To be contained by C , we mean that the point belongs to C . We are looking for a circle cover, not a disk cover. Also, we assume that arithmetic operations on point coordinates take a constant time.

Question 4. Describe briefly an algorithm for the case $k = 1$. What is its complexity?

Question 5. Same question for $k = 2$.

Question 6. Give a reduction rule showing that this problem, whenever k is a fixed parameter, admits a kernel. What is the size of the kernel? Is the reduction rule polynomial? Argue. (Hint: observe that two different circles intersect in at most two points.)

Question 7. Propose a polynomial time algorithm that either returns a set of $O(k \log k)$ circles containing all the points of P , or returns "no" to say that one need more than k circles to contained all the points of P . (Hint: combine the solution of the previous question with an approximation algorithm slightly modified.)

Tree-decomposition

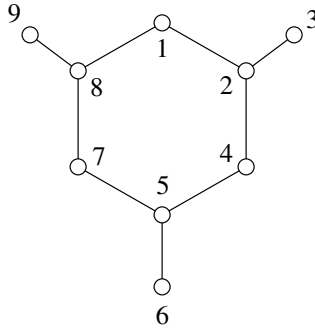
Question 8. Recall the definition of a tree-decomposition T of a graph G .

We consider a $n \times n$ grid graph G , i.e., a grid having n rows of n columns. So G has n^2 nodes in total. We also consider a path Q of $n - 1$ nodes denoted by $X_1 - X_2 - \dots - X_{n-1}$ such that each X_i is composed of all the nodes of the i th and $(i + 1)$ -th rows of G . So $|X_i| = 2n$.

Question 9. Does Q represent a tree-decomposition of G ? explain why?

Question 10. Draw a tree-decomposition of smallest possible width for the following

graph. (Hint: you can forget the degree-1 node, construct a tree-decomposition, and then update it.)



Matching

Question 11. Recall what is a perfect matching.

Let us denote by G_2 the 3×4 grid graph, i.e., the grid composed of 3 rows and 4 columns and having 12 nodes.

Question 12. Draw for G_2 a maximal matching having the minimum size. (You do not need to argue why this is the minimum possible size.)

Question 13. Draw for G_2 a maximum matching, i.e., a matching of maximum size. Argue why this is the maximum.

Question 14. Draw for G_2 a vertex cover of minimum size. Argue that this is the minimum.

Let us denote by G_n the $3 \times 2n$ grid graph. This graph has $6n$ nodes, and G_2 is a particular case of G_n with $n = 2$.

Question 15. Give the construction of a vertex cover of minimum size for G_n for every n , and give its size. Argue why this is the minimum size.

FIN.